35. Consider an American put with expiration time 2 and strike price 5 in the BAPM with $N = 2$, $\tilde{p} = \tilde{q} = 1/2$, $r = 1/4$, $u = 2$, $d = 1/2$, $S_0 = 4$. Let Y_k be the maximum of zero and the payoff if the put is exercised at k. Let X be the discounted Y process.
   (a) Find the Snell envelope Z of X.
   (b) Find the process V that satisfies $Z = \beta V$.
   (c) Find the optimal stopping time τ^* and $\tilde{\mathbb{E}}(X_{\tau^*})$.
   (d) Verify that Z is a supermartingale (but not a martingale) and Z^{τ^*} is a martingale.
36. Consider the BAPM with $N = 3$, $\tilde{p} = \tilde{q} = 1/2$, $r = 1/4$, $u = 2$, $d = 1/2$, $S_0 = 4$ and let $K = 4$.
   (a) Find V_0 for a European put with expiration time N and strike price K. Denote this value by V_0^{EP}.
   (b) Find V_0 for a European call with expiration time N and strike price K. Denote this value by V_0^{EC}.
   (c) Find V_0 for an American put with expiration time N and strike price K. Denote this value by V_0^{AP}. Also find the optimal stopping time.
   (d) Find V_0 for an American call with expiration time N and strike price K. Denote this value by V_0^{AC}. Also find the optimal stopping time.
   (e) What is the relation between V_0^{AC} and V_0^{EC}?
   (f) What is the relation between V_0^{AP} and V_0^{EP}?
   (g) Verify that $S_0 - K \leq V_0^{\text{AC}} - V_0^{\text{AP}} \leq S_0 - K\beta_3$ holds.
37. Consider the American put from the previous problem.
   (a) Find the entire value process of the American put.
   (b) Find the Doob decomposition of the discounted value process.
   (c) Find the largest optimal stopping time.
   (d) Find the process C from Definition 4.37.
   (e) To hedge the American put, find the initial wealth and the hedging portfolio process.
   (f) Verify that the wealth process resulting in (e) is the same as the value process.
38. Price an American down-and-out call with barrier level 4 and strike price 3 (usual BAPM with $N = 3$).
39. Price an American down-and-in call with barrier level 4 and strike price 3 (usual BAPM with $N = 3$).