Chapter 3

Discrete Random Variables

106

Expected Value

Let X be a discrete rv with set of possible values D and pmf p(x). The expected value or mean value of X is defined by

$$E(X) = \mu_X = \mu = \sum_{x \in D} x \cdot p(x)$$

107

Example

Roll a die and let X be the number thrown.

Find the expected value of X.

108

Example

Find the expected value of a discrete uniform random variable.

Find the expected value of a Bernoulli random variable.

109

Example

Suppose the following game is offered to you: Throw a coin three times. If you have three heads, you receive $1, with two heads $0.50, and $0.20 otherwise. Introduce a random variable X describing the payoff.

Find the expected value of X.

110

Example

It is known that two out of five certain machines are not functioning properly. The machines are put in random order and tested successively until a broken machine is identified. Let the random variable X denote the number of tests necessary to identify a broken machine.

Find the expected value of X.

111

Expected Value of a Function

Let X be a discrete rv with set of possible values D and pmf p(x). The expected value of a function h(X) is

$$E(h(X)) = \mu_{h(X)} = \sum_{x\in D} h(x)\cdot p(x)$$

112

Example

A computer store has purchased three computers at $500 a piece. It will sell them for $1000 a piece. After one month, the manufacturer will repurchase any unsold computer at $200 a piece. Let X denote the number of computers sold within this month and suppose that X has pmf given by p(0)=0.1, p(1)=0.2, p(2)=0.3, p(3)=0.4.

Find the expected profit of the computer store.

113

Properties of Expectation

1. **E(aX)=aE(X) for all numbers a**
2. **E(X+b)=E(X)+b for all numbers b**
3. **E(aX+b)=aE(X)+b for all numbers a, b**

114

Variance

Let X be a discrete rv with set of possible values D and pmf p(x). The variance of X is defined by

$$V(X) = E((X-E(X))^2) = \sigma_X^2 = \sum_{x\in D}(x-\mu)^2\cdot p(x)$$

The root of the variance is called the standard deviation (SD) of X

115

Example

Roll a die and let X be the number thrown.

Find the variance of X.

116

A Shortcut Formula for the Variance

An alternative expression for the variance is

$$V(X)=E(X^2)-(E(X))^2$$

117

Example

Find the variance of a discrete uniform random variable.

Find the variance of a Bernoulli random variable.

Example

It is known that two out of five certain machines are not functioning properly. The machines are put in random order and tested successively until a broken machine is identified. Let the random variable X denote the number of tests necessary to identify a broken machine.

Find the variance of X.

Properties of the Variance

1. **$V(aX+b)=a^2V(X)$ for all numbers a, b**
2. **$V(X+b)=V(X)$ for all numbers b**
3. **$V(aX)=a^2V(X)$ for all numbers a**
4. **$\sigma_{aX+b}=|a|\sigma_X$ for all numbers a, b**
5. **$\sigma_{aX}=|a|\sigma_X$ for all numbers a**
6. **$\sigma_{X+b}=\sigma_X$ for all numbers b**

Example

A computer store has purchased three computers at $500 a piece. It will sell them for $1000 a piece. After one month, the manufacturer will repurchase any unsold computer at $200 a piece. Let X denote the number of computers sold within this month and suppose that X has pmf given by p(0)=0.1, p(1)=0.2, p(2)=0.3, p(3)=0.4.

Find the variance of the computer store's profit.