Instructions: Each of the five problems is worth 16 points. Only responses entered in the allocated space for each problem will be graded. Present only the complete solution including all explanation (without scratch work, use the back of the assignment sheet for that purpose) neatly. You must support all of your answers in order to receive credit. Do not remove the staples. Do not turn in the assignment sheet. Grades will be posted on the web this weekend.

1. Let V be a real vector space with inner product $(\cdot,\cdot)$. Prove $\|x+y\|^2+\|x-y\|^2 = 2\left(\|x\|^2+\|y\|^2\right)$.
2. Consider an eigenvalue problem $f''+\lambda f=0$ with symmetric boundary conditions. Show that if $f(b)f'(b)-f(a)f'(a)\le 0$ for all $f:[a,b]\to\mathbb{R}$ satisfying the boundary conditions, then there is no negative eigenvalue.
3. Let $\phi(x)=-1-x$ if $x\in[-1,0)$ and $\phi(x)=1-x$ if $x\in(0,1]$ and $\phi(0)=0$.
   (a) Find the full Fourier series of ϕ in the interval $(-1,1)$.
   (b) Does the Fourier series converge in the mean square sense, pointwise, or uniformly? Give short (one line) explanations for each of your claims.
4. Solve $\Delta u=0$, $0<x<a$, $0<y<b$, $u(x,0)=0$, $u(x,b)=g(x)$, $u(0,y)=0$, $u(a,y)=0$.
5. Check whether $u(x,y)=x^3-3x^2y$ is harmonic in $\mathbb{R}^2$.