12. Show that the Riemann-Stieltjes integral is unique, if it exists.

13. Prove that \( \int f \, dg \) is linear in \( f \) and \( g \).

14. Find \( \int_a^b f \, dg \), where \( f \) is a constant function.

15. Find \( \int_a^b f \, dg \), where \( g \) is a constant function.

16. Find \( \int_a^b f \, dg \), where \( g \) is a step function.

17. Define \( f \) and \( g \) by

\[
  f(x) = \begin{cases} 
    0 & \text{if } 0 \leq x \leq 1 \\
    1 & \text{if } 1 < x \leq 2 
  \end{cases}
\]

and

\[
  g(x) = \begin{cases} 
    0 & \text{if } 0 \leq x < 1 \\
    1 & \text{if } 1 \leq x \leq 2 
  \end{cases}
\]

On which of the intervals \([0, 1], [1, 2], [0, 2]\) is \( f \in \mathcal{R}(g) \)? Evaluate each of the three integrals, if they exist.

18. Calculate each of the following integrals:

\[
\int_0^4 x^2 \, dx, \quad \int_0^\pi x \cos x \, dx, \quad \int_0^1 x^3 \, dx^2, \quad \int_{-1}^2 \sqrt{x + 2} \, dx \\
\int_0^1 x \, dx \tan x, \quad \int_0^\pi e^{\sin x} \, dx, \quad \int_0^3 \sqrt{x} \, d(x \lfloor x \rfloor).
\]

19. For \( n \in \mathbb{N} \) and \( f \in C^1[0, n] \), find \( \int_0^n f \, dx \), and use your result to derive Euler’s summation formula:

\[
\sum_{k=0}^n f(k) = \int_0^n f(x) \, dx + \frac{f(0) + f(n)}{2} + \int_0^n \left(x - \lfloor x \rfloor - \frac{1}{2}\right) f'(x) \, dx.
\]

20. Show that refining a partition decreases its upper sum.