Electricity & Magnetism

FE Review

Voltage

$$v=\frac{dw}{dq} \qquad \frac{\text{joule}}{\text{coulomb}}=\frac{J}{C}=V \quad (volts)$$

1 V ≡ 1 J/C
1 J of work must be done to move a 1-C charge through a potential difference of 1V.

Current

$$i=\frac{dq}{dt} \qquad \frac{\text{coulomb}}{\text{second}}=\frac{C}{s}=A \quad (amperes)$$

Power

$$p=\frac{dw}{dt}=\frac{dw}{dq}\cdot\frac{dq}{dt}=vi \qquad \frac{\text{joule}}{\text{second}}=\frac{J}{s}=W \quad (watts)$$

Point Charge, Q

- Experiences a force **F**=Q**E** in the presence of electric field **E**
  (**E** is a vector with units volts/meter)
- Work done in moving Q within the **E** field

$$\mathrm{W}=-\int_0^{r_1}\mathbf{F}\bullet d\mathbf{r}$$

- **E** field at distance r from charge Q in free space

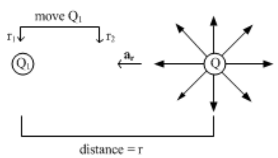
$$\mathbf{E}=\frac{Q\mathbf{a}_r}{4\pi\varepsilon_0 r^2} \qquad \varepsilon_0=8.85\times10^{-12}\ \text{F/m (permittivity of free space)}, \quad \mathbf{a}_r=\text{unit vector in direction of }\mathbf{E}$$

- Force on charge Q_1 a distance r from Q

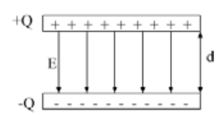
$$\mathbf{F}=Q_1\mathbf{E}=\frac{Q_1Q\mathbf{a}_r}{4\pi\varepsilon_0 r^2}$$

- Work to move Q_1 closer to Q

$$\mathrm{W}=-\int_{r_1}^{r_2}\frac{Q_1Q}{4\pi\varepsilon_0 r^2}\mathbf{a}_r\bullet\mathbf{a}_r dr=\frac{Q_1Q}{4\pi\varepsilon_0}\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$$



Parallel Plate Capacitor



Q=charge on plate
A=area of plate
ε_0=8.85x10^{-12} F/m

Electric Field between Plates

$$\mathrm{E}=\frac{Q}{\varepsilon_0 A}$$

Potential difference (voltage) between the plates

$$\mathrm{V}=\mathrm{Ed}=\frac{Qd}{\varepsilon_0 A}$$

Capacitance

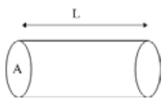
$$\mathrm{C}=\frac{\varepsilon_0 A}{d}$$

Resistivity

$$R = \frac{\rho L}{A} = \frac{L}{\sigma A}$$

L = length
A = cross-sectional area
ρ = resistivity of material
σ = conductivity of material

Example: Find the resistance of a 2-m copper wire if the wire has a diameter of 2 mm.

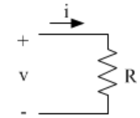


$$\rho_{Cu} = 2\times10^{-8}\,\Omega\cdot m$$

$$A = \pi r^2 = \pi(0.001)^2$$

$$R = \frac{\rho L}{A} = \frac{(2\times10^{-8}\,\Omega\cdot m)(2m)}{\pi(0.001)^2\,m^2} = 1.273\times10^{-2}\,\Omega = 12.73\ m\Omega$$

Resistor



$$v = Ri$$

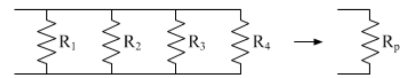
Power absorbed

$$p = vi = Ri^2 = \frac{v^2}{R}$$

Energy dissipated

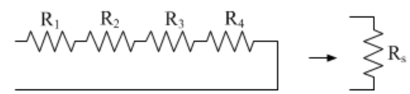
$$w = \int p\,dt = \int_{\text{one period}} p\,dt$$

Parallel Resistors



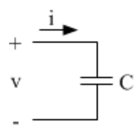
$$R_p = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}}$$

Series Resistors



$$R_s = R_1 + R_2 + R_3 + R_4$$

Capacitor



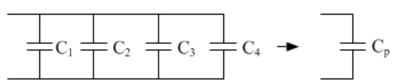
$$i = C\frac{dv}{dt}$$

$$v = \frac{1}{C}\int_{-\infty}^{t} i\,d\lambda = v(0) + \frac{1}{C}\int_{0}^{t} i\,d\lambda$$

Stores Energy

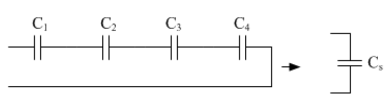
$$w = \frac{1}{2}Cv^2$$

Parallel Capacitors



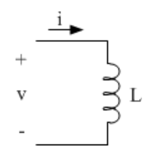
$$C_p = C_1 + C_2 + C_3 + C_4$$

Series Capacitors



$$C_s = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\frac{1}{C_4}}$$

Inductor



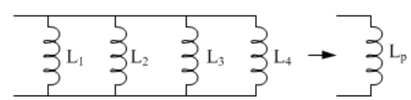
$$v = L\frac{di}{dt}$$

$$i = \frac{1}{L}\int_{-\infty}^{t} v\,d\lambda = i(0) + \frac{1}{L}\int_{0}^{t} v\,d\lambda$$

Stores Energy

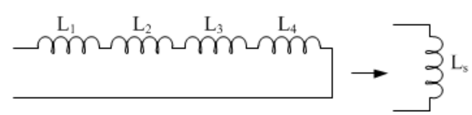
$$w = \frac{1}{2}Li^2$$

Parallel Inductors



$$L_p = \frac{1}{\frac{1}{L_1}+\frac{1}{L_2}+\frac{1}{L_3}+\frac{1}{L_4}}$$

Series Inductors



$$L_s = L_1 + L_2 + L_3 + L_4$$

KVL – Kirchhoff's Voltage Law

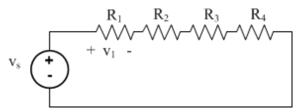
The sum of the voltage drops around a closed path is zero.

KCL – Kirchhoff's Current Law

The sum of the currents leaving a node is zero.

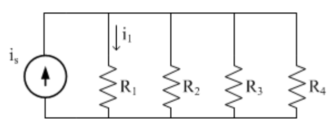
Voltage Divider

$$v_1 = \frac{R_1}{R_1 + R_2 + R_3 + R_4} v_s$$

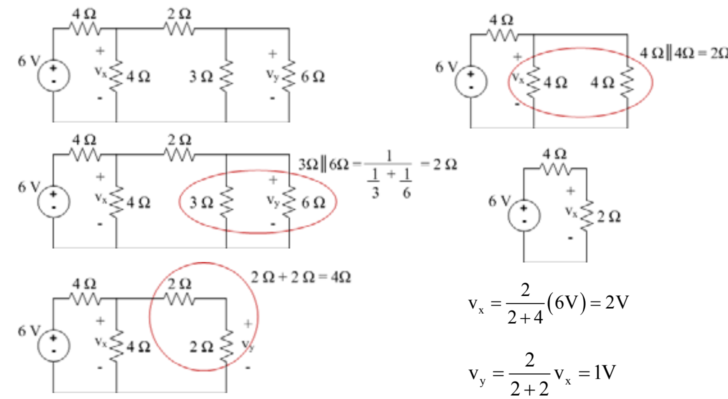


Current Divider

$$i_1 = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}} i_s$$



Example: Find v_x and v_y and the power absorbed by the 6-Ω resistor.



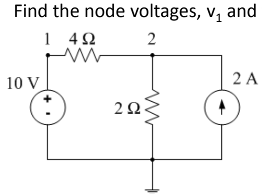
$$v_x = \frac{2}{2+4}(6\text{V}) = 2\text{V}$$

$$v_y = \frac{2}{2+2} v_x = 1\text{V}$$

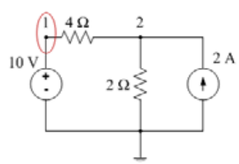
$$P_{6\Omega} = \frac{v^2}{R} = \frac{v_y^2}{6} = \frac{1}{6}\text{W}$$

Node Voltage Analysis

Find the node voltages, v_1 and v_2



Look at voltage sources first

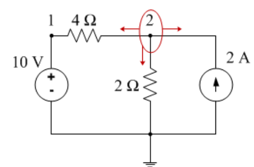


Node 1 is connected directly to ground by a voltage source → v_1 = 10 V

All nodes not connected to a voltage source are KCL equations

↓

Node 2 is a KCL equation



KCL at Node 2

$$\frac{v_2 - v_1}{4} + \frac{v_2}{2} - 2 = 0$$

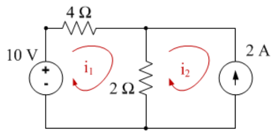
$$-\frac{1}{4}v_1 + \frac{3}{4}v_2 = 2 \quad \rightarrow \quad -v_1 + 3v_2 = 8$$

$$v_2 = \frac{8 + v_1}{3} = \frac{8+10}{3} = 6\text{V}$$

v_1 = 10 V
v_2 = 6 V

Mesh Current Analysis

Find the mesh currents i_1 and i_2 in the circuit



Look at current sources first

Mesh 2 has a current source in its outer branch

$$i_2 = -2\text{A}$$

All meshes not containing current sources are KVL equations

KVL at Mesh 1

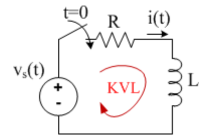
$$-10 + 4i_1 + 2\left(i_1 - i_2\right) = 0$$

$$i_1 = \frac{10 + 2i_2}{6} = \frac{10 + 2(-2)}{6} = 1\text{A}$$

Find the power absorbed by the 4-Ω resistor

$$P_{4\Omega} = i^2 R = (1)^2(4) = 4\text{W}$$

RL Circuit



$$-v_s + Ri + L\frac{di}{dt} = 0$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{1}{L}v_s$$

Put in some numbers

R = 2 Ω
L = 4 mH
$v_s(t)$ = 12 V
i(0) = 0 A

$$\frac{di}{dt} + 500i = 3000$$

$$i_n(t) = Ae^{-Rt/L} = Ae^{-500t}$$

$$i_p(t) = K$$

$$0 + 500K = 3000 \quad \rightarrow \quad K = \frac{v_s}{R} = 6$$

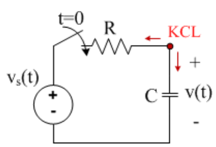
$$i(t) = Ae^{-500t} + 6$$

$$i(0) = 0 \quad \rightarrow \quad 0 = Ae^{-500(0)} + 6 \quad \rightarrow \quad A = -6$$

$$\boxed{i(t) = \frac{v_s}{R}\left(1 - e^{-Rt/L}\right) = 6\left(1 - e^{-500t}\right)\text{A}}$$

$$v(t) = L\frac{di}{dt} = 0.004\frac{d}{dt}\left(6 - 6e^{-500t}\right) = 12e^{-500t}\ \text{V}$$

RC Circuit



$$C\frac{dv}{dt} + \frac{v - v_s}{R} = 0$$

$$\frac{dv}{dt} + \frac{1}{RC}v = \frac{1}{RC}v_s$$

Put in some numbers

R = 2 Ω
C = 2 mF
$v_s(t)$ = 12 V
v(0) = 5V

$$\frac{dv}{dt} + 250v = 3000$$

$$v_n(t) = Ae^{-t/RC} = Ae^{-250t}$$

$$v_p(t) = K$$

$$0 + 250K = 3000 \quad \rightarrow \quad K = v_s = 12$$

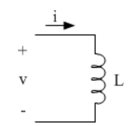
$$v(t) = Ae^{-250t} + 12$$

$$v(0) = 5 \quad \rightarrow \quad 5 = Ae^{-250(0)} + 12 \quad \rightarrow \quad A = -7$$

$$\boxed{v(t) = v_s - (v_s - v_0)e^{-t/RC} = 12 - 7e^{-250t}\ \text{V}}$$

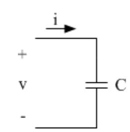
$$i(t) = C\frac{dv}{dt} = 0.002\frac{d}{dt}\left(12 - 7e^{-250t}\right) = 3.5e^{-250t}\ \text{A}$$

DC Steady-State



$$v = L\cancel{\frac{di}{dt}}^{\,0} = 0 \quad \rightarrow$$ Short Circuit

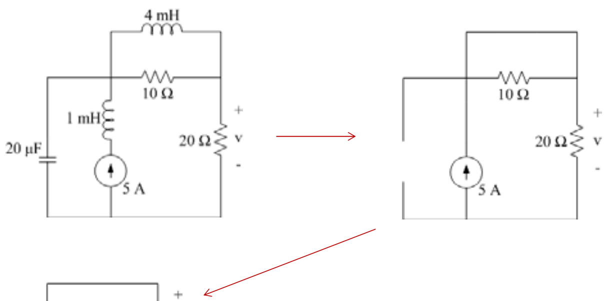
i = constant

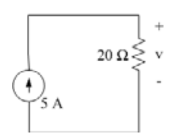


$$i = C\cancel{\frac{dv}{dt}}^{\,0} = 0 \quad \rightarrow$$ Open Circuit

v = constant

Example: Find the DC steady-state voltage, v, in the following circuit.

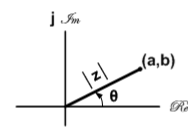




$$v = (20\ \Omega)(5\ \text{A}) = 100\ \text{V}$$

Complex Arithmetic

Rectangular	Exponential	Polar
a+jb	$Ae^{j\theta}$	$A\angle\theta$



Plot z=a+jb as an ordered pair on the real and imaginary axes

$$\tan\theta = \frac{b}{a}$$

$$|z| = |a+jb| = \sqrt{a^2+b^2}$$

Complex Conjugate

(a+jb)* = a-jb $(A\angle\theta)^* = A\angle{-\theta}$

Euler's Identity

$e^{j\theta} = \cos\theta + j\sin\theta$

Complex Arithmetic

$$z_1 = Ae^{j\theta} = A\angle\theta = a_x + ja_y$$

$$z_2 = Be^{j\varphi} = B\angle\varphi = b_x + jb_y$$

$$\frac{20\angle 40^\circ}{5\angle 60^\circ} = \frac{20}{5}\angle(40^\circ - 60^\circ) = 4\angle -20^\circ$$

Addition

$$z_1 + z_2 = (a_x + ja_y) + (b_x + jb_y) = (a_x + b_x) + j(a_y + b_y)$$

Multiplication

$$z_1 \bullet z_2 = (A\angle\theta)(B\angle\varphi) = AB\angle(\theta+\varphi)$$

Division

$$\frac{z_1}{z_2} = \frac{A\angle\theta}{B\angle\varphi} = \frac{A}{B}\angle(\theta-\varphi)$$

Phasors

A complex number representing a sinusoidal current or voltage.

$$V_m \cos(\omega t + \varphi) \rightarrow V_m\angle\varphi$$

Only for:

- Sinusoidal sources
- Steady-state

Impedance

A complex number that is the ratio of the phasor voltage and current.

$$Z = \frac{\mathbf{V}}{\mathbf{I}}$$ units = ohms (Ω)

Admittance

$$Y = \frac{\mathbf{I}}{\mathbf{V}}$$ units = Siemens (S)

Phasors

Converting from sinusoid to phasor

$20\cos(40t+15°)A \rightarrow 20\angle 15°A$

$100\cos(10^3 t)V \rightarrow 100\angle 0°V$

$6\sin(100t+10°)A = 6\cos(100t+10°-90°)A \rightarrow 6\angle -80°A$

Ohm's Law for Phasors

$\mathbf{V} = Z\mathbf{I}$

Current Divider

Ohm's Law

KVL

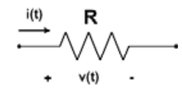
KCL

Voltage Divider

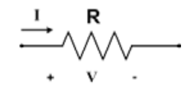
Mesh Current Analysis

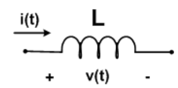
Node Voltage Analysis

Impedance

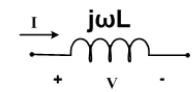


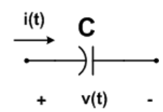
$Z = R = R\angle 0°$



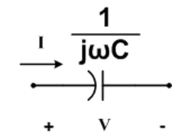


$Z = j\omega L = \omega L\angle 90°$

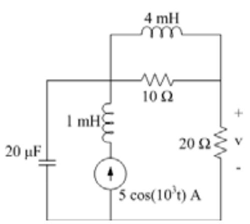


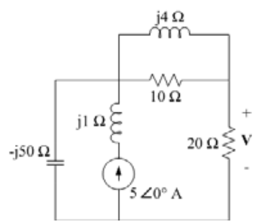


$Z = \frac{1}{j\omega C} = \frac{1}{\omega C}\angle -90°$



Example: Find the steady-state output, v(t).



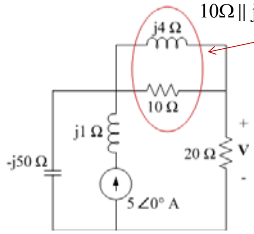


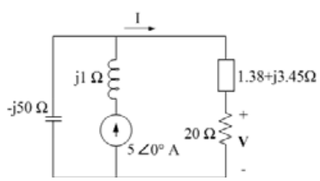
$$I = \frac{\frac{1}{20+1.38+j3.45}}{\frac{1}{-j50}+\frac{1}{20+1.38+j3.45}} 5\angle 0°$$
$$= 4.88\angle -24.67°A$$

$$V = ZI = 20(4.88\angle -24.67°)$$
$$= 97.61\angle -24.67°V$$

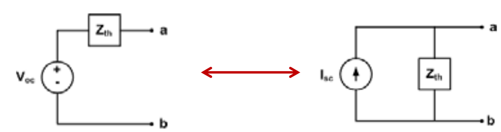
$$v(t) = 97.61\cos(10^3 t - 24.67°)V$$

$10\Omega \,||\, j4\Omega = 3.71\angle 68.20°\Omega = 1.38 + j3.45\Omega$





Source Transformations

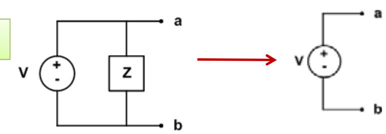


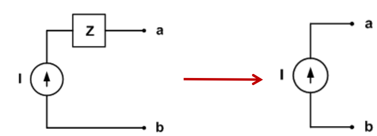
Thévenin Equivalent

Norton Equivalent

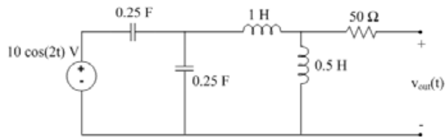
$\mathbf{V}_{oc} = \mathbf{I}_{sc} \cdot Z_{th}$

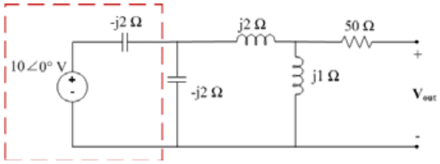
Two special cases



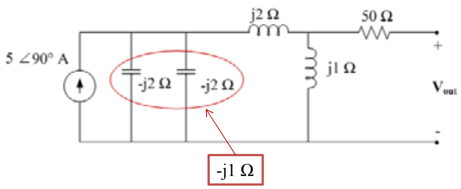


Example: Find the steady-state voltage, $v_{out}(t)$

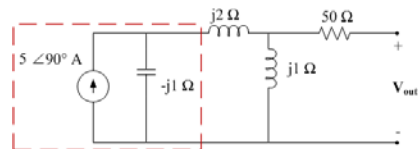




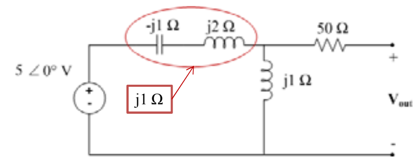
$$I = \frac{10\angle 0°}{-j2} = 5\angle 90° A$$



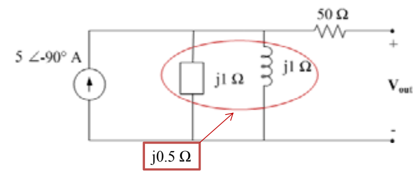
$$Z = \frac{1}{\frac{1}{-j2} + \frac{1}{-j2}} = -j1\Omega$$



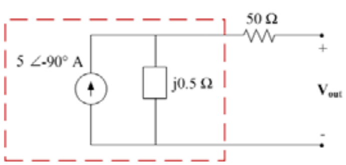
$$V = (5\angle 90°)(-j1) = 5\angle 0° V$$



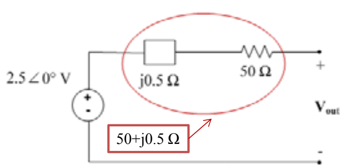
$$I = \frac{5\angle 0°}{j1} = 5\angle -90° A$$

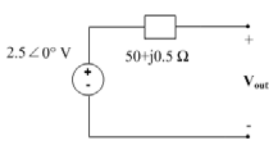


$$Z = \frac{1}{\frac{1}{j1} + \frac{1}{j1}} = j0.5\Omega$$



$$V = (5\angle -90°)(j0.5) = 2.5\angle 0° V$$





Thévenin Equivalent

No current flows through the impedance

$$\mathbf{V}_{out} = 2.5\angle 0° V$$

$$v_{out}(t) = 2.5\cos(2t) V$$

AC Power

Complex Power $\quad S = \frac{1}{2}\mathbf{VI}^* = P + jQ \quad$ units = VA (volt-amperes)

Average Power $\quad P = \frac{1}{2}V_m I_m \cos\theta \quad$ units = W (watts)

a.k.a. "Active" or "Real" Power

Reactive Power $\quad Q = \frac{1}{2}V_m I_m \sin\theta \quad$ units = VAR (volt-ampere reactive)

Power Factor $\quad PF = \cos\theta \quad$ θ = impedance angle

leading or lagging

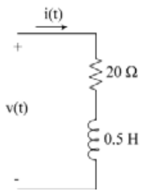
current is leading the voltage
θ<0

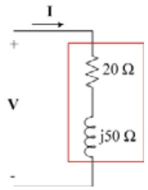
current is lagging the voltage
θ>0

Example:

v(t) = 2000 cos(100t) V

$\mathbf{V} = 2000\angle 0°\ \text{V}$





$$Z = 20 + j50\ \Omega$$
$$= 53.85\angle 68.20°\ \Omega$$

Current, **I**

$$I = \frac{V}{Z} = \frac{2000\angle 0°}{20 + j50} = 37.14\angle -68.20°\text{A}$$

Power Factor

$$PF = \cos\theta = \cos(68.20°) = 0.371 \quad \text{lagging}$$

Complex Power Absorbed

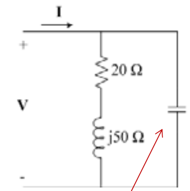
$$S = \frac{1}{2}\mathbf{V}\mathbf{I}^* = \frac{1}{2}(2000\angle 0°)(37.14\angle +68.20°)$$
$$= 37139\angle 68.20°\ \text{VA}$$
$$= 13793 + j34483\ \text{VA}$$

Average Power Absorbed

P=13793 W

Power Factor Correction

We want the power factor close to 1 to reduce the current.



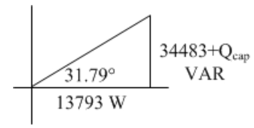
Correct the power factor to 0.85 lagging

$$\theta_{new} = \cos^{-1} 0.85 = +31.79°$$

Add a capacitor in parallel with the load.

Total Complex Power

$$S_{total} = S + S_{cap} = (13793 + j34483) + (0 + jQ_{cap})$$



$$Q_{cap} = P\tan\theta_{new} - Q = 13793\tan(31.79°) - 34483 = -24934\ \text{VAR}$$

$$Q_{cap} = -25934\ \text{VAR} \quad \rightarrow \quad S_{cap} = -j25934\ \text{VA}$$

S for an ideal capacitor

$$S = -j\frac{1}{2}\omega C V_m^2$$

v(t) = 2000 cos(100t) V

$$-j25934 = -j\frac{1}{2}(100)C(2000)^2$$

$$\boxed{C = 0.13\ \text{mF}}$$

$$S_{total} = S + S_{cap} = (13793 + j34483) + (0 - j25934)$$
$$= 13793 + j8549\ \text{VA} = 16227.5\angle 31.79°\ \text{VA}$$

Without the capacitor
$I = 37.14\angle -68.20°\text{A}$

Current after capacitor added

$$S = \frac{1}{2}\mathbf{V}\mathbf{I}^* \quad \rightarrow \quad \mathbf{I} = \left(\frac{2S}{\mathbf{V}}\right)^* = \left(\frac{2(13793 + j8549)}{2000\angle 0°}\right)^* = 16.23\angle -31.79°\ \text{A}$$

RMS Current & Voltage

a.k.a. "Effective" current or voltage

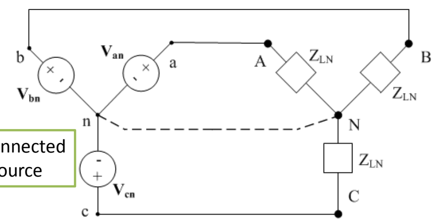
$$V_{rms} = \left(\frac{1}{T}\int_0^T v^2 dt\right)^{1/2} \qquad I_{rms} = \left(\frac{1}{T}\int_0^T i^2 dt\right)^{1/2}$$

RMS value of a sinusoid

$$A\cos(\omega t + \varphi) \quad \rightarrow \quad \frac{A}{\sqrt{2}}$$

$$\boxed{P = V_{rms} \cdot I_{rms} \cos\theta}$$

Balanced Three-Phase Systems



Y-connected source

Y-connected load

Phase Voltages

$$\mathbf{V_{an}} = V_m\angle 0°$$
$$\mathbf{V_{bn}} = V_m\angle -120°$$
$$\mathbf{V_{cn}} = V_m\angle 120°$$

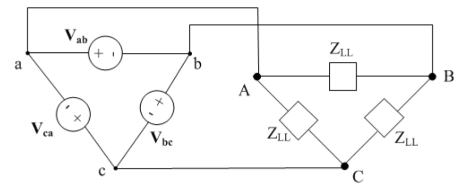
Line Currents

$$\mathbf{I_{aA}} = \mathbf{I_{AN}} = \frac{\mathbf{V_{AN}}}{Z_{LN}} = I_L\angle\varphi$$
$$\mathbf{I_{bB}} = \mathbf{I_{BN}} = \frac{\mathbf{V_{BN}}}{Z_{LN}} = I_L\angle\varphi - 120°$$
$$\mathbf{I_{cC}} = \mathbf{I_{CN}} = \frac{\mathbf{V_{CN}}}{Z_{LN}} = I_L\angle\varphi + 120°$$

Line Voltages

$$\mathbf{V_{ab}} = \mathbf{V_{an}} - \mathbf{V_{bn}} = \left(\sqrt{3}\angle 30°\right)\mathbf{V_{an}} = V_L\angle\psi$$
$$\mathbf{V_{bc}} = V_L\angle\psi - 120°$$
$$\mathbf{V_{ca}} = V_L\angle\psi + 120°$$

Balanced Three-Phase Systems



Δ-connected source

Δ-connected load

Line Voltages

$$\mathbf{V_{ab}} = V_m\angle 0°$$
$$\mathbf{V_{bc}} = V_m\angle -120°$$
$$\mathbf{V_{ca}} = V_m\angle 120°$$

Phase Currents

$$\mathbf{I_{AB}} = \frac{\mathbf{V_{AB}}}{Z_{LL}} = I_L\angle\varphi$$
$$\mathbf{I_{BC}} = \frac{\mathbf{V_{BC}}}{Z_{LL}} = I_L\angle\varphi - 120°$$
$$\mathbf{I_{CA}} = \frac{\mathbf{V_{CA}}}{Z_{LL}} = I_L\angle\varphi + 120°$$

Line Currents

$$\mathbf{I_{aA}} = \mathbf{I_{AB}} - \mathbf{I_{CA}} = \left(\sqrt{3}\angle -30°\right)\mathbf{I_{AB}} = I_L\angle\psi$$
$$\mathbf{I_{bB}} = I_L\angle\psi - 120°$$
$$\mathbf{I_{cC}} = I_L\angle\psi + 120°$$

Currents and Voltages are specified in RMS

$$S = \frac{1}{2}\mathbf{VI}^* \rightarrow S = \mathbf{VI}^* \rightarrow S = 3\mathbf{VI}^*$$

S for peak voltage & current

S for RMS voltage & current

S for 3-phase voltage & current

$$V_L = \text{line voltage} = |\mathbf{V_{ab}}|$$
$$I_L = \text{line current} = |\mathbf{I_{aA}}|$$
$$\theta = \text{impedance angle} = \angle Z$$

Average Power

$$P = \sqrt{3}V_L I_L \cos\theta$$

Complex Power for Y-connected load

$$S = 3\mathbf{V_{AN}}\mathbf{I_{AN}}^* = \sqrt{3}V_L I_L\angle\theta$$

Complex Power for Δ-connected load

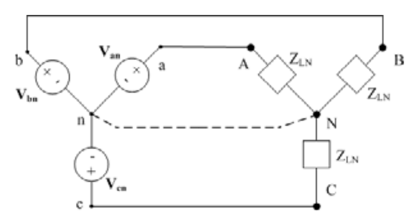
$$S = 3\mathbf{V_{AB}}\mathbf{I_{AB}}^* = \sqrt{3}V_L I_L\angle\theta$$

Power Factor

$$PF = \cos\theta \quad \text{(leading or lagging)}$$

Example:

Find the total real power supplied by the source in the balanced wye-connected circuit



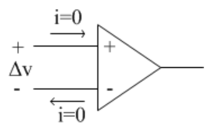
Given:

$$\mathbf{V_{an}} = 540\angle 0° \text{ V}$$
$$Z_{LN} = 270 + j270\ \Omega$$

$$\mathbf{I_{aA}} = \mathbf{I_{AN}} = \frac{\mathbf{V_{AN}}}{Z_{LN}} = \frac{540\angle 0°}{270 + j270} = 1.41\angle -45° \text{A}$$
$$S = 3\mathbf{V_{AN}}\mathbf{I_{AN}}^* = 3(540\angle 0°)(1.41\angle 45°) = 2291\angle 45° \text{VA} = 1620 + j1620 \text{VA}$$

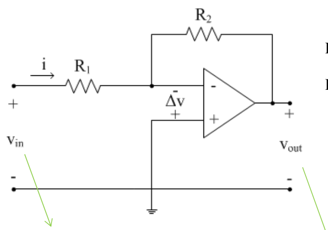
P=1620 W

Ideal Operational Amplifier (Op Amp)



With negative feedback
i=0
Δv=0

Linear Amplifier



$$\text{KVL}: \; -v_{in} + R_1 i - \overset{0}{\cancel{\Delta v}} = 0 \;\rightarrow\; i = \frac{v_{in}}{R_1}$$

$$\text{KVL}: \; -v_{in} + R_1 i + R_2 i + v_{out} = 0$$

$$-v_{in} + R_1\left(\frac{v_{in}}{R_1}\right) + R_2\left(\frac{v_{in}}{R_1}\right) + v_{out} = 0$$

$$\boxed{v_{out} = -\frac{R_2}{R_1} v_{in}}$$

mV or µV reading from sensor

0-5 V output to A/D converter

Magnetic Fields

$$\oint_S \mathbf{B} \bullet d\mathbf{s} = 0 \qquad \oint_l \mathbf{H} \bullet d\boldsymbol{l} = \oint_S \mathbf{J} \bullet d\mathbf{S} = I$$

Net magnetic flux through a closed surface is zero.

B – magnetic flux density (tesla)
H – magnetic field strength (A/m)
J - current density

$$\mathbf{B} = \mu \mathbf{H}$$

Magnetic Flux ϕ passing through a surface

$$\phi = \int_S \mathbf{B} \bullet d\mathbf{S}$$

Energy stored in the magnetic field

$$w = \frac{1}{2}\iiint_V \mu |H|^2 dV$$

Enclosing a surface with N turns of wire produces a voltage across the terminals

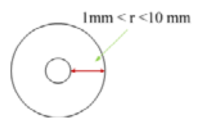
$$v = -N\frac{d\phi}{dt}$$

Magnetic field produces a force perpendicular to the current direction and the magnetic field direction

$$\mathbf{F} = I\mathbf{L} \times \mathbf{B}$$

Example:

A coaxial cable with an inner wire of radius 1 mm carries 10-A current. The outer cylindrical conductor has a diameter of 10 mm and carries a 10-A uniformly distributed current in the opposite direction. Determine the approximate magnetic energy stored per unit length in this cable. Use μ_0 for the permeablility of the material between the wire and conductor.



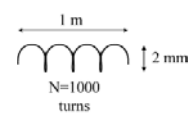
$$\int \mathbf{H} \bullet d\boldsymbol{l} = H_\varphi 2\pi r = I_0 = 10 \text{ A}$$

$$H_\varphi = \frac{10}{2\pi r} \qquad \text{for} \qquad 10^{-3}\text{m} < r < 10^{-2}\text{m}$$

$$w = \frac{1}{2}\iiint_V \mu |H|^2 dV = \frac{1}{2}\int_0^1\int_0^{2\pi}\int_{0.001}^{0.01} \mu_0 \left(\frac{10}{2\pi r}\right)^2 r\,dr\,d\varphi\,dz = 18.3\mu_0 \text{ J}$$

Example:

A cylindrical coil of wire has an air core and 1000 turns. It is 1 m long with a diameter of 2 mm so has a relatively uniform field. Find the current necessary to achieve a magnetic flux density of 2 T.



$$\int \mathbf{H} \bullet d\boldsymbol{l} = NI_0$$

$$HL = NI_0$$

$$\left(\frac{B}{\mu_0}\right) L = NI_0 \;\rightarrow\; I = \frac{BL}{N\mu_0} = \frac{(2T)(1m)}{1000\left(4\pi \times 10^{-7}\right)} = 1590 \text{ A}$$

Questions?