1. Let A, B, $C \subset X$ be sets. Prove the following:
   (a) $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$;
   (b) $A \subset B \iff A^c \supset B^c$;
   (c) $A \cup B = A \iff B \subset A$;
   (d) $(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$.
2. Let $f : X \to Y$ be a function. Assume $A, A_1, A_2 \subset X$ and $B, B_1, B_2 \subset Y$. Compare each of the following two sets (i.e., put either an "=" or "$\subset$" or "$\supset$" in between, whichever "is the best") and prove your claim.
   (a) $f(A_1 \cup A_2)$ and $f(A_1) \cup f(A_2)$;
   (b) $f(A_1 \cap A_2)$ and $f(A_1) \cap f(A_2)$;
   (c) $f^{-1}(B_1 \cup B_2)$ and $f^{-1}(B_1) \cup f^{-1}(B_2)$;
   (d) $f^{-1}(B_1 \cap B_2)$ and $f^{-1}(B_1) \cap f^{-1}(B_2)$;
   (e) $f^{-1}(f(A))$ and A;
   (f) $f(f^{-1}(B))$ and B.
3. Let X, Y be sets and $f : X \to Y$. Prove that the following are equivalent:
   (a) f is one-to-one on X;
   (b) $f(A \setminus B) = f(A) \setminus f(B)$ for all $A, B \subset X$;
   (c) $f^{-1}(f(E)) = E$ for all $E \subset X$;
   (d) $f(A \cap B) = f(A) \cap f(B)$ for all $A, B \subset X$.