20. Let $f_n(x) = x - \frac{x^n}{n}$. Determine the limit function $f$ of $\{f_n\}$ on $[0,1]$ and decide whether $f'_n \to f'$. Does $\int_0^1 f_n(x)dx \to \int_0^1 f(x)dx$ hold?

21. Let $f_n(x) = nx(1-x)^n$. Graph $f_1$, $f_2$, $f_5$, and $f_{11}$ on $[0,1]$. Determine the limit function $f$ of $\{f_n\}$ as $n \to \infty$. Is the limit function continuous? Is $f'_n \to f'$ true? How about the integral? Discuss whether $\{f_n\}$ is uniformly convergent on $[0,1]$.

22. Let $f_n(x) = nx e^{-nx}$. Is $\{f_n\}_{n \in \mathbb{N}}$ uniformly convergent on $(0, \infty)$?

23. Prove that $f_n \to f$ uniformly provided $|f_n(x) - f(x)| \leq \alpha_n$ for all $n \geq N$ and all $x \in E$, where $\{\alpha_n\}$ is assumed to be a sequence that converges to zero.

24. Let $f_n(x) = x^{2n}/(1+x^{2n})$. What is the limit function? Show that the sequence converges uniformly on $[0,q]$ with $q < 1$ and also on $[\alpha, \infty)$ with $\alpha > 1$ but not on $\mathbb{R}$.

25. Let $f_n(x) = nx/(1+n^2x^2)$ for $x \in [0,1]$. Show that the sequence converges uniformly on $[q,1]$ for any $q \in (0,1)$ but not on $[0,1]$.

26. Show that $\sum_{k=0}^\infty x^k(1-x)$ converges on $(-1,1]$, but not uniformly.

27. Let $\alpha > 1$. Show that $\sum_{k=1}^\infty \frac{\sin(kx)}{k^\alpha}$ is uniformly convergent on $\mathbb{R}$.

28. Prove: If $f_n \to f$ uniformly on $E$ and $g \in B(E)$, then $f_ng \to fg$ uniformly on $E$.

29. Prove: If $f_n \to f$ uniformly on $E$ and $|f_n(x)| \geq \alpha > 0$ for all $n \in \mathbb{N}$ and all $x \in E$, then $1/f_n \to 1/f$ uniformly on $E$. 