Chapter 2

Probability

Conditional Probability

For any two events A and B with P(B)>0, the conditional probability of A given that B has occurred is defined by

$$P(A\mid B)=\frac{P(A\cap B)}{P(B)}$$

Example

400 women in a company are engineers, 100 women are not engineers, 200 men are engineers, and 800 men are not engineers. An employee is randomly chosen each month to win a price. What is the probability that it is an engineer? If it is known that the person is a woman, what is then the probability that it is an engineer?

Properties of Conditional Probability

1. **$P(\emptyset\mid B)=0$ for any event B with $P(B)>0$**
2. **$P(B\mid B)=1$ for any event B with $P(B)>0$**
3. **$P(A'\mid B)=1-P(A\mid B)$ for any events A and B with $P(B)>0$**
4. **$P(A\cap B)=P(A\mid B)\cdot P(B)$ for any events A and B with $P(B)>0$**
5. **$P(A\cap B)=P(B\mid A)\cdot P(A)$ for any events A and B with $P(A)>0$**

Example

Among ten bulbs there are three that do not work. We select two bulbs successively (without replacing) and test them. By F_1 and F_2 we denote the events that the first and the second bulb, respectively, does not work. Find $P(F_1)$, $P(F_1')$, $P(F_2|F_1)$, $P(F_2'|F_1)$, $P(F_2|F_1')$, $P(F_2'|F_1')$, $P(F_2\cap F_1)$, $P(F_2\cap F_1')$, $P(F_2)$, and $P(F_2')$.

Example

Assume an urn contains 4 black and 6 white balls. Two balls are selected randomly (without replacement). Let B_1, W_1, B_2, W_2 be the outcomes that the selected first ball is black, white, and the second ball is black, white, respectively. Find $P(B_1|W_2)$ and $P(B_1|B_2)$.

Law of Total Probability

Let A_1, A_2, …, A_k be mutually exclusive and exhaustive events. Then, for any event B, we have

$$P(B) = \sum_{i=1}^{k} P(B \mid A_i)P(A_i)$$

Bayes' Theorem

Let A_1, A_2, …, A_k be mutually exclusive and exhaustive events. Then, for any event B, we have

$$P(A_j \mid B) = \frac{P(B \mid A_j)P(A_j)}{\sum_{i=1}^{k} P(B \mid A_i)P(A_i)}$$

Example

Among a group of people, 25% are vaccinated against flu. The probability that a person who is vaccinated gets the flu is 0.2. The probability that a person who is not vaccinated gets the flu is 0.3.

a) Suppose a person gets the flu. What is the probability that this person is vaccinated?

b) Suppose a person does not get the flu. What is the probability that this person is not vaccinated?