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Chapter 5

Joint Probability Distributions

- 5.1 Jointly Distributed Random Variables
- 5.2 Expected Values, Covariance, and Correlation
- 5.3 Statistics and Their Distributions
- 5.4 The Distribution of the Sample Mean
- 5.5 The Distribution of a Linear Combination

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Joint pmf

The **joint probability mass function** of two discrete rvs X and Y is defined for all numbers x and y by

$$p(x,y)=P(X=x \text{ and } Y=y)$$

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Example

We throw two dice. Let X and Y denote the numbers of "3" and "4", respectively.

Find the **joint pmf** of X and Y .

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Marginal pmfs

Let X and Y be two discrete rvs with joint pmf $p(x,y)$. The **marginal probability mass functions** of X and Y are defined for all numbers x and y by

$$p_X(x)=\sum_y p(x,y) \text{ and } p_Y(y)=\sum_x p(x,y).$$

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Example

We throw two dice. Let X and Y denote the numbers of "3" and "4", respectively.

Find the **marginal pmfs** of X and Y .

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Independence

Two discrete rvs X and Y with joint pmf $p(x,y)$ and marginal pmfs $p_X(x)$ and $p_Y(y)$ are called **independent** if for every pair of x and y values, we have

$$p(x,y)=p_X(x)p_Y(y).$$

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Example

We throw two dice. Let X and Y denote the numbers of "3" and "4", respectively.

Are X and Y independent?




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Conditional pmf

Let X and Y be two discrete rvs with joint pmf $p(x,y)$ and marginal pmfs $p_X(x)$ and $p_Y(y)$. Then the conditional pmf of Y given $X=x$ is defined by

$$p_{Y|X}(y|x) = p(x,y)/p_X(x).$$


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Example

We throw two dice. Let X and Y denote the numbers of "3" and "4", respectively.

For each value of x , calculate the conditional pmf of Y when $X=x$.



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