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Chapter 4

Continuous Random Variables

- 4.1 Probability Density Functions
- 4.2 Cumulative Distribution Functions and Expected Values
- 4.3 The Normal Distribution
- 4.4 The Exponential and Gamma Distributions
- 4.5 Other Continuous Distributions**

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Weibull RV

The **Weibull** rv X with parameters $\alpha > 0$ and $\beta > 0$ is the rv with the pdf (for $x \geq 0$)

$$f(x; \alpha, \beta) = \frac{\alpha}{\beta^\alpha} x^{\alpha-1} e^{-(x/\beta)^\alpha}$$

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Expectation and Variance of the Weibull RV

If X is a Weibull rv with parameters α and β , then

$$E(X) = \beta \Gamma(1 + \alpha^{-1})$$

$$V(X) = \beta^2 \{ \Gamma(1 + 2\alpha^{-1}) - (\Gamma(1 + \alpha^{-1}))^2 \}$$

$$F(x; \alpha, \beta) = 1 - e^{-(x/\beta)^\alpha} \text{ for } x \geq 0$$

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Example

Let X denote the amount of NO_x emission (g/gal) from a randomly selected four-stroke engine of a certain type. Suppose that X has a Weibull distribution with $\alpha=2$ and $\beta=10$.

What is the probability that X is

- at most 20?
- more than 40?

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Lognormal RV

A nonnegative rv X is said to be a **lognormal** rv with parameters μ and $\sigma > 0$ if $\ln(X)$ is a normal rv with parameters μ and σ .

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Expectation and Variance of the Lognormal RV

If X is a lognormal rv with parameters μ and σ , then

$$E(X) = e^{\mu + \sigma^2/2}$$

$$V(X) = e^{2\mu + \sigma^2} (e^{\sigma^2} - 1)$$

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Example

In 6 months from today, a stock price is assumed to have a lognormal distribution with parameters $\mu=3.759$ and $\sigma=0.141$.

- Find the mean and the variance of the stock price
- Find an interval such that the probability that the stock price lies in that interval is 0.95

