1. Find $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \ldots$ by first computing the Fourier series of

$$f(x) = \begin{cases} -1 & \text{if } -\pi \leq x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } 0 < x \leq \pi. \end{cases}$$

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2. Prove: If both ϕ and ψ are periodic functions with the same period p, then the solution of the wave equation $u_{tt} = c^2 u_{xx}$ (on the whole real line) together with $u(x,0) = \phi(x)$ and $u_t(x,0) = \psi(x)$ is also periodic in x with period p.

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3. Here we consider the problem

$$u_t = k u_{xx} \ \ (0 \leq x \leq 1, \ t \geq 0), \quad u(0,t) + u(1,t) = u_x(0,t) + u_x(1,t) = 0.$$

First perfom separation of variables and find all eigenvalues of the resulting eigenvalue problem. Are it's boundary conditions symmetric? Next, solve the above problem with each of the following initial data:

- $u(x,0) = \cos(\pi x)$;
- $u(x,0) = 4\cos(3\pi x) - 8\sin(9\pi x)$;
- $u(x,0) = x^2 - x$.
