Missouri University of Science and Technology — Founded 1870 | Rolla, Missouri

Section 1.2

Solutions and Initial Value Problems

Solutions of Ordinary Differential Equations

A function $y(t)$ is an explicit solution of an n^{th}-order ODE if there exists an interval such that

1) $y(t)$ has n derivatives for all t in the interval

and

2) $y(t)$ satisfies the ODE for all t in the interval.

Example 1

Verify that

$$ty' - y = t^2$$

has an explicit solution

$$y(t) = 3t + t^2$$

Example 2

Verify that

$$y(x) = \frac{1}{x}$$

is an explicit solution of

$$xy' + y = 0$$

Implicit Solutions

A relation $G(x, y) = 0$ is said to be an implicit solution of an n^{th}-order ODE if it defines one or more explicit solutions of the ODE.

Example 3

Show that

$$x^2 + y^2 = 4$$

defines an implicit solution of the differential equation

$$\frac{dy}{dx} = -\frac{x}{y}$$

Initial Value Problems

An initial value problem combines an n^{th}-order ODE with the n initial conditions

$$y(t_0) = y_0$$
$$y'(t_0) = y_1$$
$$y''(t_0) = y_2$$
$$\vdots$$
$$y^{(n-1)}(t_0) = y_{n-1}$$

A solution to the IVP must satisfy the ODE along with all n initial conditions.

Example 4

Verify that $y = (x-2)^3$ is a solution of the initial value problem

$$\frac{dy}{dx} = 3y^{2/3}\ ,\ y(2) = 0$$