1. Find the solution of the problem

\[ u_t - ku_{xx} + btu = 0, \quad u(x, 0) = \phi(x), \]

where \( b, k > 0 \). (Hint: Problems 37 and 38.)

2. Prove that the total energy for the wave equation

\[ E(t) = \frac{1}{2} \int_0^d \left\{ \frac{1}{c^2} u_x^2(x,t) + u_x^2(x,t) \right\} dx \]

is conserved when having Neumann boundary conditions.

3. Find the Fourier series of

\[ f(x) = \begin{cases} 
-3 & \text{if } -\pi \leq x < 0 \\
0 & \text{if } x = 0 \\
1 & \text{if } 0 < x \leq \pi. 
\end{cases} \]

Does the Fourier series of \( f \) converge pointwise to \( f \) in \((-\pi, \pi)\)?

4. Let \( c > 0 \) and \( N \in \mathbb{N} \). Consider the discrete problem

\[ \Delta_m u(n+1, m) = k\Delta_{nn} u(n, m), \quad u(0, m) = u(N, m) = 0 \]

(with \( \Delta_m u(n, m) = u(n, m+1) - u(n, m) \), \( \Delta_n u(n, m) = u(n+1, m) - u(n, m) \) etc.) and find solutions by separating the variables \( n \in \{0, 1, \ldots, N\} \) and \( m \in \mathbb{N}_0 \). (Hint: Problem 46.)