20. Let $f_n(x) = x - \frac{x^n}{n}$. Determine the limit function f of $\{f_n\}$ on $[0, 1]$ and decide whether $f_n' \to f'$. Does $\int_0^1 f_n(x)\mathrm{d}x \to \int_0^1 f(x)\mathrm{d}x$ hold?
21. Let $f_n(x) = nx(1 - x)^n$. Graph f_1, f_2, f_5, and f_{11} on $[0, 1]$. Determine the limit function f of $\{f_n\}$ as $n \to \infty$. Is the limit function continuous? Is $f_n' \to f'$ true? How about the integral? Discuss whether $\{f_n\}$ is uniformly convergent on $[0, 1]$.
22. Let $f_n(x) = nxe^{-nx}$. Is $\{f_n\}_{n\in\mathbb{N}}$ uniformly convergent on $(0, \infty)$?
23. Prove that $f_n \to f$ uniformly provided $|f_n(x) - f(x)| \le \alpha_n$ for all $n \ge N$ and all $x \in E$, where $\{\alpha_n\}$ is assumed to be a sequence that converges to zero.
24. Let $f_n(x) = x^{2n}/(1+x^{2n})$. What is the limit function? Show that the sequence converges uniformly on $[0, q]$ with $q < 1$ and also on $[\alpha, \infty)$ with $\alpha > 1$ but not on $\mathbb{R}$.
25. Let $f_n(x) = nx/(1 + n^2x^2)$ for $x \in [0, 1]$. Show that the sequence converges uniformly on $[q, 1]$ for any $q \in (0, 1)$ but not on $[0, 1]$.
26. Show that $\sum_{k=0}^{\infty} x^k(1-x)$ converges on $(-1, 1]$, but not uniformly.
27. Let $\alpha > 1$. Show that $\sum_{k=1}^{\infty} \frac{\sin(kx)}{k^\alpha}$ is uniformly convergent on $\mathbb{R}$.
28. Prove: If $f_n \to f$ uniformly on E and $g \in \mathrm{B}(E)$, then $f_n g \to fg$ uniformly on E.
29. Prove: If $f_n \to f$ uniformly on E and $|f_n(x)| \ge \alpha > 0$ for all $n \in \mathbb{N}$ and all $x \in E$, then $1/f_n \to 1/f$ uniformly on E.