

MISSOURI S&T MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY

Chapter 2

Probability

- 2.1 Sample Spaces and Events
- 2.2 Axioms, Interpretations, and Properties of Probability
- 2.3 Counting Techniques
- 2.4 Conditional Probability**
- 2.5 Independence

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Conditional Probability

For any two events A and B with $P(B) > 0$, the **conditional probability of A given that B has occurred** is defined by

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

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Example

400 women in a company are engineers, 100 women are not engineers, 200 men are engineers, and 800 men are not engineers. An employee is randomly chosen each month to win a prize. What is the probability that it is an engineer? If it is known that the person is a woman, what is then the probability that it is an engineer?

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Properties of Conditional Probability

1. $P(\emptyset | B) = 0$ for any event B with $P(B) > 0$
2. $P(B | B) = 1$ for any event B with $P(B) > 0$
3. $P(A' | B) = 1 - P(A | B)$ for any events A and B with $P(B) > 0$
4. $P(A \cap B) = P(A | B) \cdot P(B)$ for any events A and B with $P(B) > 0$
5. $P(A \cap B) = P(B | A) \cdot P(A)$ for any events A and B with $P(A) > 0$

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Example

Among ten bulbs there are three that do not work. We select two bulbs successively (without replacing) and test them. By F_1 and F_2 we denote the events that the first and the second bulb, respectively, does not work. Find $P(F_1)$, $P(F_2 | F_1)$, $P(F_2 | F_1')$, $P(F_2 \cap F_1)$, $P(F_2 \cap F_1')$, $P(F_2)$, and $P(F_2')$.

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Example

Assume an urn contains 4 black and 6 white balls. Two balls are selected randomly (without replacement). Let B_1, W_1, B_2, W_2 be the outcomes that the selected first ball is black, white, and the second ball is black, white, respectively. Find $P(B_1 | W_2)$ and $P(B_1 | B_2)$.

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Law of Total Probability

Let A_1, A_2, \dots, A_k be mutually exclusive and exhaustive events. Then, for any event B , we have

$$P(B) = \sum_{i=1}^k P(B | A_i)P(A_i)$$

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Bayes' Theorem

Let A_1, A_2, \dots, A_k be mutually exclusive and exhaustive events. Then, for any event B , we have

$$P(A_j | B) = \frac{P(B | A_j)P(A_j)}{\sum_{i=1}^k P(B | A_i)P(A_i)}$$

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Example

Among a group of people, 25% are vaccinated against flu. The probability that a person who is vaccinated gets the flu is 0.2. The probability that a person who is not vaccinated gets the flu is 0.3.

- Suppose a person gets the flu. What is the probability that this person is vaccinated?
- Suppose a person does not get the flu. What is the probability that this person is not vaccinated?

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