1. An oil concern produces three different kinds of crude oil, which are used to produce regular and premium gasoline. The amount of sulfur contained in each of the three kinds, the daily producing capacity of each of the three kinds, and the costs for producing each of the three kinds are given in the table below. A barrel of regular gasoline is allowed to contain at most 200 g sulfur, and its sales price is \$22. A barrel of premium gasoline can't contain more than 100 g sulfur, and its sales price is \$26. The problem is how much regular and how much premium gasoline should be produced in order to maximize the daily profit of the concern. Formulate a linear program that solves this problem and give four nonfeasible sets of amounts of production quantities, as well as four feasible sets including their corresponding daily profits.

Crude oil	sulfur contained g/barrel	capacity barrels/day	cost \$/barrel
1	300	5000	6
2	100	3000	8
3	50	4000	12.

2. Prove that matrix multiplication (with matrix addition) is distributive.
3. Manhattan Beach Brewing Co. is brewing a bock beer and an export beer. The bock beer sells for \$5 per case and the export beer for \$2 per case. To produce a case of bock beer, 2 pounds of hops and 5 pounds of barley are needed, for export beer 1 pound hops and 2 pounds barley are needed (per case). Currently, 60 pounds barley and 25 pounds hops are available.
   (a) Rewrite the problem as an LP.
   (b) Solve the LP graphically.
4. Prove that if x and y are optimal solutions of an LP, then every point on the line segment $\overline{xy}$ is also an optimal solution of that LP.
5. Find all basic solutions and all bfs of $Ax = b$ with
$$A = \begin{bmatrix} 2 & 1 & 1 & 1 & 1 \\ 2 & 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}.$$
6. Sunco Oil has three different processes that can be used to manufacture various types of gasoline. Each process involves blending oils in the company's catalytic cracker. Running process 1 for an hour costs \$5 and requires 2 barrels of crude oil 1 and 3 barrels of crude oil 2. The output from running process 1 for an hour is 2 barrels of gas 1 and 1 barrel of gas 2. Running process 2 for an hour costs \$4 and requires 1 barrel of crude 1 and 3 barrels of crude 2. The output from running process 2 for an hour is 3 barrels of gas 2. Running process 3 for an hour costs \$1 and requires 2 barrels of crude 2 and 3 barrels of gas 2. The output from running process 3 for an hour is 2 barrels of gas 3. Each week, 200 barrels of crude 1, at \$2/barrel, and 300 barrels of crude 2, at \$3/barrel, may be purchased. All gas produced can be sold at the following per-barrel prices: gas 1, \$9; gas 2, \$10, gas 3, \$24. Formulate an LP whose solution will maximize revenues less costs. Assume that only 100 hours of time on the catalytic cracker are available each week.