

MISSOURI S&T MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY

## Chapter 3

# Discrete Random Variables

- 3.1 Random Variables
- 3.2 Probability Distributions
- 3.3 Expected Values**
- 3.4 The Binomial Probability Distribution
- 3.5 Hypergeometric and Negative Binomial Distributions
- 3.6 The Poisson Probability Distribution

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## Expected Value

Let  $X$  be a discrete rv with set of possible values  $D$  and pmf  $p(x)$ . The **expected value** or **mean value** of  $X$  is defined by

$$E(X) = \mu_X = \mu = \sum_{x \in D} x \cdot p(x)$$

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## Example

Roll a die and let  $X$  be the number thrown.  
Find the **expected value** of  $X$ .

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## Example

Find the **expected value** of a discrete uniform random variable.

Find the **expected value** of a Bernoulli random variable.

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## Example

Suppose the following game is offered to you: Throw a coin three times. If you have three heads, you receive \$1, with two heads \$0.50, and \$0.20 otherwise. Introduce a random variable  $X$  describing the payoff.  
Find the **expected value** of  $X$ .

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## Example

It is known that two out of five certain machines are not functioning properly. The machines are put in random order and tested successively until a broken machine is identified. Let the random variable  $X$  denote the number of tests necessary to identify a broken machine.  
Find the **expected value** of  $X$ .

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### Expected Value of a Function

Let  $X$  be a discrete rv with set of possible values  $D$  and pmf  $p(x)$ . The **expected value** of a function  $h(X)$  is

$$E(h(X)) = \mu_{h(X)} = \sum_{x \in D} h(x) \cdot p(x)$$

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### Example

A computer store has purchased three computers at \$500 a piece. It will sell them for \$1000 a piece. After one month, the manufacturer will repurchase any unsold computer at \$200 a piece. Let  $X$  denote the number of computers sold within this month and suppose that  $X$  has pmf given by  $p(0)=0.1, p(1)=0.2, p(2)=0.3, p(3)=0.4$ .

**Find the expected profit of the computer store.**

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### Properties of Expectation

1.  $E(aX)=aE(X)$  for all numbers  $a$
2.  $E(X+b)=E(X)+b$  for all numbers  $b$
3.  $E(aX+b)=aE(X)+b$  for all numbers  $a, b$

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### Variance

Let  $X$  be a discrete rv with set of possible values  $D$  and pmf  $p(x)$ . The **variance** of  $X$  is defined by

$$V(X) = E((X - E(X))^2) = \sigma_X^2 = \sum_{x \in D} (x - \mu)^2 \cdot p(x)$$

The root of the variance is called the **standard deviation (SD)** of  $X$

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### Example

Roll a die and let  $X$  be the number thrown.

Find the **variance** of  $X$ .

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### A Shortcut Formula for the Variance

An alternative expression for the variance is

$$V(X) = E(X^2) - (E(X))^2$$

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### Example

Find the **variance** of a discrete uniform random variable.

Find the **variance** of a Bernoulli random variable.

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### Example

It is known that two out of five certain machines are not functioning properly. The machines are put in random order and tested successively until a broken machine is identified. Let the random variable  $X$  denote the number of tests necessary to identify a broken machine.

Find the **variance** of  $X$ .

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### Properties of the Variance

1.  $V(aX+b)=a^2V(X)$  for all numbers  $a, b$
2.  $V(X+b)=V(X)$  for all numbers  $b$
3.  $V(aX)=a^2V(X)$  for all numbers  $a$
4.  $\sigma_{aX+b}=|a|\sigma_X$  for all numbers  $a, b$
5.  $\sigma_{aX}=|a|\sigma_X$  for all numbers  $a$
6.  $\sigma_{X+b}=\sigma_X$  for all numbers  $b$

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### Example

A computer store has purchased three computers at \$500 a piece. It will sell them for \$1000 a piece. After one month, the manufacturer will repurchase any unsold computer at \$200 a piece. Let  $X$  denote the number of computers sold within this month and suppose that  $X$  has pmf given by  $p(0)=0.1, p(1)=0.2, p(2)=0.3, p(3)=0.4$ .

Find the **variance of the computer store's profit**.

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