Chapter 9

Properties of Stock Options

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Proposition 9.1

If we let c, p, C, P be the prices of a European call, European put, American call, American put, respectively (all on the same stock, same K and same T), then

$0 \le c \le C$ and $0 \le p \le P$.

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Theorem 9.2

For a stock paying no dividends,

$(S_0 - Ke^{-rT})^+ \le c < S_0$.

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Example 9.3

For a European call on a non-dividend paying stock with $S_0=51$, $K=50$, $T=1/2$, $r=0.12$, find the bounds on c.

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Theorem 9.4

For a stock paying no dividends,

$(Ke^{-rT} - S_0)^+ \le p < Ke^{-rT}$.

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Example 9.5

For a European put on a non-dividend paying stock with $S_0=38$, $K=40$, $T=1/4$, $r=0.1$, find the bounds on p.

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Remark 9.6

- Theorems 9.2 and 9.4 can be graphically summarized
- Note also that by Corollary 5.12 (first part), $S_0-Ke^{-rT}=V(0)$ is the value of a long forward contract on the non-dividend paying stock

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Theorem 9.7

Put-call parity holds:

$c-p=S_0-Ke^{-rT}$.

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Example 9.8

Suppose a stock paying no dividends trades at $15.60 a share. European calls on the stock with strike price $15 and exercise date in 3 months are trading at $2.83, r=6.72% (cc). What is the price of a European put on the same stock with the same strike price and the same exercise date?

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Theorem 9.9

For a stock paying no dividends,

$C=c$.

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Corollary 9.10

For a stock paying no dividends,

$(S_0-Ke^{-rT})^+\leq C<S_0$.

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Example 9.11

Suppose S=10, strike price of American put expiring in one year is $80, and r=0.16. Here it is better to do early exercise.

Theorem 9.12

For a stock paying no dividends,

$(K-S_0)^+ \le P < K$.

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Theorem 9.13

Put-call parity estimates hold:

$S_0-K \le C-P \le S_0-Ke^{-rT}$

(for a stock paying no dividends).

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Example 9.14

Suppose $S_0=19$, $K=20$, $T=5/12$, $r=0.1$, $C=1.50$. Find the bounds on P.

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Theorem 9.15

For a stock paying dividends with present value D,

- $(S_0-D-Ke^{-rT})^+ \le c < S_0-D$
- $(Ke^{-rT}+D-S_0)^+ \le p < Ke^{-rT}$
- $c-p=S_0-D-Ke^{-rT}$

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Theorem 9.15 (continued)

- $S_0-D-K \le C-P \le S_0-Ke^{-rT}$
- $\max\{0,S_0-D-Ke^{-rT},S_0-K\} \le C < S_0$
- $\max\{0,Ke^{-rT}+D-S_0,K-S_0\} \le P < K$

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Remark 9.16

In the rest of this chapter, we discuss the dependence of c, p, C, P on K, S, T, and r (varying only one variable while keeping the others constant; all on stocks paying no dividends).

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Proposition 9.17

c is nonincreasing in K while p is nondecreasing in K, i.e.,

$c(K) \geq c(K')$ and $p(K) \leq p(K')$

for

$K \leq K'$.

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Theorem 9.18

c and p are Lipschitz continuous in K, namely,

$|c(K)-c(K')| \leq e^{-rT}|K-K'|$

and

$|p(K)-p(K')| \leq e^{-rT}|K-K'|$.

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Theorem 9.19

c and p are convex in K, i.e.,

$K \leq K'$ and $0<\alpha<1$

implies

$c(\alpha K+(1-\alpha)K') \leq \alpha c(K)+(1-\alpha)c(K')$

and

$p(\alpha K+(1-\alpha)K') \leq \alpha p(K)+(1-\alpha)p(K')$.

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Theorem 9.20

If $S \leq S'$, then

- $c(S) \leq c(S')$ and $p(S) \geq p(S')$
- $c(S')-c(S) \leq S'-S$
- $p(S)-p(S') \leq S'-S$
- c and p are convex in S.

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Theorem 9.21

For American options, Proposition 9.17 holds, Theorem 9.18 (with T=0 there) holds, and Theorems 9.19 and 9.20 hold. Finally, if $T \leq T'$, then

$C(T) \leq C(T')$ and $P(T) \leq P(T')$.

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Remark 9.22

Summary of the effect on the stock option price when the parameter is increased while the other ones are kept constant:

	EC	EP	AC	AP
K	-	+	-	+
S	+	-	+	-
T	?	?	+	+
r	+	-	+	-
D	-	+	-	+
σ	+	+	+	+

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