54. Use Fourier series to solve the following boundary value problems:

(a) \( u_t = u_{xx} \) (\( 0 < x < 1, \ t > 0 \)), \( u(0,t) = u(1,t) = 0, u(x,0) = x \).

(b) \( u_{tt} = u_{xx} \) (\( 0 < x < \pi, \ t > 0 \)), \( u(0,t) = u(\pi,t) = 0, u(x,0) = 0, u_t(x,0) = x^2(\pi - x)^2 \).

(c) \( u_t = u_{xx} \) (\( 0 < x < \pi, \ t > 0 \)), \( u(0,t) = u(\pi,t) = 0, u(x,0) = x(\pi - x) \).

(d) \( u_t = u_{xx} \), (\( 0 < x < \pi, \ t > 0 \)), \( u_x(0,t) = u_x(\pi,t) = 0, u(x,0) = \cos^4 x \).

(e) \( 9u_{tt} = u_{xx} \) (\( 0 < x < \pi, \ t > 0 \)), \( u(0,t) = u(\pi,t) = 0, u(x,0) = 0, u_t(x,0) = x(x - \pi) \).

55. Find the Fourier sine series in \((0, \pi)\) of \( f(x) = \cos x \).

56. Find the Fourier cosine series in \((0, \pi)\) of \( f(x) = \cos^3 x \).

57. Find the Fourier coefficients of \( f \) on \([-l, l]\) if \( f \) is

(a) even;

(b) odd.

58. Find the Fourier coefficients of \( f \) on \([-\pi, \pi]\) for

(a) \( f(x) = x \);

(b) \( f(x) = |x| \);

(c) \( f(x) = |\sin x| \);

(d) \( f(x) = x^2 \);

(e) \( f(x) = \cosh(\alpha x), \ \alpha \neq 0 \);

(f) \( f(x) = -3 \) if \(-\pi \leq x < 0, \ f(x) = 0 \) if \( x = 0 \), and \( f(x) = 1 \) if \( 0 < x \leq \pi \).

59. Use the previous problem to find the following infinite series:

(a) \( 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \ldots \);

(b) \( 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \frac{1}{9^2} + \ldots \);

(c) \( \frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \ldots \);

(d) \( 1 - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \frac{1}{9^2} - \frac{1}{11^2} + \ldots \);

(e) \( \frac{1}{\alpha} + \sum_{n=1}^{\infty} \frac{2\alpha}{\alpha^2+n^2} \).

60. Use the previous problem to determine the value of \( \sum_{n=1}^{\infty} \frac{1}{n^2} \).

61. Find the complex form of the Fourier series of \( f(x) = e^x \).