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An innovative approach to designing unknown-input observers in Takagi–Sugeno systems

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Abstract

The main objective of this research is to introduce an innovative and advanced methodology for the design of unknown-input observers adapted to continuous-time Takagi–Sugeno (T–S) systems. We focus on the development of functional observers capable of handling the unknown inputs present in the state and output equations. The design and analysis of these observers are strongly based on the principles of Lyapunov–Krasovskiĭ stability theory, providing a robust and theoretically powerful background. The convergence criteria for these observers are structured according to the formulation of linear matrix inequalities, providing a strict basis for stability analysis. In order to underline the effectiveness of the proposed approach, we offer full validation through simulation results derived from two numerical examples. These examples serve as specific demonstrations of the performance of the designed observers, highlighting their effectiveness in both reduced-order and full-order scenarios. Through this detailed exploration, we aim to highlight the applicability in actual applications and the reliability of our methodology introduced in the field of unknown-input observers for T–S systems.

Keywords Takagi–Sugeno (T–S) · Functional observer (FO) · Unknown input · Reduced order · Full-order · Lyapunov–Krasovskiĭ stability · Linear matrix inequalities (LMIs)

Mathematics Subject Classification Primary 93B53, 93B51 · Secondary 93C42, 93D30

1 Introduction

Takagi–Sugeno (T–S) systems, introduced in [1], represent a class of fuzzy systems designed to approximate nonlinear systems with known complex models. Demonstrating their usefulness in a variety of control problems [2–4], T–S systems utilize the modeling concept in which a nonlinear system is characterized by a collection of local linear models, seamlessly interconnected by nonlinear functions. T–S models are recognized as universal approximators [5, 6], highlighting their ability to effectively capture the complex dynamics of nonlinear systems. An advantageous feature of T–S models is their linearity in parts of the results. This linearity allows classical techniques designed for linear models to be adapted to meet a myriad of challenges, including control design, stability analysis, observation and filtering. The versatility and efficiency of T–S systems make them an invaluable tool for solving a variety of problems in systems analysis and control.

A functional observer directly estimates the state function, a design challenge that has been the subject of active research for several decades. Its distinctive ability to estimate state functions in a single step, as opposed to a two-step process, has received particular attention. In addition, functional observers help to reduce the order of observers. Recently, approaches to design functional observers have been established in [7–21]. For linear systems, [22] derived a finite-time functional observer. A unique linear functional observer for LTI systems was introduced in [23]. The use of functional observers in the design of output feedback controllers for T–S systems can be found, for example, in [24], where the closed-loop stability criteria are expressed in terms of LMIs.

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A revised description of necessary and sufficient criteria in the presence of functional observers has been provided in [25].

This paper studies the presence of functional observers for nonlinear systems in the context of unknown inputs affecting both state and output. The study uses the T–S fuzzy model, in particular for scenarios where some inputs are measurable - a common condition in practical applications. Many real-world systems are susceptible to external disturbances that manifest themselves in the form of unknown inputs. The design of unknown-input observers (UIOs) has received considerable research attention, as evidenced by works such as [26–28]. In particular, the application of fuzzy functional unknown-input observers has demonstrated its effectiveness in contexts ranging from the landing of quadrotor aerial robots to wastewater treatment plants, as highlighted in [29].

Stability analysis of Takagi–Sugeno (T–S) systems with unknown inputs has been the subject of various investigations, as shown by works such as [30, 31]. Remarkably, [32] revealed that models with unknown inputs can be transformed into T–S models. Furthermore, in [33], a design methodology was proposed, combining a proportional multiple integral (PMI) observer with the Lipschitz approach for T–S systems with unmeasurable premise variables. The work presented in [34] introduced an H_∞ unified dynamic observer (DO) applicable to a class of linear systems with unknown inputs and disturbances.

Distinctly, [35, 36] addressed the design of observers to T–S systems with delays. Notably, the authors assumed bounded delays to derive their results. In our present work, we contribute by designing a full-order unknown-input observer, extending our discussion to encompass the special case of reduced-order observers without the presence of delays. This extension of our study aims to provide a comprehensive understanding of the dynamics associated with unknown inputs in both full-order and reduced-order observer scenarios.

The principal purpose of this work is to develop a technique for designing functional observers with unknown inputs for continuous-time nonlinear systems, using the T–S system representation. Based on Lyapunov theory, we establish necessary and sufficient criteria expressed in terms of linear matrix inequalities (LMIs), which can be solved with tools such as the Yalmip/MATLAB toolbox. We extend our approach to cover both reduced-order and full-order observers, providing a multi-purpose setting for practical implementation and design considerations.

The remainder of this paper is structured as follows. Section 2 presents the design problem of the observer under study, as well as some preliminary results. Section 3 describes the design approach of a fuzzy functional observer with unknown input. In Sect. 4, we study the special case of a reduced-order observer. Section 5 presents the design of a full-order fuzzy observer. Section 6 provides simulation examples to test the theoretical convergence of the observer. Conclusions are presented in Sect. 7.

2 Preliminaries and problem setting

Let us consider the class of nonlinear systems defined by the following continuous-time T–S model, see e.g., [1].

Plant rule i:

If $\theta_1(t)$ is M_i^1 and … and $\theta_i(t)$ is M_k^i.

Then

$$\dot{x}(t) = A_i x(t) + B_i u(t) + R_i h(t), \quad i = 1, \ldots, m, \tag{1}$$

where $\theta_1(t), \ldots, \theta_l$ are the premise variables, supposed to be measurable, $M_1^i, \ldots, M_i^k$ are the fuzzy sets for $\theta_k(t)$, k is the number of premise variables, r is the number of **IF-THEN** rules. The state vector is represented by $x(t) \in \mathbb{R}^n$. The input vector is represented by $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^p$ is the output vector, $h(t) \in \mathbb{R}^q$ is the unknown input vector. This model is represented compactly by

$$\begin{cases} \dot{x}(t) &= \sum_{i=1}^{m} \mu_i(\theta(t))\{A_i x(t) + B_i u(t) + R_i h(t)\} \\ y(t) &= Cx(t) + Sh(t) \\ z(t) &= Ex(t) \\ x(t_0) &= \rho_0, \end{cases} \tag{2}$$

where $z(t) \in \mathbb{R}^r$ is the vector to be estimated, with $r \leq n$. $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $R_i \in \mathbb{R}^{n\times q}$, $i = 1, \ldots, m$, $C \in \mathbb{R}^{p\times n}$, $S \in \mathbb{R}^{p\times q}$ and $E \in \mathbb{R}^{r\times n}$ are known constant matrices with compatible dimensions, and ρ_0 is the initial datum. Without losing generality, it is supposed that rank $C = p$ and rank $E = r$. The fuzzy basis functions are represented by

$$\mu_i(\theta(t)) = \frac{\prod_{j=1}^{k} \psi_{ij}(\theta_j(t))}{\sum_{i=1}^{m} \prod_{j=1}^{k} \psi_{ij}(\theta_j(t))} \tag{3}$$

for $i = 1, \ldots, m$, where $\theta(t) = [\theta_1(t), \ldots, \theta_l(t)]^T$ and $\psi_{ij}(\theta_j(t)), i = 1, \ldots, m, j = 1, \ldots, k$ is the grade of membership of $\theta_j(t)$ in F_i^j. For simplicity, we shall remove the parameter $\mu_i(\theta(t))$ in the following. The fuzzy basis functions verify by definition.

$$0 \leq \mu_i(\theta(t)) \leq 1, \quad \forall i = 1, \ldots, m, \quad \sum_{i=1}^{m} \mu_i(\theta(t)) = 1. \tag{4}$$

Before reconstructing the state function, we need first to define the following functional observer

$$\begin{cases} \dot{\nu}(t) = \sum_{i=1}^{m} \mu_i(\theta(t))\{N_i \nu(t) + J_i y(t) + H_i u(t)\} \\ \hat{z}(t) = \nu(t) + G y(t), \end{cases} \tag{5}$$

where $\nu \in \mathbb{R}^r$ is the state vector of the observer, $\hat{z}(t) \in \mathbb{R}^r$ is the estimate of $z(t)$, N_i, J_i, H_i, $i = 1, \ldots, m$ and G are unknown and constant matrices of appropriate dimensions, to be determined such that $\hat{z}(t)$ asymptotically converges to $z(t)$.

The design of an unknown-input functional observer (UIFO) can be specified using the above notation.

Problem description

Select the observer parameters N_i, J_i, H_i for $i = 1, \ldots, m$ and G, such that

$$\lim_{t \to +\infty} [z(t) - \hat{z}(t)] = 0, \tag{6}$$

for any initial functions.

Before proceeding, we recall the following assumption and proposition.

Assumption 1. The pairs (A_i, B_i) and (A_i, C) are observable and detectable.
Proposition 1. The pair (Λ, Γ) is observable.

Proof The observability of the pair (Λ, Γ) implies

$$\text{rank} \begin{bmatrix} sI - \Gamma \\ \Lambda \end{bmatrix} = n, \qquad s \in \mathbb{C},\ \text{Re}(s) \geq 0, \tag{7}$$

according to [37], and this completes the proof. □

Notations

In the following, the symbols I and O denote, respectively, the identity and zero matrices.

3 Functional observer design

The observation error vector is defined as the difference between $z(t)$ and its estimate $\hat{z}(t)$ by

$$\varepsilon(t) = z(t) - \hat{z}(t) = Tx(t) - \nu(t) - GSh(t), \tag{8}$$

with

$$T = E - GC. \tag{9}$$

In Proposition 3.1 below, we give the conditions needed to prove the existence and stability of the functional observer (5).

Proposition 3.1 *For any set of initial conditions, the estimate $\hat{z}(t)$ converges asymptotically to $z(t)$. The initial conditions $x(0)$ and $\hat{z}(0)$ are appropriate for any $h(t)$ and any $u(t)$ if the following conditions hold.*

(1) $\dot{\varepsilon}(t) = \sum_{i=1}^{m} \mu_i(\theta(t)) N_i \varepsilon(t)$ is asymptotically stable,
(2) $TA_i - N_i T - J_i C = 0$ for all $i = 1, \ldots, m$,
(3) $TR_i - N_i GS - J_i S = 0$ for all $i = 1, \ldots, m$,
(4) $GS = 0$,
(5) $H_i = TB_i$ for all $i = 1, \ldots, m$.

Proof Using the same reasoning as for [12] and from (8), the error dynamics is as follows:

$$\dot{\varepsilon}(t) = T\dot{x}(t) - \dot{\nu}(t) \tag{10}$$

Using (2) and (5), relation (10) can be rewritten as

$$\begin{aligned} \dot{\varepsilon}(t) &= \sum_{i=1}^{m} \mu_i(\theta(t))\{N_i \varepsilon(t) + TA_i - N_i T - J_i C)x(t) \\ &\quad + (TR_i - N_i GS - J_i S)h(t) - GS\dot{h}(t) \\ &\quad + (TB_i - H_i)u(t)\}. \end{aligned} \tag{11}$$

Now, if conditions (2)–(5) are verified, the estimation error dynamics (11) becomes

$$\dot{\varepsilon}(t) = \sum_{i=1}^{m} \mu_i(\theta(t)) N_i \varepsilon(t). \tag{12}$$

Then, we can see that if condition (3) is satisfied, then $\hat{z}(t) \to z(t)$. This concludes the proof. □

The design of the functional observer is now simplified to finding the gain matrices N_i, J_i, H_i, $i = 1, \ldots, m$, T and G such that Proposition 3.1 is satisfied. By substituting T in conditions (2) and (3) in Proposition 3.1, we have

$$N_i E = EA_i - \begin{bmatrix} G & F_i \end{bmatrix} \begin{bmatrix} CA_i \\ C \end{bmatrix}, \tag{13}$$

where $F_i = J_i - N_i G$, $i = 1, \ldots, m$. Now, considering that E has full-row rank, let $D \in \mathbb{R}^{(n-r)\times n}$, $M_1 \in \mathbb{R}^{n\times r}$ and $M_2 \in \mathbb{R}^{n\times(n-r)}$ such that

$$\begin{bmatrix} E \\ D \end{bmatrix} = [\, M_1 \quad M_2 \,]^{-1}. \tag{14}$$

Post-multiplying (13) by (14), we get,

$$N_i = EA_iM_1 - \begin{bmatrix} G & F_i \end{bmatrix} \begin{bmatrix} CA_i \\ C \end{bmatrix} M_1, \tag{15}$$

where $\begin{bmatrix} G & F_i \end{bmatrix}$ is an unknown matrix that satisfies the condition

$$\begin{bmatrix} G & F_i \end{bmatrix} \xi_i = \varphi_i \tag{16}$$

with

$$\xi_i = \begin{bmatrix} CA_iM_2 & CR_i & S \\ CM_2 & S & O \end{bmatrix}$$

and

$$\varphi_i = EA_iM_2$$

for $i = 1, \ldots, m$. According to the above equations, knowing F_i and G is both necessary and sufficient to determine N_i, J_i, and H_i, $i = 1, \ldots, m$.

Necessary and sufficient conditions for the existence of a solution to (16) are given in the following lemma.

Lemma 3.1 *There are matrices G and F_i that satisfy (16) if and only if*

$$\text{rank} \begin{bmatrix} EA_i & ER_i & O \\ CA_i & CR_i & S \\ C & S & O \\ E & O & O \end{bmatrix} = \text{rank} \begin{bmatrix} CA_i & CR_i & S \\ C & S & O \\ E & O & O \end{bmatrix} \tag{17}$$

for $i = 1, \ldots, m$.

Proof A solution to (16) exists according to the general solution of linear algebraic equations [38] if and only if

$$\varphi_i(I - \xi^+\xi_i) = 0, \tag{18}$$

where ξ_i^+ is a generalized inverse of matrix ξ_i satisfying $\xi_i\xi_i^+\xi_i = \xi_i$ and (17) are satisfied for $i = 1, \ldots, m$. Equation (18) can also be written as

$$\text{rank} \begin{bmatrix} \xi_i \\ \varphi_i \end{bmatrix} = \text{rank}\,[\xi_i]\,, \tag{19}$$

for $i = 1, \ldots, m$. Now we define the matrix

$$W_1 = \begin{bmatrix} M_1 & M_2 & O & O \\ O & O & I & O \\ O & O & O & I \end{bmatrix}.$$

Then, we have

$$\begin{aligned} \text{rank} \begin{bmatrix} EA_i & ER_i & O \\ CA_i & CR_i & S \\ C & S & O \\ E & O & O \end{bmatrix} &= \text{rank} \begin{bmatrix} EA_i & ER_i & O \\ CA_i & CR_i & S \\ C & S & O \\ E & O & O \end{bmatrix} W_1 \\ &= r + \text{rank} \begin{bmatrix} \xi_i \\ \varphi_i \end{bmatrix}. \end{aligned} \tag{20}$$

On the other hand, we have

$$\begin{aligned} \text{rank} \begin{bmatrix} CA_i & CR_i & S \\ C & S & O \\ E & O & O \end{bmatrix} &= \text{rank} \begin{bmatrix} CA_i & CR_i & S \\ C & S & O \\ E & O & O \end{bmatrix} W_1 \\ &= r + \text{rank}\,[\xi_i]\,. \end{aligned} \tag{21}$$

So, from (20), (21) and considering the equality (19), we can conclude the proof. □

We assume that (17) is satisfied. Hence, the general solution of (16) is given by

$$\begin{bmatrix} G & F_i \end{bmatrix} = \varphi_i\xi_i^+ - X_i(I - \xi_i\xi_i^+), \tag{22}$$

for $i = 1, \ldots, m$. Equivalently,

$$G = \gamma_i - X_i\delta_i \quad \text{and} \quad F_i = \Gamma_i - X_i\Delta_i, \tag{23}$$

where

$$\gamma_i = \varphi_i\xi_i^+ \begin{bmatrix} I \\ O \end{bmatrix}, \quad \delta_i = (I - \xi_i\xi_i^+) \begin{bmatrix} I \\ O \end{bmatrix} \tag{24}$$

and

$$\Gamma_i = \varphi_i\xi_i^+ \begin{bmatrix} O \\ I \end{bmatrix}, \quad \Delta_i = (I - \xi_i\xi_i^+) \begin{bmatrix} O \\ I \end{bmatrix} \tag{25}$$

for $i = 1, \ldots, m$, where X_i, $i = 1, \ldots, m$, is an arbitrary matrix of appropriate dimension that will be determined in the sequel using the LMI approach. By replacing the matrix $\begin{bmatrix} G & F_i \end{bmatrix}$ giving by (22) in (15), we get

$$N_i = \alpha_i - X_i\beta_i, \tag{26}$$

where

$$\alpha_i = EA_iM_1 - \varphi_i\xi_i^+ \begin{bmatrix} CA_i \\ C \end{bmatrix} M_1 \tag{27}$$

and

$$\beta_i = (I - \xi_i\xi_i^+) \begin{bmatrix} CA_i \\ C \end{bmatrix} M_1 \tag{28}$$

for $i = 1, \ldots, m$.

Remark 3.1 By a suitable selection of matrices X_i, $i = 1,\ldots,m$, it is necessary and sufficient that the pairs (α_i,β_i), $i = 1,\ldots,m$, are observable. If (α_i,β_i), $i = 1,\ldots,m$, is not observable, then a matrix X_i, $i = 1,\ldots,m$, can still be found such that the observer is asymptotically stable if and only if the pair (α_i,β_i), $i = 1,\ldots,m$, is detectable.

The expressions providing the matrices J_i and H_i, $i = 1,\ldots,m$, are

$$J_i = F_i + N_i E, \qquad \forall i = 1,\ldots,m, \tag{29}$$

$$H_i = (E - GC)B_i, \qquad \forall i = 1,\ldots,m. \tag{30}$$

The dynamics of the estimation error under the conditions of Proposition 3.1 are provided by

$$\dot{\varepsilon}(t) = \sum_{i=1}^{m} \mu_i(\theta(t))\{\alpha_i - X_i\beta_i\}\varepsilon(t). \tag{31}$$

Therefore, the design of the functional observer (5) is simplified to determine the matrices X_i that satisfy condition (1) of Proposition 3.1.

In the next result, we give necessary and sufficient conditions for N_i, $i = 1,\ldots,m$ to be stable.

Lemma 3.2 *The matrices N_i, $i = 1,\ldots,m$, given by (26) are Hurwitz if and only if*

$$\operatorname{rank}\begin{bmatrix} sE - EA_i & -ER_i & O \\ CA_i & CR_i & S \\ C & S & O \end{bmatrix} = \operatorname{rank}\begin{bmatrix} CA_i & CR_i & S \\ C & S & O \\ E & O & O \end{bmatrix} \tag{32}$$

for all $s \in \mathbb{C}$, $\operatorname{Re}(s) \geq 0$, $i = 1,\ldots,m$.

Proof The detectability of the pair (α_i,β_i), $i = 1,\ldots,m$, which is equivalent to (7), is again equivalent to the stability of N_i, $i = 1,\ldots,m$. The left-hand side of (32) is equivalent to

$$\begin{aligned}
&\operatorname{rank}\begin{bmatrix} sE - EA_i & -ER_i & O \\ CA_i & CR_i & S \\ C & S & O \end{bmatrix} \\
&= \operatorname{rank}\begin{bmatrix} sE - EA_i & -ER_i & O \\ CA_i & CR_i & S \\ C & S & O \end{bmatrix} W_1 \\
&= \operatorname{rank}\ W_2 \begin{bmatrix} sE - EA_iM_1 & -\varphi_i \\ \begin{bmatrix} CA_iM_1 \\ CA_iM_2 \end{bmatrix} & \xi_i \end{bmatrix}.
\end{aligned}$$

We now define the full column matrix by

$$W_{2i} = \begin{bmatrix} I & -\varphi_i\xi_i^+ \\ O & I - \xi_i\xi_i^+ \\ O & \xi_i\xi_i^+ \end{bmatrix}, \quad i = 1,\ldots,m,$$

and the full row matrix by

$$W_{3i} = \begin{bmatrix} I & O \\ \xi_i\xi_i^+\begin{bmatrix} CA_iM_1 \\ CA_iM_2 \end{bmatrix} & I \end{bmatrix}, \quad i = 1,\ldots,m.$$

Then, one has

$$\begin{aligned}
&\operatorname{rank}\begin{bmatrix} sE - EA_i & -ER_i & O \\ CA_i & CR_i & S \\ C & S & O \end{bmatrix} \\
&= \operatorname{rank}\ W_2 \begin{bmatrix} sE - EA_iM_1 & -\varphi_i \\ \begin{bmatrix} CA_iM_1 \\ CA_iM_2 \end{bmatrix} & \xi_i \end{bmatrix} \\
&= \operatorname{rank}\begin{bmatrix} \begin{bmatrix} sI - \alpha_i \\ \beta_i \end{bmatrix} & O \\ \xi_i\xi_i^+\begin{bmatrix} CA_iM_1 \\ CA_iM_2 \end{bmatrix} & \xi_i \end{bmatrix} W_3 \\
&= \operatorname{rank}\begin{bmatrix} sI - \alpha_i \\ \beta_i \end{bmatrix} + \operatorname{rank}\ \xi_i,
\end{aligned}$$

using the result of Lemma 3.1, we obtain (32). □

For the computation of matrices X_i, the following theorem is given.

Theorem 3.1 *There exist matrices X_i, $i = 1,\ldots,m$, such that condition (1) of Proposition 3.1 holds if and only if there exist a symmetric positive definite matrix P and Y_i, $i = 1,\ldots,m$, fulfilling the condition*

$$\begin{bmatrix} \alpha_i P + P\alpha_i^T - Y_i\beta_i - \beta_i^T Y_i^T & O \\ O & -I \end{bmatrix} < O \tag{33}$$

for $i = 1,\ldots,m$. In this situation, the matrices X_i are given by $X_i = P^{-1}Y_i$, $i = 1,\ldots,m$.

Proof Using the Lyapunov function $V(t) = \varepsilon^T P\varepsilon$ with $P = P^T > 0$, its derivative is given by

$$\dot{V}(t) = \dot{\varepsilon}^T P\varepsilon + \varepsilon^T P\dot{\varepsilon} = \varepsilon^T(N_i^T P + PN_i)\varepsilon.$$

Clearly, $\dot{V}(t) < 0$ if and only if

$$N_i^T P + PN_i < O. \tag{34}$$

Replacing N_i, $i = 1,\ldots,m$, by its value (26), LMI (34) is equivalent to

$$\alpha_i P + P\alpha_i^T - Yi\beta_i - \beta_i^T Y_i^T < O, \tag{35}$$

where $Y_i^T = PX_i$. These conditions are equivalent to the LMIs (33) by applying the Schur lemma [39]. Then, the proof is complete. □

Remark 3.2 If $S = 0$, then conditions (17) and (32) are simplified as

$$\text{rank}\begin{bmatrix} EA_i & ER_i \\ CA_i & CR_i \\ C & O \\ E & O \end{bmatrix} = \text{rank}\begin{bmatrix} CA_i & CR_i \\ C & O \\ E & O \end{bmatrix}, \tag{36}$$

$$\text{rank}\begin{bmatrix} sE - EA_i & -ER_i \\ CA_i & CR_i \\ C & O \end{bmatrix} = \text{rank}\begin{bmatrix} CA_i & CR_i \\ C & O \\ E & O \end{bmatrix} \tag{37}$$

for $i = 1, \ldots, m$.

We propose the following design approach for the obtained observer.

Algorithm 1 Design steps of unknown input functional observers

1: **Input:** Select a matrix $E \in \mathbb{R}^{(n-r)\times n}$ to make $\begin{bmatrix} E \\ C \end{bmatrix}$ non-singular.
2: **if** rank conditions (17) and (32) are satisfied, **then**
3: Compute the matrices M_1 and M_2 from (14).
4: Deduce the values of matrices α_i and β_i, $i = 1, \ldots, m$, by using (27) and (28).
5: Solve the LMI (33), and compute X_i, $i = 1, \ldots, m$.
6: Compute N_i, $i = 1, \ldots, m$, from equation (26).
7: Compute G and F_i, $i = 1, \ldots, m$, from equation (23).
8: Compute J_i and H_i, $i = 1, \ldots, m$, from (29) and (30).
9: **end if**

Algorithm 1 provides all observer parameters.

Remark 3.3 If $r = n - p$, then the observer is of reduced order, in which case the proposed design amounts to transforming the system into an equivalent system (2), as discussed in the following section.

4 Reduced-order observer design

In this section, we present a special case of a reduced-order observer and we give the conditions under which it is asymptotically stable.

If $r = n - p$, then conditions (17) and (32) are equivalent to

$$\text{rank}\begin{bmatrix} S & O \\ CR_i & S \\ ER_i & O \end{bmatrix} = \text{rank}\begin{bmatrix} S & O \\ CR_i & S \end{bmatrix}, \tag{38}$$

$$\text{rank}\begin{bmatrix} sI_r - A_i & -R_i & O \\ C & S & O \\ CA_i & CR_i & S \end{bmatrix} = r + \text{rank}\begin{bmatrix} S & 0 \\ CR_i & S \end{bmatrix} \tag{39}$$

for $i = 1, \ldots, m$. These are the conditions developed for the full-order observer for linear systems, see [40]. Then, (15) and (16) can be written as

$$\begin{bmatrix} N_i & F_i \end{bmatrix}\begin{bmatrix} E \\ C \end{bmatrix} + GCA_i = LA_i, \tag{40}$$

$$F_iS + GCR_i = LR_i, \tag{41}$$

and

$$GS = 0 \tag{42}$$

for $i = 1, \ldots, m$. Now, let $\begin{bmatrix} E \\ C \end{bmatrix}^{-1} = \begin{bmatrix} D_1 & D_2 \end{bmatrix}$. Then the general solution of (42) is expressed by

$$G = \Theta_i\Sigma_i^+ - \mathbb{Z}_i(I - \Sigma_i\Sigma_i^+), \tag{43}$$

$$F_i = EA_iD_2 - \Theta_i\Sigma_i^+CA_iD_2 + \mathbb{Z}_i(I - \Sigma_i\Sigma_i^+)CA_iD_2 \tag{44}$$

for $i = 1, \ldots, m$, with $\Sigma_i = \begin{bmatrix} CR_i - CA_iD_2S & S \end{bmatrix}$ and $\theta_i = \begin{bmatrix} ER_i - EA_iD_2S & O \end{bmatrix}$, $i = 1, \ldots, m$. Then,

$$N_i = \Lambda_i - \mathbb{Z}_i\Gamma_i, \quad i = 1, \ldots, m \tag{45}$$

with $\Lambda_i = EA_iD_1 - \Theta_i\Sigma_i^+CA_iD_1$ and $\Gamma_i = (I - \Sigma_i\Sigma_i^+)CA_iD_1$, $i = 1, \ldots, m$.

The matrices $\mathbb{Z}_i$, $i = 1, \ldots, m$, can be determined from the following theorem.

Theorem 4.1 *The reduced-order observer (5) is asymptotically stable if there exist symmetric matrices $\mathbb{P}$ and $\mathbb{X}_i$, $i = 1, \ldots, m$, satisfying the inequalities*

$$\begin{bmatrix} \Lambda_i\mathbb{P} + \mathbb{P}\Lambda_i^T - \mathbb{X}_i\Gamma_i - \Gamma_i^T\mathbb{X}_i^T & O \\ O & -I \end{bmatrix} < O. \tag{46}$$

The matrices $\mathbb{Z}_i$ are determined by $\mathbb{Z}_i = \mathbb{P}^{-1}\mathbb{X}_i$, $i = 1, \ldots, m$.

The proof of Theorem 4.1 is similar to the proof of Theorem 3.1, thus it is omitted.

Remark 4.1 In case where $E = I$, then the associated necessary and sufficient conditions for the existence of the full-order fuzzy observer (5) are reduced to the following section.

5 Full-order observer design when $E = I$

This section is devoted to the design of the observer (5) when $E = I$, in which case full state estimation is possible. In such a case, the observer dynamics of system (2) when $z(t) = x(t)$ is described by

$$\begin{cases} \dot{v}(t) = \sum_{i=1}^{m} \mu_i(\theta(t))\{N_i v(t) + J_i y(t) + H_i u(t)\} \\ \hat{x}(t) = v(t) + Gy(t). \end{cases} \tag{47}$$

The state estimation error in this case is defined by

$$\varepsilon(t) = x(t) - \hat{x}(t). \tag{48}$$

Now, (9) becomes

$$T = I - GC. \tag{49}$$

Using the new form of T given in (49), (15) and (16) can be written as

$$N_i = A_i + \begin{bmatrix} G & F_i \end{bmatrix} \begin{bmatrix} CA_i \\ C \end{bmatrix}, \tag{50}$$

$$\begin{bmatrix} G & F_i \end{bmatrix} \Xi_i = \Pi_i \tag{51}$$

with

$$F_i = -J_i - N_i G,$$

where

$$\Xi_i = \begin{bmatrix} CR_i & S \\ S & O \end{bmatrix}$$

and

$$\Pi_i = \begin{bmatrix} -R_i & O \end{bmatrix}$$

for $i = 1, \ldots, m$.

The following result specifies the necessary and sufficient criteria for (51) to have a solution.

Lemma 5.1 *There exists a solution of (51) if and only if*

$$\text{rank} \begin{bmatrix} CR_i & S \\ S & O \end{bmatrix} = \text{rank}\ \ G + \text{rank} \begin{bmatrix} R_i \\ S \end{bmatrix}, \tag{52}$$

for $i = 1, \ldots, m$.

Proof A solution to (51) exists if and only if

$$\Xi_i \Pi_i^+ \Pi_i = \Xi_i \tag{53}$$

for $i = 1, \ldots, m$, where Ξ_i^+ is a generalized inverse of the matrix Ξ_i satisfying $\Xi_i \Xi_i^+ \Xi_i = \Xi_i$, or equivalently

$$\text{rank} \begin{bmatrix} \Xi_i \\ \Pi_i \end{bmatrix} = \text{rank}\left[\Xi_i\right], \quad i = 1, \ldots, m, \tag{54}$$

which is equivalent to

$$\text{rank} \begin{bmatrix} I & O & C \\ O & I & O \\ O & O & I \end{bmatrix} \begin{bmatrix} \Xi_i \\ \Pi_i \end{bmatrix} = \text{rank} \begin{bmatrix} O & S \\ S & O \\ -R_i & O \end{bmatrix} = \text{rank}[\Xi_i],$$

which is exactly condition (52). Now, under condition (52), the general solution of (55) is given by

$$\begin{bmatrix} G & F_i \end{bmatrix} = \Pi_i \Xi_i^+ - \mathbb{X}_i (I - \Xi_i \Xi_i^+) \tag{55}$$

for $i = 1, \ldots, m$. In this case, $\mathbb{X}_i$, $i = 1, \ldots, m$, are arbitrary matrices of suitable size that are found using an LMI technique in the sequel. N_i, $i = 1, \ldots, m$, are supplied from (55) by

$$N_i = \mathbb{A}_i - \mathbb{X}_i \mathbb{B}_i, \tag{56}$$

where

$$\mathbb{A}_i = A_i + \Pi_i \Xi_i^+ \begin{bmatrix} CA_i \\ C \end{bmatrix} \tag{57}$$

and

$$\mathbb{B}_i = (I - \Xi_i \Xi_i^+) \begin{bmatrix} CA_i \\ C \end{bmatrix} \tag{58}$$

for $i = 1, \ldots, m$. □

Now we state the result below, which will be utilized in the sequel.

Lemma 5.2 *The matrices* N_i, $i = 1, \ldots, m$, *are Hurwitz if and only if*

$$\text{rank} \begin{bmatrix} sI - A_i & -R_i \\ C & S \end{bmatrix} = n + \text{rank} \begin{bmatrix} R_i \\ S \end{bmatrix} \tag{59}$$

for $i = 1, \ldots, m$.

Proof We have

$$\begin{aligned} &\text{rank} \begin{bmatrix} sI - A_i & \Pi_i \\ \begin{bmatrix} CA_i \\ C \end{bmatrix} & \Xi_i \end{bmatrix} \\ &= \text{rank} \begin{bmatrix} I & O & O \\ C & I & -sI \\ O & O & I \end{bmatrix} \begin{bmatrix} sI - A_i & \Pi_i \\ \begin{bmatrix} CA_i \\ C \end{bmatrix} & \Xi_i \end{bmatrix} \\ &= \text{rank} \begin{bmatrix} sI - A_i & -R_i \\ C & S \end{bmatrix} + \text{rank}\ \ S. \end{aligned}$$

Now, considering the matrices

$$W_{4i} = \begin{bmatrix} I & -\Pi_i \Xi_i^+ \\ O & I - \Xi_i \Xi_i^+ \\ O & \Xi_i \Xi_i^+ \end{bmatrix}$$

and

$$W_{5i} = \begin{bmatrix} I & O \\ -\Xi_i^+ \begin{bmatrix} A_i \\ C \end{bmatrix} & I \end{bmatrix}$$

for $i = 1, \ldots, m$, we have

$$\text{rank} \begin{bmatrix} sI - A_i & \Pi_i \\ \begin{bmatrix} CA_i \\ C \end{bmatrix} & \Xi_i \end{bmatrix} = W_{4i} \begin{bmatrix} sI - A_i & \Pi_i \\ \begin{bmatrix} CA_i \\ C \end{bmatrix} & \Xi_i \end{bmatrix} W_{5i}$$
$$= \text{rank} \begin{bmatrix} sI - A_i \\ B_i \end{bmatrix} + \text{rank } \Xi_i.$$

Then, by using Assumption 2, the proof is completed. □

The determination of the matrices $\mathbb{X}_i$, $i = 1, \ldots, m$, can be performed using the following theorem, which is also a corollary of Theorem 3.1.

Theorem 5.1 *The full-order observer (5) is asymptotically stable if there exist symmetric matrices $\mathbb{P}$ and $\mathbb{Y}_i$, $i = 1, \ldots, m$, satisfying the inequalities*

$$\begin{bmatrix} \mathbb{A}_i\mathbb{P} + \mathbb{P}\mathbb{A}_i^T - \mathbb{Y}_i\mathbb{B}_i - \mathbb{B}_i^T\mathbb{Y}_i^T & O \\ O & -I \end{bmatrix} < O \tag{60}$$

for $i = 1, \ldots, m$. The matrix $\mathbb{X}_i$ can be determined by $\mathbb{X}_i = \mathbb{P}^{-1}\mathbb{Y}_i$, $i = 1, \ldots, m$.

Proof The proof is identical to the proof of Theorem 3.1. □

Given the preceding results, the suggested observer can be designed as follows.

Algorithm 2 Design steps of full-order unknown input observers

1: Verify that the rank conditions (52) and (59) are satisfied.
2: Compute the matrices $\mathbb{A}_i$ and $\mathbb{B}_i$, $i = 1, \ldots, m$, by using (57) and (58).
3: Solve the LMI (60) and compute X_i, $i = 1, \ldots, m$.
4: Compute N_i, $i = 1, \ldots, m$ using (56).
5: Compute G and F_i, $i = 1, \ldots, m$, respectively using (55).
6: From (29) and (30), compute J_i and H_i, $i = 1, \ldots, m$.

To validate the theoretical results, the next section is designed to give some interesting numerical examples.

6 Numerical examples

Two examples are provided in this section to demonstrate the observer design techniques presented in this paper.

Example 6.1 Let us consider the system presented in Sect. 2, with the following matrices borrowed from [33]:

$$A_1 = \begin{bmatrix} -0.0035 & -22.5 & 0 & -32.2 \\ 0 & -0.094 & 1 & 0 \\ 0 & -1.94 & -0.188 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -0.04 & -22.1 & 0 & -32.4 \\ 0 & -0.1 & 1 & 0 \\ 0 & -1.77 & 0.22 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -8.83 \\ -0.0196 \\ -2.02 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -8.7 \\ -0.03 \\ -1.89 \\ 0 \end{bmatrix},$$

$$R_1 = 0.05 * \begin{bmatrix} 0.5 & 0.7 \\ 0.5 & 1 \\ 0.5 & 0.5 \\ 0.5 & 0.4 \end{bmatrix}, \quad R_2 = R_1,$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad S = 0.$$

The membership functions are

$$\mu_1(\theta(t)) = 0.5(1 - \tanh(\theta(t))),$$
$$\mu_2(\theta(t)) = 1 - \mu_1(\theta(t))$$

In this example, we select the matrix $E = I$, so we are dealing with full-order observers (Sect. 5). We then use Algorithm 2 in order to model an observer of the form (5). We first check the rank conditions (52) and (59). The solutions from Theorem 5.1 are then obtained using the Yalmip toolbox [41] and the MATLAB solver.

$$\mathbb{P} = 187.4267 * \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbb{Y}_1 = \begin{bmatrix} 123.1629 & -49.2651 & -73.8977 & 0 & 93.7163 & -1.5927 & -0.0359 & 0 \\ -58.5997 & 23.4411 & 35.1586 & 0 & -1.5927 & -103.8652 & -72.5625 & 0 \\ -80.6756 & 32.2666 & 48.4029 & 0 & -0.0359 & -72.5625 & 91.7944 & 0 \\ -81.8252 & 32.7277 & 49.0975 & 0 & -4.7720 & 72.1135 & 50.9836 & 0 \end{bmatrix},$$

$$\mathbb{Y}_2 = \begin{bmatrix} 123.1629 & -49.2651 & -73.8977 & 0 & 93.7104 & -1.6885 & -0.1257 & 0 \\ -57.8153 & 23.1297 & 34.6916 & 0 & -1.6885 & -84.6692 & -47.8521 & 0 \\ -78.3285 & 31.3326 & 46.9959 & 0 & -0.1257 & -47.8521 & 91.4052 & 0 \\ -81.8072 & 32.7277 & 49.0915 & 0 & -4.7960 & 58.3721 & 33.1887 & 0 \end{bmatrix},$$

$$\mathbb{X}_1 = \begin{bmatrix} 0.6571 & -0.2628 & -0.3943 & 0 & 0.5000 & -0.0085 & -0.0002 & 0 \\ -0.3126 & 0.1251 & 0.1876 & 0 & -0.0085 & -0.5542 & -0.3871 & 0 \\ -0.4304 & 0.1722 & 0.2583 & 0 & -0.0002 & -0.3871 & 0.4898 & \\ -0.4366 & 0.1746 & 0.2620 & 0 & -0.0255 & 0.3848 & 0.2720 & 0 \end{bmatrix},$$

$$\mathbb{X}_2 = \begin{bmatrix} 0.6571 & -0.2629 & -0.3943 & 0 & 0.5000 & -0.0090 & -0.0007 & 0 \\ -0.3085 & 0.1234 & 0.1851 & 0 & -0.0090 & -0.4517 & -0.2553 & 0 \\ -0.4179 & 0.1672 & 0.2507 & 0 & -0.0007 & -0.2553 & 0.4877 & 0 \\ -0.4365 & 0.1746 & 0.2619 & 0 & -0.0256 & 0.3114 & 0.1771 & 0 \end{bmatrix}.$$

The associated observer parameters are then

$$N_1 = \begin{bmatrix} -0.5000 & -0.0083 & -0.0001 & -0.0254 \\ 0.0083 & -0.5000 & 0.3729 & -1.5937 \\ 0.0001 & -0.3729 & -0.5000 & -1.1491 \\ 0.0254 & 1.5937 & 1.1491 & -0.5000 \end{bmatrix},$$

$$N_2 = \begin{bmatrix} -0.5000 & -0.0072 & 0.0003 & -0.0250 \\ 0.0072 & -0.5000 & 0.2312 & -1.4685 \\ -0.0003 & -0.2312 & -0.5000 & -0.7507 \\ 0.0250 & 1.4685 & 0.7507 & -0.5000 \end{bmatrix},$$

$$J_1 = \begin{bmatrix} 0.3289 & 0.0058 & 0.0003 & 0.0170 \\ -0.0051 & 0.3289 & -0.2447 & 1.0476 \\ 0.0003 & 0.2454 & 0.3289 & 0.7554 \\ -0.0164 & -1.0469 & -0.7548 & 0.3289 \end{bmatrix},$$

$$J_2 = \begin{bmatrix} 0.3289 & 0.0050 & 0.0001 & 0.0167 \\ -0.0044 & 0.3289 & -0.1516 & 0.9653 \\ 0.0005 & 0.1522 & 0.3289 & 0.4936 \\ -0.0161 & -0.9647 & -0.4930 & 0.3289 \end{bmatrix},$$

$$H_1 = \begin{bmatrix} -16.6719 \\ -10.8825 \\ -12.1970 \\ -10.8696 \end{bmatrix}, \quad H_2 = \begin{bmatrix} -16.3368 \\ -10.6397 \\ -11.8619 \\ -10.6200 \end{bmatrix},$$

$$G = 0.6571, \quad F_1 = -0.0578, \quad F_2 = -0.0565.$$

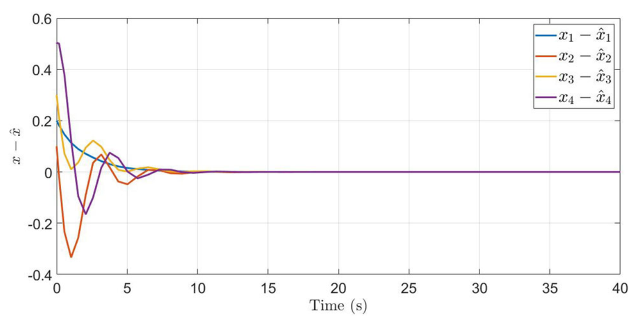

Fig. 1 Evolution of the estimation errors (full-order observer)

Figure 1 shows the evolution of estimation errors of the state variables $x_1(t)$, $x_2(t)$, $x_3(t)$ and $x_4(t)$ from the initial condition $\varepsilon(0) = [0.2\ 0.1\ 0.3\ 0.5]^T$. It is seen that the designed full-order observer ensures the convergence to zero of all state estimation errors, demonstrating the effectiveness of the proposed approach.

Example 6.2 The T–S system presented below is now used to demonstrate the synthesis techniques and to validate the stability criteria specified in Theorem 3.1.

$$A_1 = \begin{bmatrix} -0.4 & -1 \\ 0.4 & -0.1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.6 & -0.1 \\ 2 & -0.2 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.4 \\ 0.3 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.5 \\ 0.2 \end{bmatrix}, \quad C = \begin{bmatrix} 0.5 & -1 \end{bmatrix},$$

$$S = 0, \quad E = \begin{bmatrix} 1 & 0.5 \end{bmatrix},$$
$$R_1 = \begin{bmatrix} 0.2 \\ -0.3 \end{bmatrix}, \quad R_2 = \begin{bmatrix} -0.5 \\ 0.4 \end{bmatrix}.$$

The membership functions for this system are assumed to be

$$\mu_1(\theta(t)) = \left[1 - \left(\frac{1}{1+\exp(3(-x_1(t)) - 0.5\pi)}\right)\right] \times \left(\frac{1}{1+\exp(3(-x_1(t)) - 0.5\pi)}\right),$$
$$\mu_2(\theta(t)) = 1 - \mu_1(\theta(t))$$

Let us apply step-by-step Algorithm 1. First, conditions (17) and (32) should be checked:

$$\text{rank}\begin{bmatrix} EA_i & ER_i & O \\ CA_i & CR_i & S \\ C & S & O \\ E & O & O \end{bmatrix} = \text{rank}\begin{bmatrix} CA_i & CR_i & S \\ C & S & O \\ E & O & O \end{bmatrix} = 3,$$
$$\text{rank}\begin{bmatrix} sE - EA_i & -ER_i & O \\ CA_i & CR_i & S \\ C & S & O \end{bmatrix} = \text{rank}\begin{bmatrix} C_iA_i & CR_i & S \\ C & S & O \\ E & O & O \end{bmatrix} = 3$$

for $i = 1, 2$. Hence, conditions (17) and (32) hold, ensuring the existence of a stable observer of the form (5). Then, matrices M_1 and M_2 can be evaluated as

$$M_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}.$$

By using the Yalmip toolbox [41] and MATLAB, we obtain the solutions to LMI (33) as

$$P = 219.3352, \quad Y_1 = [0.7472 \quad 0.9940], \quad Y_2 = [76.7348 \; -9$$

Then, the free gain matrices X_1 and X_2 are computed as

$$X_1 = [0.0034 \quad 0.0045], \quad X_2 = [867.2226 \quad -21.7727].$$

Following that, the functional observer parameters are

$$N_1 = -1, \quad N_2 = -0.0719, \quad J_1 = 0.6250,$$
$$J_2 = 0.7500, \quad H_1 = 0.5625, \quad H_2 = 0.5938,$$
$$G = 0.1250, \quad F_1 = 0.7500, \quad F_2 = 0.7500.$$

The observer behavior is illustrated in Figs. 2, 3, 4, 5 and 6, with the initial conditions $x(0) = 0.001$ and $v(0) = 0.1$. Figures 2 and 3 show the system states and their estimates. Clearly, these states are well estimated. Figure 4 shows that the corresponding estimation errors do converge to zero. Figures 5 and 6 show the control input u and the unknown input h.

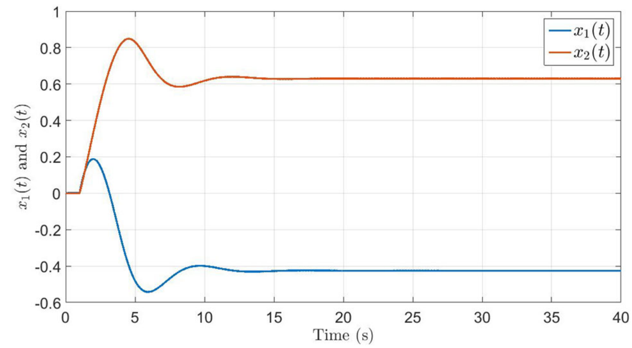

Fig. 2 Evolution of the states

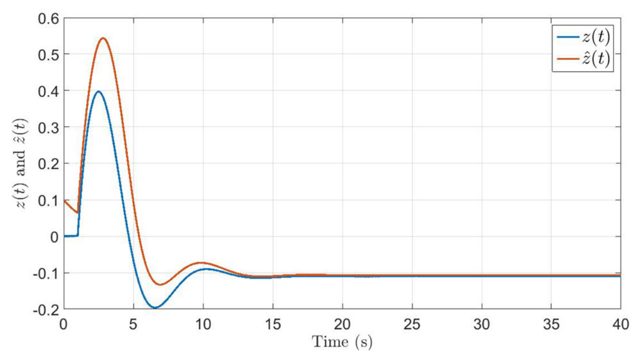

Fig. 3 Evolution of the output $z(t)$ and its estimation $\hat{z}(t)$

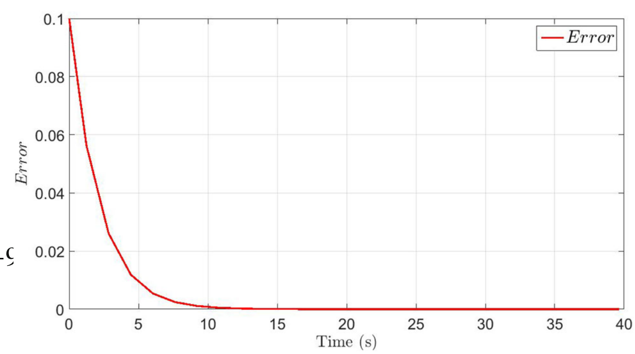

Fig. 4 Evolution of the estimation error

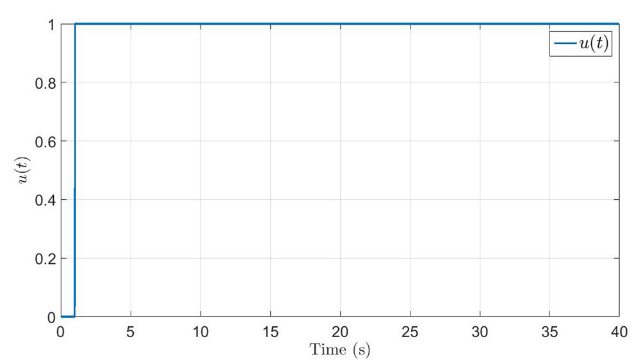

Fig. 5 The known input u

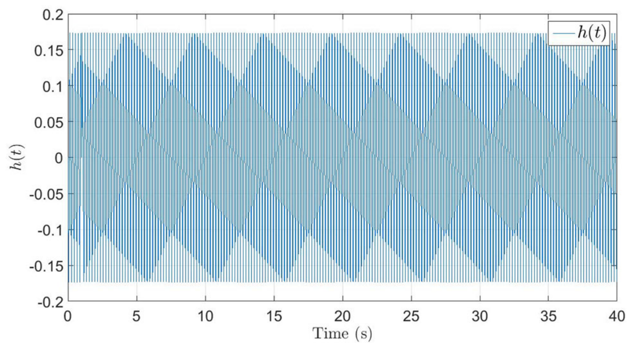

Fig. 6 The unknown input h

7 Conclusion

To conclude, our research has addressed the complex problem of designing functional observers suitable for Takagi–Sugeno systems, particularly when confronted with unknown system inputs. Through our exhaustive efforts, we have not only developed a systematic design approach, but also established strict stability conditions in the form of linear matrix inequalities (LMIs) with equality constraints. The practical implementation of our theoretical framework is facilitated by the introduction of algorithms, which effectively provide observer gains. In particular, we use the Yalmip toolbox for numerical solutions, which guarantees a robust and efficient computational process. Further highlighting the practical applicability and significance of our contributions, we have carried out extensive numerical simulations. The results obtained validate the relevance and effectiveness of our theoretical results, as embedded in Theorems 3.1, 4.1, and 5.1, as well as the practical algorithms proposed. Thanks to this dual validation process—theoretical and applied—our work not only advances the field of functional observers for Takagi–Sugeno systems, but also builds a bridge between theoretical ideas and real-world applications. The study can be extended to the case of discrete systems or fuzzy systems with time-varying delays [21, 42].

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