Math 5737/Econ 5337 Financial Mathematics

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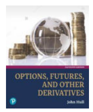

1

Literature

- Options, Futures, and other Derivatives, John C. Hull, Prentice Hall, 2022 (11th edition)
- Mathematics of Finance, Marek Capiński and Tomasz Zastawniak, Springer, 2011 (2nd edition)
- Risk-Neutral Valuation, Nick Bingham and Rüdiger Kiesel, Springer, 2004 (2nd edition)

2

Chapter 1

Introduction

3

Definition 1.1

A derivative (derivative security, contingent claim) is a financial instrument whose value depends on (derives from, is contingent on) the values of other, more basic, underlying variables (the underlying).

4

Example 1.2

- Forwards
- Futures
- Swaps
- Options

5

Remark 1.3

Market forms for derivatives are

- Exchange-traded markets, e.g., NYSE 1792, CBOT 1848, CME 1919, NASDAQ 1971, CBOE 1973
  - open outcry system
  - electronic
- Over-the-counter markets

6

Definition 1.4

There are three types of traders:

- Hedgers attempt to reduce exposure to risk a company already faces
- Speculators invest available funds opportunistically in the hope of making a profit
- Arbitrageurs try to lock in riskless profit

arbitrage: risk-free profit with no initial investment

no-arbitrage principle: arbitrage opportunities are absent

7

Definition 1.5

- A forward contract is an agreement to sell or buy an asset at a fixed date in the future (delivery time) for a price specified in advance (forward price, delivery price).
- The party selling the asset assumes what is termed a short (forward) position, while the party buying the asset enters into a long (forward) position.

8

Definition 1.5 (continued)

- It costs nothing to enter a forward contract. A forward contract can be contrasted to a spot contract, which is an agreement to buy or sell an asset today.

9

Example 1.6 (a)

Suppose that

- The spot price of ⅛ oz gold is $420
- The 1-year forward price of ⅛ oz gold is $450
- The 1-year interest rate is 5% p.a.

10

Example 1.6 (b)

Suppose that

- The spot price of ⅛ oz gold is $420
- The 1-year forward price of ⅛ oz gold is $430
- The 1-year interest rate is 5% p.a.

11

Example 1.7

Foreign exchange quote for GBP in USD
on August 15, 2025

	buy	sell
spot	1.35584	1.35617
1-month forward	1.35627	1.35661
3-month forward	1.35660	1.35698
6-month forward	1.35652	1.35695

12

Example 1.7 (continued)

(a) Say it is August 15, 2025, and Import Co, a company based in the US, knows that it will have to pay GBP 10 million on November 15, 2025, for goods purchased from a British supplier.

(b) Export Co exports goods to UK and knows it will receive GBP 10 million in three months.

13

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Definition 1.8

- A European call option is a contract giving the holder the right (but no obligation) to buy an asset (the underlying) for a price fixed in advance (exercise price, strike price) at a specified future time (exercise time, expiry time, maturity).
- A European put option is a contract giving the right to sell an asset for a certain strike price at a certain exercise time.

14

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Definition 1.8 (continued)

- Since payoffs are nonnegative, a premium (the market price of the option) must be paid when acquiring an option.
- An American put or call option can be exercised any time up to and including the expiry time.

15

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Example 1.9

Suppose an investor owns 1000 Microsoft shares on August 15, 2025, \$522 per share is the current price. Suppose the investor is concerned about a possible share price decline in the next 2 months and wants protection. The investor could buy 1000 put options with strike price \$520 and expiration time October 17, 2025. Assume each option price is \$14. Ignore the time value of money.

16

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Example 1.10

On August 15, 2025, European calls on Microsoft stock with K=515 USD and T=October 17, 2025 were traded at 21.04 USD at NASDAQ. Ignoring the time value of money, when will the investment bring profit?

17