Instructions: Each of the ten problems is worth 21 points. Only responses entered in the allocated space for each problem will be graded. Present only the complete solution including all explanation (without scratch work, use the back of the assignment sheet for that purpose) neatly. You must support all of your answers in order to receive credit. Do not remove the staples. Do not turn in the assignment sheet. Grades will be posted on the web this afternoon.

93. Find the general solution of $2u_x - \frac{u_t}{2} = 0$.

94. Find the solution of $xu_x + tu_t = 0$ that satisfies $u(x,1) = \sin(\cos(x))$.

95. "Factor the operator" to find the solution of $u_{xx} - u_{xt} - 2u_{tt} = 0$, $u(x,0) = x^2$, $u_t(x,0) = 0$.

96. Solve the diffusion equation (on the whole line) with initial condition $u(x,0) = 1$.

97. Consider a metal rod $(0 < x < l)$, insulated along its sides but not at its ends, which is initially at temperature two everywhere. Suddenly both ends are plunged into a bath of temperature zero. Write the differential equation, boundary conditions, and initial conditions. Write the formula for the temperature $u(x,t)$ at later times. In this problem, you can use the infinite series expansion $\sum_{n=1}^{\infty} \frac{1}{2n-1} \sin \frac{(2n-1)\pi x}{l} = \frac{\pi}{4}$.

98. Find all eigenvalues and eigenfunctions of $f'' + \lambda f = 0$, $f(0) = 0$, $f'(a) = 0$.

99. Solve $u_{tt} = c^2 u_{xx}$ $(0 < x < l,\ t > 0)$, $u(0,t) = u(l,t) = 0$, $u(x,0) = x$, $u_t(x,0) = 0$.

100. Let $\phi(x) = -1$ if $x \in [-1,0)$ and $\phi(x) = 1$ if $x \in (0,1]$ and $\phi(0) = 0$. Find the full Fourier series of ϕ in the interval $(-1,1)$. Say (without explanation) whether the Fourier series converges in the mean square sense, pointwise, or uniformly.

101. Solve $\Delta u = 0$ $(0 < x < a,\ 0 < y < b)$, $u(0,y) = 0$, $u(a,y) = 0$, $u_y(x,0) = 0$, $u(x,b) = h(x)$.

102. Use centered differences to approximate the harmonic function in $S = \{(x,y) :\ 0 < x < 1,\ 0 < y < 1\}$ that satisfies $u(x,0) = 9x(1-x)$ for $0 \le x \le 1$ and vanishes at all other points of the boundary of S (use step size $\frac{1}{3}$ for both x and y).