56. Find the Fourier coefficients of \( f \) on \([-l, l] \) if \( f \) is
   (a) even;
   (b) odd.

57. Find the Fourier coefficients of \( f \) on \([-\pi, \pi] \) for
   (a) \( f(x) = x \);
   (b) \( f(x) = |x| \);
   (c) \( f(x) = |\sin x| \);
   (d) \( f(x) = x^2 \);
   (e) \( f(x) = \cosh(\alpha x), \alpha \neq 0 \).

58. For the set of real-valued polynomials on \([-1, 1] \), show that \( p \)
defined by \( p(x) = x \) is orthogonal to every constant function.
Next, find a quadratic polynomial that is orthogonal to both \( p \)
and the constant functions. Finally, find a cubic polynomial that
is orthogonal to all quadratic polynomials. Hence construct an
orthonormal set with three vectors.

59. Find \( \sum_{k=1}^{n} \sin(k\theta) \).

60. For \(|a| < 1\), find
   (a) \( \sum_{n=0}^{\infty} a^n \cos(n\theta) \);
   (b) \( \sum_{n=1}^{\infty} a^n \sin(n\theta) \).

61. Let \( e_n(x) = \frac{1}{\sqrt{2\pi}} e^{inx} \), where \( x \in (-\pi, \pi) \). Let \( f \) be continuous and
   2\( \pi \)-periodic on \( \mathbb{R} \). Put \( f_m = \sum_{n=-m}^{m} \langle f, e_n \rangle e_n \) and \( F_m = \frac{1}{m+1} \sum_{k=0}^{m} f_k \).
   (a) Establish the formula \( F_m(y) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) K_m(y-x) dx \), where
       \( K_m(\theta) = \frac{1}{m+1} \sum_{k=0}^{m} \sum_{n=-k}^{k} e^{in\theta} \) is the so-called Fejér kernel.
   (b) Show that \( K_m(\theta) = \frac{1}{m+1} \frac{\sin \frac{(m+1)\theta}{2}}{\sin \frac{\theta}{2}} \) if \( \theta \neq 2\pi n \) for some \( n \in \mathbb{Z} \).
   (c) Prove: \( F_m(y) - f(y) = \frac{1}{2\pi} \int_{y-\pi}^{y+\pi} [f(x) - f(y)] K_m(y-x) dx \).
   (d) Draw the graph of \( K_m \) for \( m \in \{2, 5, 8\} \).