21. Problems from the Textbook: 1, 3, 5, 7, 9, 11, 13, 17, 27, 29 (2.8); 1, 5, 7, 9, 11, 13, 15, 25, 27, 31, 37, 39, 41, 45, 49, 51, 54 (2.9); 1, 5, 7, 9, 17, 21, 23, 25, 29, 31 (2.10); 1, 4, 11, 12, 20, 23, 24 (2.PP); 4, 8, 11, 16, 19, 24, 30, 36, 41, 43, 47, 52, 53, 56, 60, 65 (3.1).

22. Find all points $x_0$ such that $f$ has a tangent at $x_0$ parallel to the $x$-axis. Also, find both the linear and quadratic approximations of $f$ at $a$. Use them to approximate $f(b)$.

    (a) $f(x) = \frac{x^3}{3} + \frac{x^2-20x+100001}{48}$, $a = 2$, $b = 2.1$,

    (b) $f(x) = \frac{x-1}{x^2+2}$, $a = 3$, $b = 3.3$,

    (c) $f(x) = (x^3-1)^8(3x^2+4x)^7$, $a = 1$, $b = 0.9$,

    (d) $f(x) = \sqrt{x}\sqrt[4]{x^2+1}$, $a = 2$, $b = 2.2$,

    (e) $f(x) = \alpha x^2 + \beta x + \gamma$, where $\alpha$, $\beta$, $\gamma$ are real numbers, $a = 0$, $b = 0.5$.

23. Let $f(x) = x^3 - x^2 - 2$.

    (a) Show that $f$ has a zero between 1 and 2, pick an $x_0$, and find the zero of the tangent of $f$ at $x_0$.

    (b) Apply Newton's method until the first 7 decimal places of the zero are correct.

24. Let $f(x) = 8.5x^3 - 9.5x^2 - 7.5x + 1$.

    (a) Let $x_0 = 0$ and apply Newton's method until the first 7 decimal places of the zero are correct.

    (b) Give this zero exactly.

    (c) Find the remaining two zeros of $f$.

25. Let $a$ be a real number. We want to compute $\sqrt[3]{a}$.

    (a) Find a function $f(x)$ that has $\sqrt[3]{a}$ as its zero.

    (b) Compute $x - \frac{f(x)}{f'(x)}$ and simplify.

    (c) Use Newton's method to find the first 7 decimal places of $\sqrt[3]{2}$.

    (d) Use Newton's method to find the first 7 decimal places of $\sqrt[3]{3}$.

26. Find the smallest and largest values of $f$ on $D$:

    (a) $f(x) = x^2$, $D = [-4, 5]$;

    (b) $f(x) = 5x^2 - 6x + 2$, $D = (-\infty, 1]$;

    (c) $f(x) = -x^3$, $D = (1, 5]$;

    (d) $f(x) = \frac{x^3}{x}$, $D = [-2, 2] \setminus \{0\}$;

    (e) $f(x) = \frac{1}{x}$, $D = [-6, \frac{1}{2}] \setminus \{0\}$;

    (f) $f(x) = 2x^3 - 9x^2 + 12x - 30$, $D = [-3, 3]$;

    (g) $f(x) = x^4 - 4x^3 + 6x^2 - 4x$, $D = [-2, 1]$;

    (h) $f(x) = ax^2 + bx + c$, $D = \mathbb{R}$.