## Outline of Math 3304: Nagle, Saff, and Snider

A list of sections and topics to cover – and representative online homework problems for the topics

The homework problems corresponding to a topic are listed in parentheses after the topic

## Nagle, Saff, and Snider sections

<u>Chapter 1 (Introduction)</u>

1.1 Background: basic modeling, solve, definitions

- Basic example of deriving a differential equation to model something (14, 15, 17)
- Solve via separation of variables
- Definitions and classifications (1, 3, 5, 8, 10)

1.2 What is a solution?

- Definition of explicit and implicit solutions
- Show that a function is an explicit solution of an ODE (3, 21)
- [Note: 1.2 in Nagle contains nonlinear existence/uniqueness theory, but this is not required]

1.3 Direction fields

- Create basic direction fields: rough sketch by hand, or create using software (5)
- Interpret a given direction field (3, 5)

<u>Chapter 2 (First Order Equations)</u>

2.2 Separable equations

- Solve (8, 15, 25, 26)
- [Note: Here and below, “solve” indicates find general solutions and/or solve initial value problems]

2.3 Linear equations: integrating factors and theory

- Solve using integrating factors (9, 13, 15, 17)
- Existence and uniqueness of solutions

<u>Chapter 3 (First Order Equations – Mathematical Models)</u>

3.2 Compartment analysis: mixing and population models

- Mixing models (1, 4)
- Population models (14, 19)
- [Note: Other modeling is covered in the homework (25), but I will not cover this in class]

<u>Chapter 4 (Second Order Equations)</u>

4.2 Homogeneous case: real distinct and repeated roots, theory

- Solve (4, 7, 11, 17)
- Existence and uniqueness of solutions [Note: Homework problems come later, in Section 4.7]
- Linear independence and the Wronskian (28)

[Note: Reduction of order is covered in 4.7; only the solution procedure for the repeated roots case is give here]

4.3 Homogeneous case: complex roots
Solve (4, 12, 22, 23)

4.4, 4.5 Nonhomogeneous case: MUC and theory
Find the particular solution (4.4: 10, 13, 15, 21)
Find the general solution/solve initial value problems (4.5: 18, 25)
Existence and uniqueness of solutions; superposition [Note: Homework problems in Section 4.7]

4.6 Nonhomogeneous case: VofP
Solve: constant coefficient problems (2, 4, 18)
Solve: variable coefficient problems (23)

4.7 Variable coefficient equations [Cauchy-Euler equations, theory] and reduction of order
Solve: homogeneous case (10, 12, 20)
Solve: nonhomogeneous case (37)
Existence and uniqueness of solutions (1, 8)
Reduction of order (43)

4.1, 4.9 Mass-spring systems: modeling and unforced vibrations
Derive and discuss the model
[Note: In Nagle, the mass-spring system is set up horizontally and to the right is the positive direction]
Write down the model given information about the mass-spring system (7, 9)
Solve and find information about the solution (7, 9)

4.10 Mass-spring systems: forced vibrations
Write down the model given information about the mass-spring system (9)
Solve and find the steady state solution [this is the particular solution when the system is damped] (9)
[Note: In Nagle, a resonance frequency is defined for both the undamped and damped cases]

## Chapter 6 (Higher Order Equations)

6.1 Theory
Existence and uniqueness of solutions (4)
Linear independence and the Wronskian (8, 14)

6.2 Homogeneous case: constant coefficients
Solve (1, 4, 13)
[Note: Calculators are not allowed on exams, and so factoring relatively simple polynomials is necessary]

6.3 Nonhomogeneous case: MUC
Solve (6, 9)
[Note: 6.3 also covers the annihilator method, but this is not required]

## Chapter 7 (The Laplace Transform)

7.2 Definition, linearity, theory
Find the Laplace transform of a function using the definition (4, 12)
Linearity and basic theory

Find the Laplace transform of basic functions using a Laplace transform table (13, 17)
[Note: Please give the students the Laplace transform table that we will include on the exams]

7.3 Properties: translation, Laplace transform of derivatives, derivatives of the Laplace transform
Use a table and properties to find Laplace transforms of functions (5, 8, 10)
[Note: We did not cover the transform of t^n f(t) in the past, but we will require it now]

7.4 Inverse transform
Definition and linearity: find the inverse transform of basic functions (4, 6, 7, 8)
Use the partial fraction decomposition to find the inverse transform of functions (21, 23, 25, 26)
[Note: I have not avoided any type of partial fraction decomposition in the homework]

7.5 Solving IVPs
Solve initial value problems with constant coefficients (6, 8)
[Note: Variable coefficient problems are also covered here, but this is not required – although it would be fun]

7.6 Transforms of discontinuous functions and solving IVPs
Express discontinuous functions in terms of the unit step and rectangular window functions (5, 9)
Find the Laplace transform of discontinuous functions using the translation property (5, 9)
Find inverse Laplace transforms using the translation property (14, 15)
Solve IVPs with discontinuous forcing (20, 22, 28)
[Note: Different notation here for the unit step function: u(t-a)]

7.8 Convolution
Use the convolution theorem to find the inverse Laplace transform of functions (6)
Solve integral and integro-differential equations (18, 21, 22)

7.9 Dirac delta function
Rough definition and integral definition: evaluate integrals involving the Delta function (2)
Laplace transform of the Dirac delta function and other generalized functions (10)
Solve IVPs with impulsive forcing (14, 20)

<u>Chapter 9 (Linear Systems of Differential Equations)</u>

[Note: Nagle does not avoid systems with 3 x 3 matrices, so generally we will not avoid these in the homework – however, I will avoid homework that requires inverting 3 x 3 matrices]

9.3 Review of matrices and vectors
Computations: matrix addition, scalar multiplication, matrix multiplication (2, 8)
Compute the inverse of a 2 x 2 matrix (10)
Compute derivatives and integrals of matrices with function entries (31, 39)

5.1 / 9.1 Modeling (two fluid tank mixing) and introduction
Derive a DE model for a two fluid tank mixing problem [Note: no online homework for this]
Express a system of differential equations in matrix vector notation (1, 4, 5, 11)
Express a higher order scalar differential equation in matrix vector notation (8)
[*Note*: 9.1 uses matrices and vectors – therefore, it seems useful to do the review in 9.3 first]

9.2 Linear systems of algebraic equations
Find all solutions (1, 6, 10)

[*Note*: 9.2 does Gaussian elimination without matrices – I will use matrices, but proceed as you like]

9.4 Theory

Express linear differential equations in normal form x’ = A x + f (4, 8)
Express a DE in normal form as a system of coupled scalar differential equations (12)
Existence, uniqueness, and representation of solutions of an IVP in normal form
Determine if a set of vector functions is linearly independent on an interval (13, 14)
Determine if a set of vector functions is a fundamental set of solutions for x’ = A x (22)
Given solutions of x’ = A x, form a fundamental matrix (22)

9.5 Solve homogeneous problem: real eigenvalues

Solve: real eigenvalues, distinct or repeated, with “enough” linearly independent eigenvectors (11, 14, 31)
Solve t x’ = A x (43) [Note: they are told to follow a procedure in another problem; I will not cover this in class]
[Note: The case of real repeated eigenvalues with not “enough” linearly independent eigenvectors is in 9.8]

9.6 Solve homogeneous problem: complex eigenvalues

Solve (1, 6, 14)
Find a fundamental matrix [covered earlier in 9.4] (6)

9.7 Solve nonhomogeneous problem: MUC and VofP

Solve via MUC (2, 5)
Solve via VofP (12, 15)

[*] 9.8 The matrix exponential and real repeated eigenvalues

Find the matrix exponential using the Cayley-Hamilton theorem (2, 6)
Solve x’ = A x using generalized eigenvectors [this completes the real eigenvalue case started in 9.5] (17)
[*Note: I suspect we may run out of time and we will not require this – we will see]