26. Consider an American put with expiration time 2 and strike price 5 in the BAPM with \( N = 2, \tilde{p} = \tilde{q} = 1/2, r = 1/4, u = 2, d = 1/2, S_0 = 4 \). Let \( Y_k \) be the maximum of zero and the payoff if the put is exercised at \( k \). Let \( X \) be the discounted \( Y \) process.

(a) Find the Snell envelope \( Z \) of \( X \).
(b) Find the process \( V \) that satisfies \( Z = \beta V \).
(c) Find the optimal stopping time \( \tau^* \) and \( \bar{E}(X_{\tau^*}) \).
(d) Verify that \( Z \) is a supermartingale (but not a martingale) and \( Z_{\tau^*} \) is a martingale.

27. Consider the BAPM with \( N = 3, \tilde{p} = \tilde{q} = 1/2, r = 1/4, u = 2, d = 1/2, S_0 = 4 \) and let \( K = 4 \).

(a) Find \( V_0 \) for a European put with expiration time \( N \) and strike price \( K \). Denote this value by \( V_{0\text{EP}} \).
(b) Find \( V_0 \) for a European call with expiration time \( N \) and strike price \( K \). Denote this value by \( V_{0\text{EC}} \).
(c) Find \( V_0 \) for an American put with expiration time \( N \) and strike price \( K \). Denote this value by \( V_{0\text{AP}} \). Also find the optimal stopping time.
(d) Find \( V_0 \) for an American call with expiration time \( N \) and strike price \( K \). Denote this value by \( V_{0\text{AC}} \). Also find the optimal stopping time.
(e) What is the relation between \( V_{0\text{AC}} \) and \( V_{0\text{EC}} \)?
(f) What is the relation between \( V_{0\text{AP}} \) and \( V_{0\text{EP}} \)?
(g) Verify that \( S_0 - K \leq V_{0\text{AC}} - V_{0\text{AP}} \leq S_0 - K \beta_3 \) holds.

28. Consider the American put from the previous problem.

(a) Find the entire value process of the American put.
(b) Find the Doob decomposition of the discounted value process.
(c) Find the largest optimal stopping time.
(d) Find the process \( C \) from Definition 4.37.
(e) To hedge the American put, find the initial wealth and the hedging portfolio process.
(f) Verify that the wealth process resulting in (e) is the same as the value process.

29. Price an American down-and-out call with barrier level 4 and strike price 3 (usual BAPM with \( N = 3 \)).

30. Price an American down-and-in call with barrier level 4 and strike price 3 (usual BAPM with \( N = 3 \)).