24. Transform the equation $u_{xx} + 2u_{xt} + u_{tt} = 2u$ into standard form. Solve the obtained standard PDE. Then use the transformation to obtain the solution of the original PDE.

25. Transform the following PDEs into standard form. For each step, write down exactly the transformation that is needed. Also, determine whether the equation is hyperbolic, elliptic, or parabolic.

    (a) $3u_{xx} + 4u_{tt} - u = 0$;

    (b) $4u_{xx} + u_{xt} + 4u_{tt} + u = 0$;

    (c) $u_{xx} + u_{tt} + 3u_x - 4u_t + 25u = 0$;

    (d) $u_{xx} - 3u_{xt} + u_{tt} + 2u = 0$;

    (e) $u_{xx} + 2u_{xt} + u_{tt} + u_x - 2u_t + u = 0$.

26. Determine α so that a rotation with α degrees transforms $u_{xt} = 0$ into $u_{xx} - u_{tt} = 0$. Find the general solution of the first PDE. Use this together with the transformation to find the general solution of the second PDE.

27. Determine a transformation that transforms $u_{xx} - u_{tt} = 0$ into $u_{tt} = du_{xx}$, where $d > 0$. Use the general solution of the first PDE (from the previous problem) together with the transformation to find the general solution of the second PDE.

28. Use transformations (and the previous problem) to obtain the solution of $3u_{xx} + 10u_{xt} + 3u_{tt} = 0$.

29. Find all second-order linear homogeneous PDEs in x and t that are left unchanged by any rotation of axes.

30. Show that none of the three transformations introduced in Example 1.7 (i.e., rotation of axes, change of dependent variable, and change of scale) changes the type of the PDE when applied to a second order equation with constant coefficients.