22. Is the TAPM complete?

23. Let $S_0 = 4$, $u = 2$, $d = 1/2$, $r = 1/4$, $p = 2/3$, $q = 1/3$ in the BAPM. Use Theorem 3.9 to price each of the following options.

   (a) A European put option with strike price $K = 5$ and expiration time 3.

   (b) A lookback option that expires at time 3 and pays off $\max\{S_0, S_1, S_2, S_3\} - S_3$.

   (c) An Asian call option with strike price $K = 4$ and expiration time 3 (i.e., whose payoff at time three is $((S_0 + S_1 + S_2 + S_3)/4 - K)^+)$.

24. For one of the three options from Problem 23, determine the entire value process and the hedging portfolio process.

25. Let C_N, P_N, and F_N be the payoffs of a European call, European put, and long forward, respectively, each with strike price K.

   (a) Show $C_N = F_N + P_N$.

   (b) Show $C_n = F_n + P_n$ for all $0 \le n \le N$.

   (c) Find F_0.

   (d) Calculate the forward price (i.e., the value of K that makes $F_0 = 0$).

   (e) Prove put-call parity (i.e., the price of a call struck at the forward price is the same as the price of a put struck at the forward price).

26. Let $1 \le m \le N - 1$ and $K > 0$. A chooser option is a contract sold at time zero that confers on its owner the right to receive either a European call or a European put at time m, with strike price K and expiration time N. Show that the time-zero price of a chooser option is the sum of the time-zero price of a put, expiring at time N and having strike price K, and a call, expiring at time m and having strike price $K\beta_N/\beta_m$.

27. For one of the three options from Problem 23 (not the one you did in Problem 24), price the option using Theorem 3.18.

28. For the remaining one of the three options from Problem 23, determine the entire value process using Theorem 3.18.