8. The *resolvent* $\lambda \mapsto r_\lambda := (\lambda 1 - x)^{-1}$ is defined on the resolvent set of x (in a unital Banach algebra). Suppose $\lambda_0 \in \rho(x)$ with $|\lambda - \lambda_0| < 1/\|r_{\lambda_0}\|$ and show:
   (a) $\lambda \in \rho(x)$;
   (b) $r_\lambda = \sum_{n=0}^\infty (-1)^n (\lambda - \lambda_0)^n r_{\lambda_0}^{n+1}$;
   (c) $\|r_\lambda - r_{\lambda_0}\| \le \frac{|\lambda-\lambda_0| \|r_{\lambda_0}\|^2}{1-|\lambda-\lambda_0|\|r_{\lambda_0}\|}$.
9. An x (in a complex unital Banach algebra) is called *quasi-nilpotent* if $r(x) = 0$. Prove:
   (a) x is quasi-nilpotent iff $\sigma(x) = \{0\}$;
   (b) If x is not quasi-nilpotent, then there exists an angle α with $\lim_{h\to 0+} \frac{r(x+h1)-r(x)}{h} = \cos\alpha$, and at least one of the points $r(x)\mathrm{e}^{\mathrm{i}\alpha}$, $r(x)\mathrm{e}^{-\mathrm{i}\alpha}$ is in $\sigma(x)$, such that, moreover, this point is the closest to $r(x)$ on the circle around 0 with radius $r(x)$.