8. The resolvent \( \lambda \mapsto r_\lambda := (\lambda 1 - x)^{-1} \) is defined on the resolvent set of \( x \) (in a unital Banach algebra). Suppose \( \lambda_0 \in \rho(x) \) with 
\( |\lambda - \lambda_0| < 1/\|r_{\lambda_0}\| \) and show:
(a) \( \lambda \in \rho(x) \);
(b) \( r_\lambda = \sum_{n=0}^{\infty} (-1)^n (\lambda - \lambda_0)^n r_{\lambda_0}^{n+1} \);
(c) \( \|r_\lambda - r_{\lambda_0}\| \leq \frac{\|\lambda - \lambda_0\| \|r_{\lambda_0}\|^2}{1 - \|\lambda - \lambda_0\| \|r_{\lambda_0}\|} \).

9. An \( x \) (in a complex unital Banach algebra) is called quasi-nilpotent if \( r(x) = 0 \). Prove:
(a) \( x \) is quasi-nilpotent iff \( \sigma(x) = \{0\} \);
(b) If \( x \) is not quasi-nilpotent, then there exists an angle \( \alpha \) with 
\( \lim_{h \to 0^+} \frac{r(x+\frac{h}{2}) - r(x)}{\frac{h}{2}} = \cos \alpha \), and at least one of the points 
\( r(x)e^{i\alpha}, r(x)e^{-i\alpha} \) is in \( \sigma(x) \), such that, moreover, this point is the closest to \( r(x) \) on the circle around 0 with radius \( r(x) \).