PDE – Math 583 Midterm Examination

Each problem is ten points

April 11, 2011

1. State the forms for the following equations:
   (a) Heat equation.
   (b) Korteweg-de Vries equation.
   (c) Minimal surface equation.
   (d) Black–Scholes equation.
2. State the local existence theorem concerning the Cauchy problem for the first-order quasilinear PDE.
3. Use the method of characteristics to solve the problem $u_x + u_y = x$, $u(x,0) = h(x)$.
4. What is a characteristic strip of the general first-order equation?
5. Use the method of characteristics to solve the problem $u_x u_y = u$, $u(0,x) = x^2$.
6. State the general form of the second-order quasilinear PDE and name and define the three classifications for this PDE.
7. Reduce the equation $u_{xx} + x^2 u_{yy} = y u_y$ for $x < 0$ to standard form.
8. Solve the equation $y^2 u_{xx} - 2xy u_{xy} + x^2 u_{yy} - \frac{y^2}{x} u_x - \frac{x^2}{y} u_y = 0$ by reducing it to standard form.
9. State d'Alembert's formula.
10. Solve $u_{tt} = u_{xx}$, $u(x,0) = 2\sin(x) - \sin(2x)$, $u_t(x,0) = 0$, $u(0,t) = u(\pi,t) = 0$.