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**Math 5737/Econ 5337
Financial Mathematics**

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1

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Literature

- Options, Futures, and other Derivatives, John C. Hull, Prentice Hall, 2022 (11th edition)
- Mathematics of Finance, Marek Capiński and Tomasz Zastawniak, Springer, 2011 (2nd edition)
- Risk-Neutral Valuation, Nick Bingham and Rüdiger Kiesel, Springer, 2004 (2nd edition)




2

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Chapter 1

Introduction




3

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Definition 1.1

A **derivative** (**derivative security, contingent claim**) is a financial instrument whose value depends on (derives from, is contingent on) the values of other, more basic, underlying variables (**the underlying**).




4

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Example 1.2

- Forwards
- Futures
- Swaps
- Options




5

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Remark 1.3

Market forms for derivatives are

- Exchange-traded markets, e.g., NYSE 1792, CBOT 1848, CME 1919, NASDAQ 1971, CBOE 1973
 - open outcry system
 - electronic
- Over-the-counter markets



6


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Definition 1.4

There are three types of traders:

- **Hedgers** attempt to reduce exposure to risk a company already faces
- **Speculators** invest available funds opportunistically in the hope of making a profit
- **Arbitrageurs** try to lock in riskless profit

arbitrage: risk-free profit with no initial investment
no-arbitrage principle: arbitrage opportunities are absent




7

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Definition 1.5

- A **forward contract** is an agreement to sell or buy an asset at a fixed date in the future (**delivery time**) for a price specified in advance (**forward price, delivery price**).
- The party selling the asset assumes what is termed a **short (forward) position**, while the party buying the asset enters into a **long (forward) position**.




8

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Definition 1.5 (continued)

- It costs nothing to enter a forward contract. A forward contract can be contrasted to a **spot contract**, which is an agreement to buy or sell an asset today.




9

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Example 1.6 (a)

Suppose that

- The spot price of $\frac{1}{4}$ oz gold is \$300
- The 1-year forward price of $\frac{1}{4}$ oz gold is \$340
- The 1-year interest rate is 5% p.a.




10

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Example 1.6 (b)

Suppose that

- The spot price of $\frac{1}{4}$ oz gold is \$300
- The 1-year forward price of $\frac{1}{4}$ oz gold is \$310
- The 1-year interest rate is 5% p.a.




11

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Example 1.7

Foreign exchange quote for GBP in USD
on August 14, 2023

	buy	sell
spot	1.26527	1.26536
1-month forward	1.26549	1.26560
3-month forward	1.26557	1.26573
6-month forward	1.26558	1.26575



12

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Example 1.7 (continued)

(a) Say it is August 14, 2023, and Import Co, a company based in the US, knows that it will have to pay GBP 10 million on November 14, 2023, for goods purchased from a British supplier.

(b) Export Co exports goods to UK and knows it will receive GBP 10 million in three months.




13

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Definition 1.8

- A **European call option** is a contract giving the holder the right (but no obligation) to buy an asset (**the underlying**) for a price fixed in advance (**exercise price, strike price**) at a specified future time (**exercise time, expiry time, maturity**).
- A **European put option** is a contract giving the right to sell an asset for a certain strike price at a certain exercise time.




14

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Definition 1.8 (continued)

- An **American put or call option** can be exercised any time up to and including the **expiry time**.
- Since payoffs are nonnegative, a premium (the market price of the option) must be paid when acquiring an option.




15

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Example 1.9

Suppose an investor owns 1000 Microsoft shares on August 14, 2023, \$321 per share is the current price. Suppose the investor is concerned about a possible share price decline in the next 2 months and wants protection. The investor could buy 10 put option contracts with strike price \$320 and expiration time October 20, 2023. Assume each option price is \$12. Ignore the time value of money.




16

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Example 1.10

On August 14, 2023, European calls on Microsoft stock with $K=320$ USD and $T=$ October 20, 2023 were traded at 15.93 USD at NASDAQ. Ignoring the time value of money, when will the investment bring profit?



17