

Financial Mathematics

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Preface

These are the lecture notes for the Math 6737 / Econ 6337 lecture “Financial Mathematics II” which are held during the Spring Semester 2023.

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