

MISSOURI S&T MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY

Chapter 5

Joint Probability Distributions

- 5.1 Jointly Distributed Random Variables
- 5.2 Expected Values, Covariance, and Correlation**
- 5.3 Statistics and Their Distributions
- 5.4 The Distribution of the Sample Mean
- 5.5 The Distribution of a Linear Combination

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Expectation

Let X and Y be discrete rvs with joint pmf $p(x,y)$. Then the **expected value** of a function $h(X,Y)$ is defined by

$$E(h(X,Y)) = \sum_x \sum_y h(x,y)p(x,y).$$

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Example

We throw two dice. Let X and Y denote the numbers of "3" and "4", respectively.

Calculate the **expectations** of

- $X+Y$
- $(X+Y)^2$
- XY

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Covariance

The **covariance** of two discrete rvs X and Y is defined by

$$\text{Cov}(X,Y) = E((X-E(X))(Y-E(Y))).$$

The shortcut formula is

$$\text{Cov}(X,Y) = E(XY) - E(X)E(Y).$$

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Sum of Rvs

For two rvs X and Y , we have

1. $E(X+Y) = E(X) + E(Y)$
2. $V(X+Y) = V(X) + V(Y) + 2\text{Cov}(X,Y)$

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Correlation Coefficient

The **correlation coefficient** of two discrete rvs X and Y is defined by

$$\text{Corr}(X,Y) = \text{Cov}(X,Y) / (\sigma_X \sigma_Y).$$

When $\text{Corr}(X,Y) = 0$, we say X and Y are **uncorrelated**.

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Example

We throw two dice. Let X and Y denote the numbers of "3" and "4", respectively.

Calculate the **covariance** and the **correlation coefficient** of X and Y .

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Correlation

- $\text{Corr}(aX+b, cY+d) = \text{Corr}(X, Y)$ if $ac > 0$
- $-1 \leq \text{Corr}(X, Y) \leq 1$
- $|\text{Corr}(X, Y)| = 1$ iff $Y = aX + b$ with $a \neq 0$
- independent implies uncorrelated, but not necessarily vice versa.

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Example

The probabilities for husband and wife to die within a year and within a certain age group are given as follows: Both survive the year with probability 0.984, and the probability that only the husband survives equals to the probability that only the wife survives equals to 0.0078. Each, husband and wife, pay \$100 for a life insurance (value \$10,000). Let X and Y denote the net profits for the insurance company for both contracts.

- Find the joint pmf of X and Y
- Find the marginal pmfs of X and Y
- Find the pmf and the expected value of $X+Y$
- Are X and Y independent?
- Find the covariance and the correlation coefficient of X and Y

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