

50. Find the Fourier series of f is 2π -periodic and on $[-\pi, \pi]$,

(a) f is even;

(b) f is odd;

(c) $f(x) = x$;

(d) $f(x) = |x|$;

(e) $f(x) = \cos(x/2)$;

(f) $f(x) = x^2$;

(g) $f(x) = \begin{cases} -1 & \text{if } x \in [-\pi, 0) \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x \in (0, \pi]. \end{cases}$

51. For $|a| < 1$, find

(a) $\sum_{n=0}^{\infty} a^n \cos(n\theta)$;

(b) $\sum_{n=1}^{\infty} a^n \sin(n\theta)$.

52. Work on Problem 12 of Chapter 8 in the textbook.

53. Work on Problem 13 of Chapter 8 in the textbook.

54. Work on Problem 15 of Chapter 8 in the textbook.

55. Show that the Γ function is well defined.

56. Find $\Gamma((n+1)/2)$ and $\Gamma(n/2+1)$ for all $n \in \mathbb{N}$.

57. Express $\int_{-1}^1 (1-t^2)^{(n-1)/2} dt$ in terms of Gamma functions.

58. Work on Problem 30 of Chapter 8 in the textbook.

59. Work on Problem 31 of Chapter 8 in the textbook.