

Direction Fields

Consider an ODE of the form

$$\frac{dy}{dx} = f(x, y)$$

This equation tells us that the rate of change (slope) of the unknown function y at the point (x, y) is f(x, y).

f(x, y) is often called a slope function or rate function.

We can generate a direction field (or slope field) by plotting the slope f(x,y) at a selection of points (x,y).

Example 1

Sketch a direction field for the differential equation

$$\frac{dy}{dt} = y^2 - 1$$

Then, sketch the solution curve with initial value

$$y(0) = -\frac{1}{4}$$

| Slope function $f = y^2 - 1$ |
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Sketch a direction field for the differential equation $\frac{dy}{dt} = y^2 - t$ with initial condition y(0) = -1