1. Find the solution (for positive time $t$) of

$$4xu_x + 2tu_t = xt, \qquad u(x,1) = \phi(x).$$

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2. Transform the equation

$$u_{xx} + 2u_{xt} + u_{tt} = 2u$$

into standard form. Solve the obtained standard PDE. Then use your transformation to obtain the solution of the original PDE.

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3. Suppose $\phi$ and $\psi$ are even functions and $u$ is the solution (with $c > 0$) of

$$u_{tt} = c^2 u_{xx}, \quad u(x,0) = \phi(x), \quad u_t(x,0) = \psi(x).$$

Show that $u$ is an even function in $x$ for all times $t$.

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4. Suppose $u$ solves the equation (with a given function $h$ and $c > 0$)

$$u_{tt} + 2cu_{xt} + c^2 u_{xx} = h(x - ct).$$

Introduce $v = u_t + cu_x$ and calculate $v_t + cv_x$ to obtain a PDE of first order for $v$. Solve this PDE using the geometric method. Thus obtain a PDE of first order for $u$. Solve this PDE using the geometric method. Finally, present the solution of the initial value problem

$$u_{tt} + 2cu_{xt} + c^2 u_{xx} = h(x - ct), \quad u(x,0) = \phi(x), \quad u_t(x,0) = \psi(x).$$

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