69. What is the price of a European call option on a non-dividend-paying stock when the stock price is \$52, the strike price is \$50, the risk-free interest rate is 12%, the volatility is 30%, and the time to maturity is 3 months?

70. What is the price of a European put option on a non-dividend-paying stock when the stock price is \$69, the strike price is \$70, the risk-free interest rate is 5%, the volatility is 35%, and the time to maturity is 6 months?

71. Calculate the price of a 3-month at-the-money European put option on a non-dividend-paying stock when the stock is at \$50, the risk-free interest rate is 10%, and the volatility is 30%.

72. Assume that a certain security pays off a dollar amount equal to $\ln(S(T))$ at time T, where $S(T)$ denotes the value of the stock price at time T.
    (a) Use risk-neutral valuation to calculate the price of the security at time t in terms of the stock price S at time t.
    (b) Confirm that your price satisfies the Black–Scholes–Merton differential equation.

73. Answer the previous question if $\ln(S(T))$ is replaced by $(S(T))^2$.

74. Consider a derivative that pays off $(S(T))^n$ at time T, where $S(T)$ is the stock price (following geometric Brownian motion) at that time. In view of the previous problem, we assume that the price of the derivative at time $t \leq T$ has the form $h(t,T)S^n$, where S is the stock price at time t and h is a function of t and T.
    (a) By substituting into the Black–Scholes–Merton partial differential equation, derive an ordinary differential equation for h.
    (b) What is the boundary condition for the differential equation for h?
    (c) Solve the problem for h and hence find the price of the derivative.

75. Use risk-neutral valuation to find the price at time $t \in [0,T]$ for a European
    (a) *cash-or-nothing call option* that pays $C > 0$ if the stock price at time T exceeds the level K (otherwise it pays zero);
    (b) *asset-or-nothing call option* that pays $S(T)$ if the stock price $S(T)$ at time T exceeds the level K (otherwise it pays zero).