Problems #1, Math 315, Dr. M. Bohner. Jan 10, 2005. Due Jan 19, 2 pm.

1. Are the following functions in $\mathrm{BV}[a,b]$? If so, find their total variations on $[a,b]$.
$$f(x)=\begin{cases}0 & \text{if } x\le 1\\ 1 & \text{if } x>1\end{cases} \quad\text{and}\quad g(x)=\begin{cases}0 & \text{if } x\ne 1\\ 1 & \text{if } x=1.\end{cases}$$
2. Find $\bigvee_0^{3\pi}\sin$.
3. Show that $f\in\mathrm{BV}[0,1]$, where $f$ is defined by
$$f(x)=\begin{cases}x^2\sin(1/x) & \text{if } x\ne 0\\ 0 & \text{if } x=0.\end{cases}$$
4. Show that $f\in\mathrm{C}[0,1]\setminus\mathrm{BV}[0,1]$, where
$$f(x)=\begin{cases}x\sin\frac{\pi}{2x} & \text{if } x\ne 0\\ 0 & \text{if } x=0.\end{cases}$$
Also show that $g$, defined by $g(x)=f(x^2)$, is differentiable but not of bounded variation on $[0,1]$.
5. Prove that Lipschitz functions are of bounded variation on $[a,b]$.
6. Prove that any step function is of bounded variation on $[a,b]$.
7. Prove that if $f,g\in\mathrm{BV}[a,b]$, then $f+g, f-g, fg\in\mathrm{BV}[a,b]$.
8. If $f:[a,b]\to\mathbb{R}$ is a monotonic function, find $v_f$.
9. Find $v_f$ for $f$ defined by $f(x)=\begin{cases}x+1 & \text{if } -1\le x<0\\ x & \text{if } 0\le x<1\\ 1-x & \text{if } 1\le x\le 2.\end{cases}$
10. Find $v_{\sin}:[0,2\pi]\to\mathbb{R}$.
11. Show $\mathrm{BV}[a,b]\subset\mathrm{R}[a,b]$.