Problems #4, Math 315, Dr. M. Bohner.Feb 4, 2005. Due Feb 11, 2 pm.

33. Suppose that $\{f_n\}$ is a monotonic function sequence that converges pointwise on $[a, b]$ to f. If f and every f_n are continuous on $[a, b]$, show that $\{f_n\}$ converges uniformly on $[a, b]$ to f.

34. Find examples that show that in each of Theorems 2.8, 2.12, and 2.14 the uniform convergence is not a necessary condition for the claims of the theorems to hold.

35. If $\mathcal{F}$ is a family of functions that are Lipschitz continuous with the same Lipschitz constant, show that $\mathcal{F}$ is equicontinuous.

36. If $\mathcal{F}$ is a family of differentiable functions on $[a, b]$ with uniformly bounded derivatives, show that $\mathcal{F}$ is equicontinuous.

37. Suppose $\mathcal{F}$ is a family of uniformly bounded and Riemann integrable functions on $[a, b]$. Show that the family of functions defined by $\int_a^x f(t)\mathrm{d}t$, where $f \in \mathcal{F}$, is equicontinuous.

38. For a function f defined on $[0, 1]$ we introduce the nth Bernstein polynomial as

$$B_n(f; x) = \sum_{k=0}^{n} f\left(\frac{k}{n}\right)\binom{n}{k} x^k (1-x)^{n-k}.$$

Show the following:

(a) $B_n(1; x) \equiv 1$;

(b) $B_n(x; x) = x$;

(c) $B_n(x^2; x) = x^2 + \frac{x(1-x)}{n}$.

39. Show that the polynomials in the "Weierstraß Approximation Theorem" can be chosen as Bernstein polynomials.

40. Find the third Bernstein polynomial for $\sin(\pi x/2)$.

41. Find the fourth Bernstein polynomial for $\sqrt{x}$.