Electricity & Magnetism

FE Review

Voltage

$$v = \frac{dw}{dq} \qquad \frac{\text{joule}}{\text{coulomb}} = \frac{J}{C} = V \quad (volts)$$

1 V ≡ 1 J/C
1 J of work must be done to move a 1-C charge through a potential difference of 1V.

Current

$$i = \frac{dq}{dt} \qquad \frac{\text{coulomb}}{\text{second}} = \frac{C}{s} = A \quad (amperes)$$

Power

$$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = vi \qquad \frac{\text{joule}}{\text{second}} = \frac{J}{s} = W \quad (watts)$$

Point Charge, Q

- Experiences a force **F**=Q**E** in the presence of electric field **E**
  (**E** is a vector with units volts/meter)

- Work done in moving Q within the **E** field

$$W = -\int_0^{r_1} \mathbf{F} \bullet d\mathbf{r}$$

- **E** field at distance r from charge Q in free space

$$\mathbf{E} = \frac{Q\mathbf{a}_r}{4\pi\varepsilon_0 r^2}$$

$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m (permittivity of free space)

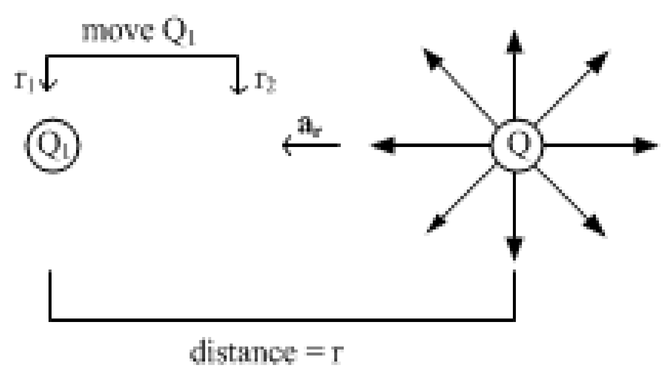
$\mathbf{a}_r$ = unit vector in direction of **E**

- Force on charge Q_1 a distance r from Q

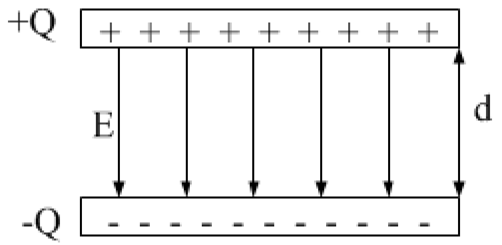
$$\mathbf{F} = Q_1\mathbf{E} = \frac{Q_1 Q\mathbf{a}_r}{4\pi\varepsilon_0 r^2}$$

- Work to move Q_1 closer to Q

$$W = -\int_{r_1}^{r_2} \frac{Q_1 Q}{4\pi\varepsilon_0 r^2}\mathbf{a}_r \bullet \mathbf{a}_r dr = \frac{Q_1 Q}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$



Parallel Plate Capacitor



Q=charge on plate
A=area of plate
$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m

Electric Field between Plates

$$E = \frac{Q}{\varepsilon_0 A}$$

Potential difference (voltage) between the plates

$$V = Ed = \frac{Qd}{\varepsilon_0 A}$$

Capacitance

$$C = \frac{\varepsilon_0 A}{d}$$

Resistivity

$$R = \frac{\rho L}{A} = \frac{L}{\sigma A}$$

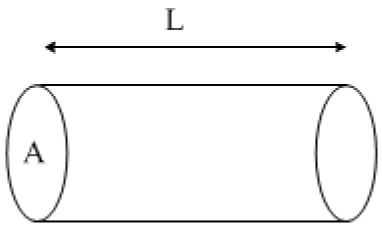
L = length

A = cross-sectional area

ρ = resistivity of material

σ = conductivity of material

Example: Find the resistance of a 2-m copper wire if the wire has a diameter of 2 mm.

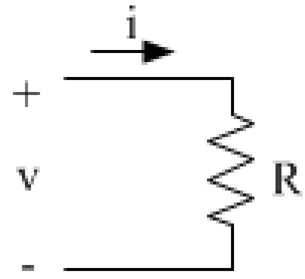


$$\rho_{Cu} = 2 \times 10^{-8}\,\Omega \cdot m$$

$$A = \pi r^2 = \pi (0.001)^2$$

$$R = \frac{\rho L}{A} = \frac{(2 \times 10^{-8}\,\Omega \cdot m)(2m)}{\pi (0.001)^2\, m^2} = 1.273 \times 10^{-2}\,\Omega = 12.73\ m\Omega$$

Resistor



$$v = Ri$$

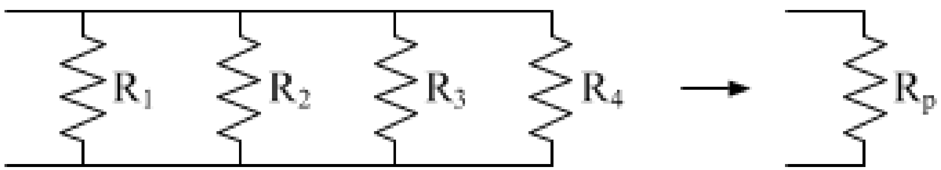
Power absorbed

$$p = vi = Ri^2 = \frac{v^2}{R}$$

Energy dissipated

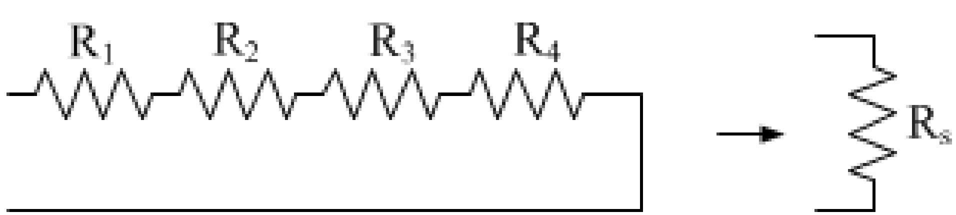
$$w = \int p \, dt = \int_{\text{one period}} p \, dt$$

Parallel Resistors



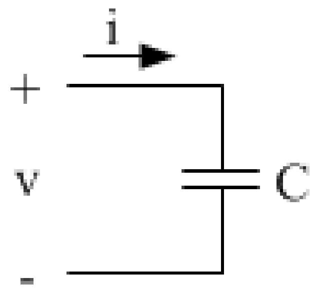
$$R_p = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}}$$

Series Resistors



$$R_s = R_1 + R_2 + R_3 + R_4$$

Capacitor



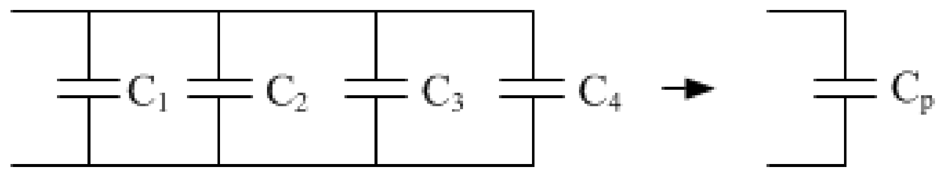
$$i = C\frac{dv}{dt}$$

$$v = \frac{1}{C}\int_{-\infty}^{t} i \, d\lambda = v(0) + \frac{1}{C}\int_{0}^{t} i \, d\lambda$$

Stores Energy

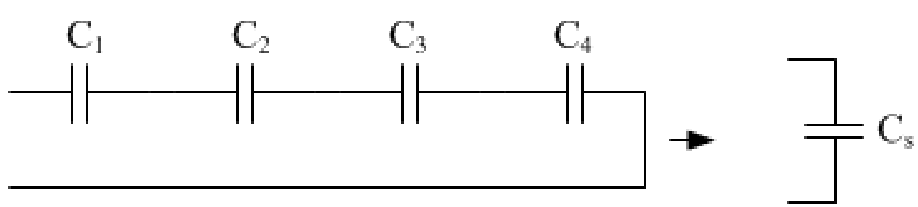
$$w = \frac{1}{2}Cv^2$$

Parallel Capacitors



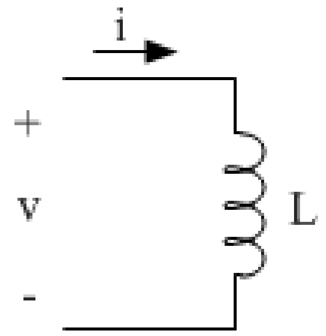
$$C_p = C_1 + C_2 + C_3 + C_4$$

Series Capacitors



$$C_s = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4}}$$

Inductor



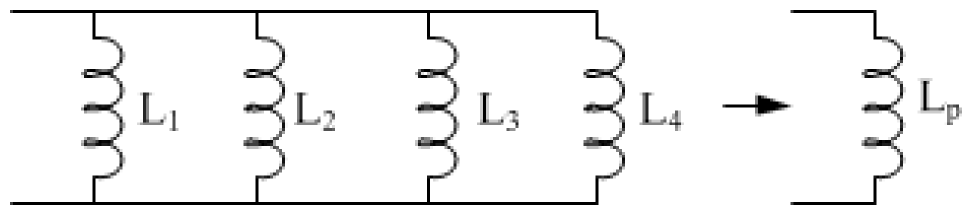
$$v = L\frac{di}{dt}$$

$$i = \frac{1}{L}\int_{-\infty}^{t} v\, d\lambda = i(0) + \frac{1}{L}\int_{0}^{t} v\, d\lambda$$

Stores Energy

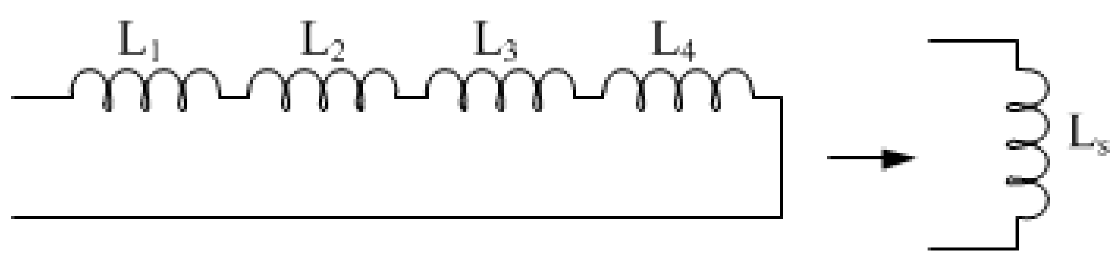
$$w = \frac{1}{2}Li^2$$

Parallel Inductors



$$L_p = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \frac{1}{L_4}}$$

Series Inductors



$$L_s = L_1 + L_2 + L_3 + L_4$$

KVL – Kirchhoff's Voltage Law

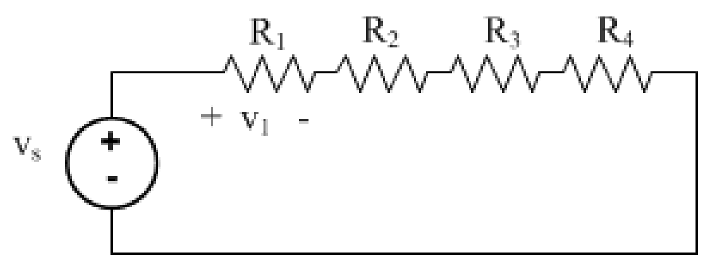
The sum of the voltage drops around a closed path is zero.

KCL – Kirchhoff's Current Law

The sum of the currents leaving a node is zero.

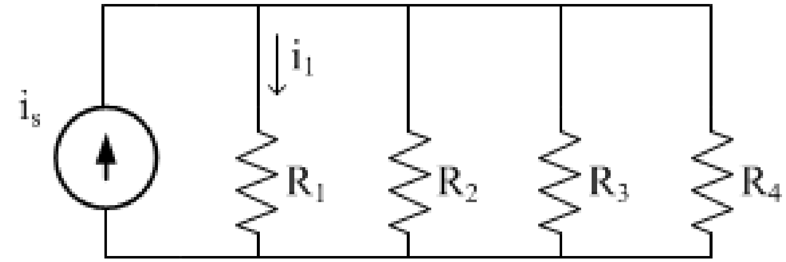
Voltage Divider

$$v_1 = \frac{R_1}{R_1 + R_2 + R_3 + R_4} v_s$$

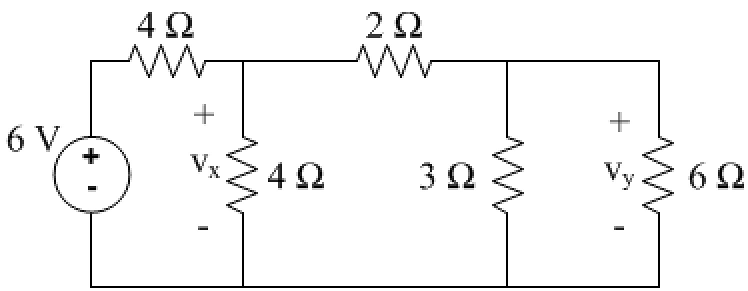


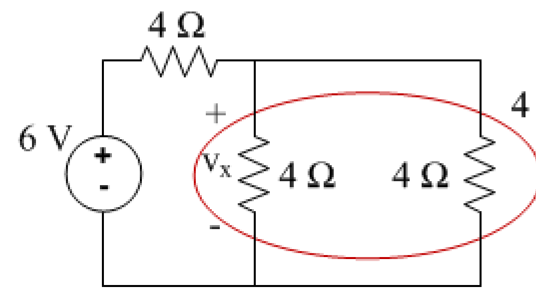
Current Divider

$$i_1 = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}} i_s$$

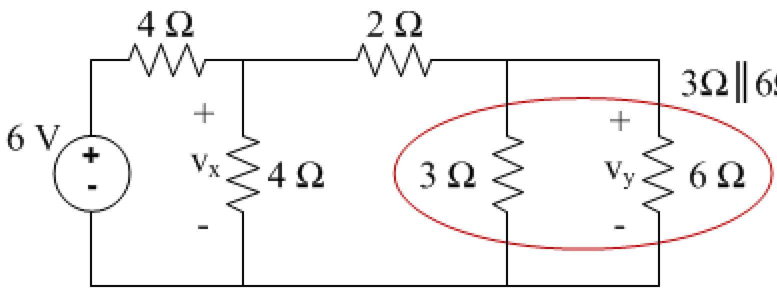


Example: Find v_x and v_y and the power absorbed by the 6-Ω resistor.

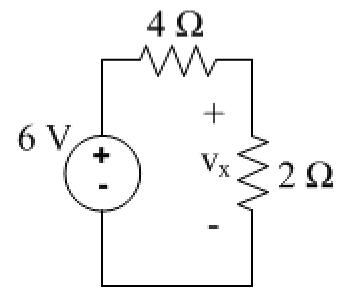


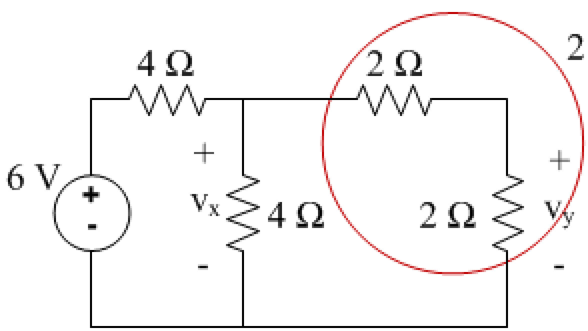


$$4\,\Omega \parallel 4\Omega = 2\Omega$$



$$3\Omega \parallel 6\Omega = \frac{1}{\frac{1}{3} + \frac{1}{6}} = 2\,\Omega$$





$$2\,\Omega + 2\,\Omega = 4\Omega$$

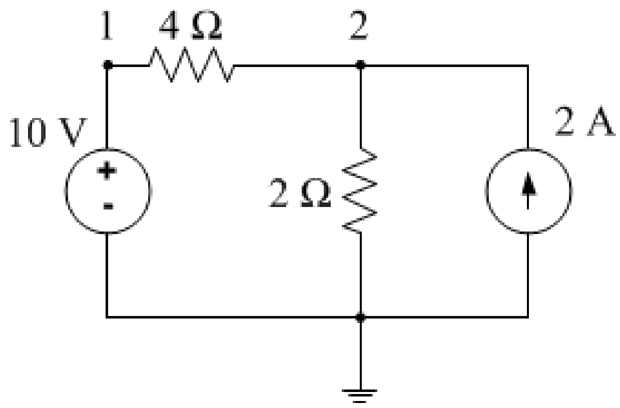
$$v_x = \frac{2}{2+4}\left(6\,\text{V}\right) = 2\,\text{V}$$

$$v_y = \frac{2}{2+2} v_x = 1\text{V}$$

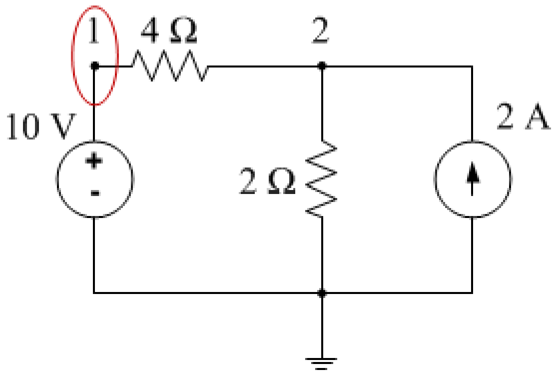
$$P_{6\Omega} = \frac{v^2}{R} = \frac{v_y^2}{6} = \frac{1}{6}\,\text{W}$$

Node Voltage Analysis

Find the node voltages, v_1 and v_2



Look at voltage sources first

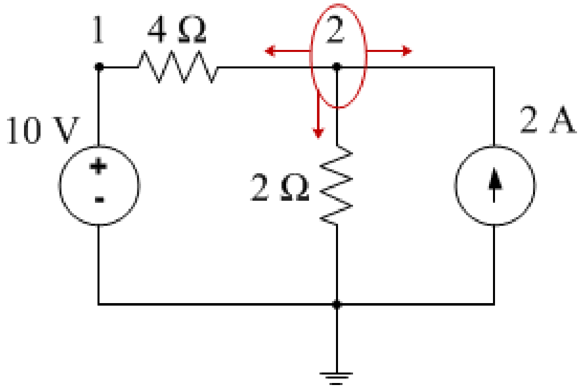


Node 1 is connected directly to ground by a voltage source → v_1 = 10 V

All nodes not connected to a voltage source are KCL equations

↓

Node 2 is a KCL equation



KCL at Node 2

$$\frac{v_2 - v_1}{4} + \frac{v_2}{2} - 2 = 0$$

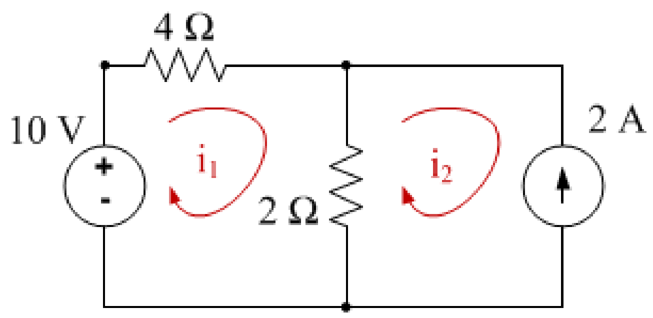
$$-\frac{1}{4}v_1 + \frac{3}{4}v_2 = 2 \quad \rightarrow \quad -v_1 + 3v_2 = 8$$

$$v_2 = \frac{8 + v_1}{3} = \frac{8+10}{3} = 6\,\text{V}$$

$v_1 = 10$ V
$v_2 = 6$ V

Mesh Current Analysis

Find the mesh currents i_1 and i_2 in the circuit



Look at current sources first

Mesh 2 has a current source in its outer branch

$$i_2 = -2A$$

All meshes not containing current sources are KVL equations

KVL at Mesh 1

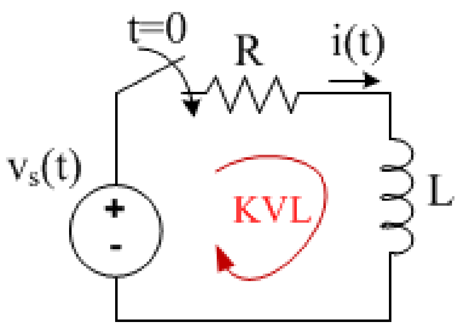
$$-10 + 4i_1 + 2(i_1 - i_2) = 0$$

$$i_1 = \frac{10 + 2i_2}{6} = \frac{10 + 2(-2)}{6} = 1A$$

Find the power absorbed by the 4-Ω resistor

$$P_{4\Omega} = i^2R = (1)^2(4) = 4W$$

RL Circuit



$$-v_s + Ri + L\frac{di}{dt} = 0$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{1}{L}v_s$$

Put in some numbers

R = 2 Ω
L = 4 mH
$v_s(t)$ = 12 V
i(0) = 0 A

$$\frac{di}{dt} + 500i = 3000$$

$$i_n(t) = Ae^{-Rt/L} = Ae^{-500t}$$

$$i_p(t) = K$$

$$0 + 500K = 3000 \quad \rightarrow \quad K = \frac{v_s}{R} = 6$$

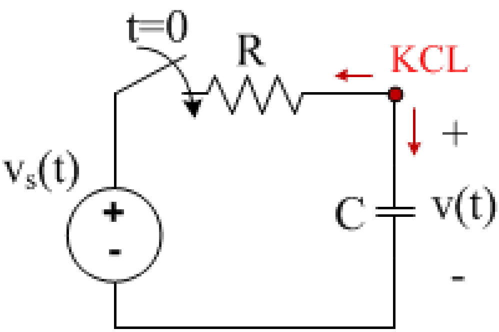
$$i(t) = Ae^{-500t} + 6$$

$$i(0) = 0 \quad \rightarrow \quad 0 = Ae^{-500(0)} + 6 \quad \rightarrow \quad A = -6$$

$$\boxed{i(t) = \frac{v_s}{R}\left(1 - e^{-Rt/L}\right) = 6\left(1 - e^{-500t}\right) \text{A}}$$

$$v(t) = L\frac{di}{dt} = 0.004\frac{d}{dt}\left(6 - 6e^{-500t}\right) = 12e^{-500t} \text{ V}$$

RC Circuit



$$C\frac{dv}{dt} + \frac{v - v_s}{R} = 0$$

$$\frac{dv}{dt} + \frac{1}{RC}v = \frac{1}{RC}v_s$$

Put in some numbers

R = 2 Ω
C = 2 mF
$v_s(t)$ = 12 V
v(0) = 5V

$$\frac{dv}{dt} + 250v = 3000$$

$$v_n(t) = Ae^{-t/RC} = Ae^{-250t}$$

$$v_p(t) = K$$

$$0 + 250K = 3000 \quad \rightarrow \quad K = v_s = 12$$

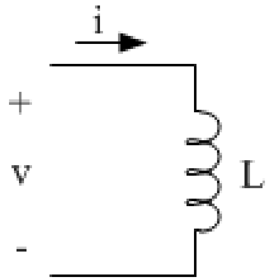
$$v(t) = Ae^{-250t} + 12$$

$$v(0) = 5 \quad \rightarrow \quad 5 = Ae^{-250(0)} + 12 \quad \rightarrow \quad A = -7$$

$$\boxed{v(t) = v_s - (v_s - v_0)e^{-t/RC} = 12 - 7e^{-250t} \ \text{V}}$$

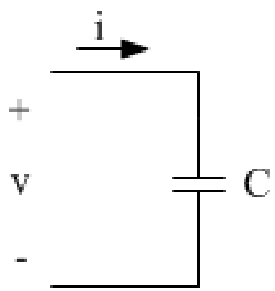
$$i(t) = C\frac{dv}{dt} = 0.002\frac{d}{dt}\left(12 - 7e^{-250t}\right) = 3.5e^{-250t} \ \text{A}$$

DC Steady-State



$$v = L\cancelto{0}{\frac{di}{dt}} = 0 \longrightarrow \boxed{\text{Short Circuit}}$$

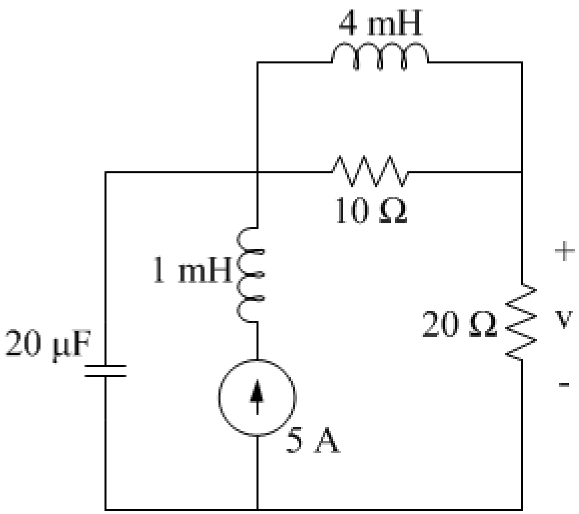
i = constant

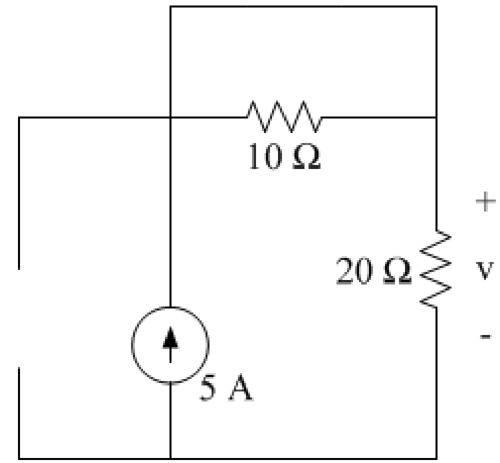


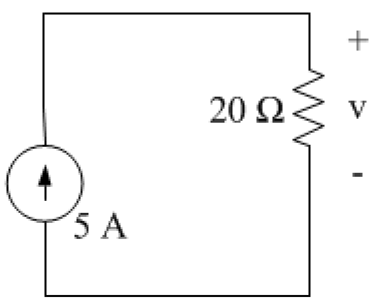
$$i = C\cancelto{0}{\frac{dv}{dt}} = 0 \longrightarrow \boxed{\text{Open Circuit}}$$

v = constant

Example: Find the DC steady-state voltage, v, in the following circuit.



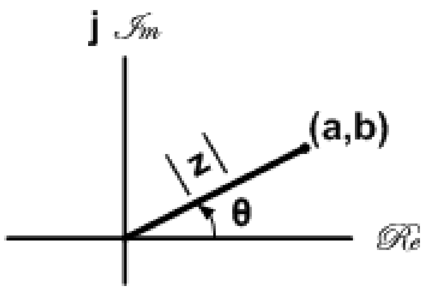




$$v = (20\ \Omega)(5\ \mathrm{A}) = 100\ \mathrm{V}$$

Complex Arithmetic

Rectangular	Exponential	Polar
$a+jb$	$Ae^{j\theta}$	$A\angle\theta$



Plot z=a+jb as an ordered pair on the real and imaginary axes

$$\tan\theta = \frac{b}{a}$$

$$|z| = |a+jb| = \sqrt{a^2+b^2}$$

Complex Conjugate

$(a+jb)^* = a-jb$ $\quad$ $(A\angle\theta)^* = A\angle-\theta$

Euler's Identity

$$e^{j\theta} = \cos\theta + j\sin\theta$$

Complex Arithmetic

$$\frac{20\angle 40^\circ}{5\angle 60^\circ} = \frac{20}{5}\angle\left(40^\circ - 60^\circ\right)$$
$$= 4\angle -20^\circ$$

$$z_1 = Ae^{j\theta} = A\angle\theta = a_x + ja_y$$

$$z_2 = Be^{j\varphi} = B\angle\varphi = b_x + jb_y$$

Addition

$$z_1 + z_2 = \left(a_x + ja_y\right) + \left(b_x + jb_y\right) = \left(a_x + b_x\right) + j\left(a_y + b_y\right)$$

Multiplication

$$z_1 \bullet z_2 = \left(A\angle\theta\right)\left(B\angle\varphi\right)$$
$$= AB\angle\left(\theta + \varphi\right)$$

Division

$$\frac{z_1}{z_2} = \frac{A\angle\theta}{B\angle\varphi}$$
$$= \frac{A}{B}\angle\left(\theta - \varphi\right)$$

Phasors

A complex number representing a sinusoidal current or voltage.

$$V_m \cos(\omega t + \varphi) \quad \rightarrow \quad V_m \angle \varphi$$

<u>Only for</u>:

- Sinusoidal sources
- Steady-state

Impedance

A complex number that is the ratio of the phasor voltage and current.

$$Z = \frac{\mathbf{V}}{\mathbf{I}}$$

units = ohms (Ω)

Admittance

$$Y = \frac{\mathbf{I}}{\mathbf{V}}$$

units = Siemens (S)

Phasors

Converting from sinusoid to phasor

$$20\cos(40t + 15°)\,A \quad \rightarrow \quad 20\angle 15°A$$

$$100\cos(10^3 t)\,V \quad \rightarrow \quad 100\angle 0°V$$

$$6\sin(100t + 10°)\,A = 6\cos(100t + 10° - 90°)A \quad \rightarrow \quad 6\angle -80°A$$

Ohm's Law for Phasors

$$\mathbf{V} = Z\mathbf{I}$$

Current Divider

Ohm's Law

KVL

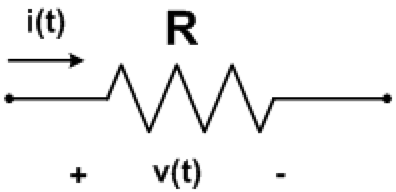
KCL

Voltage Divider

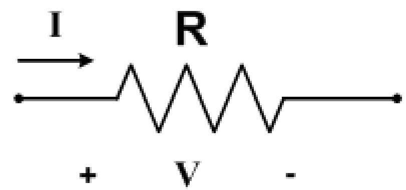
Mesh Current Analysis

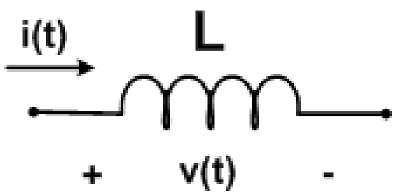
Node Voltage Analysis

Impedance

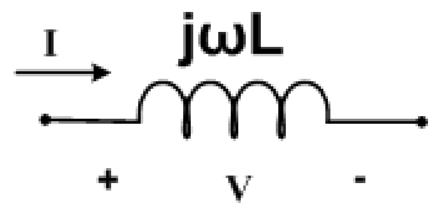


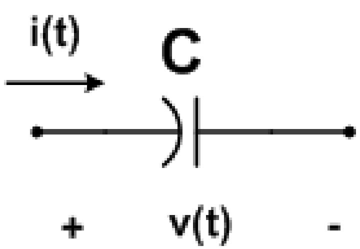
$$Z = R = R\angle 0°$$



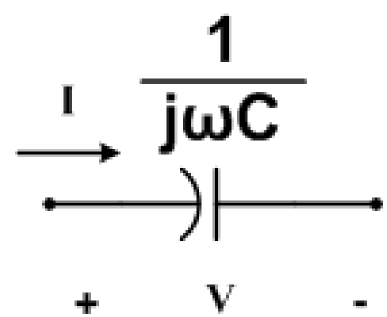


$$Z = j\omega L = \omega L\angle 90°$$

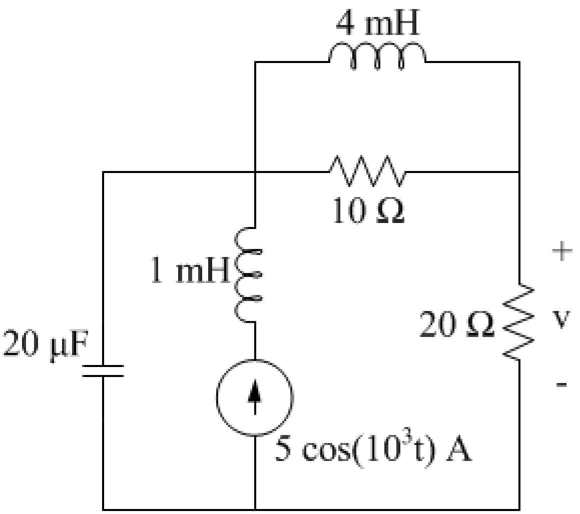


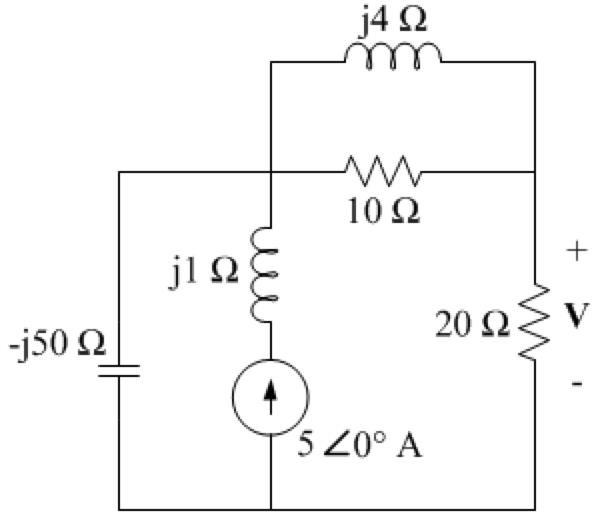


$$Z = \frac{1}{j\omega C} = \frac{1}{\omega C}\angle -90°$$



Example: Find the steady-state output, v(t).





$$I = \frac{\dfrac{1}{20+1.38+j3.45}}{\dfrac{1}{-j50}+\dfrac{1}{20+1.38+j3.45}}5\angle 0°$$

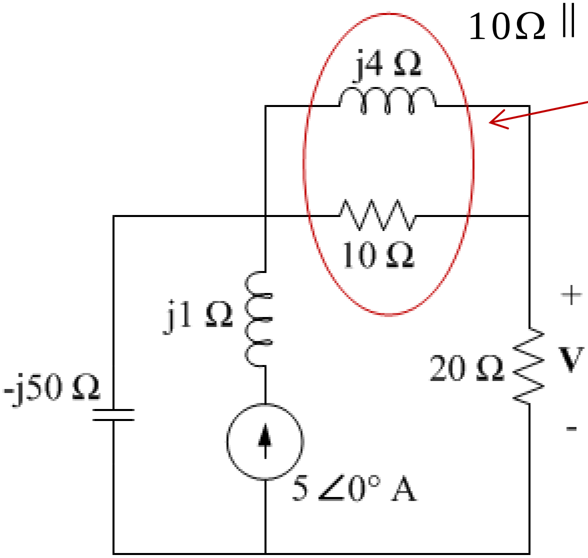
$$= 4.88\angle -24.67°\text{A}$$

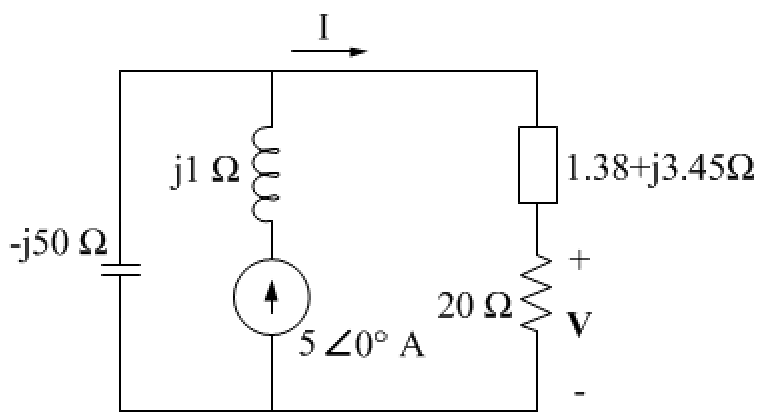
$$V = ZI = 20\left(4.88\angle -24.67°\right)$$

$$= 97.61\angle -24.67°\text{V}$$

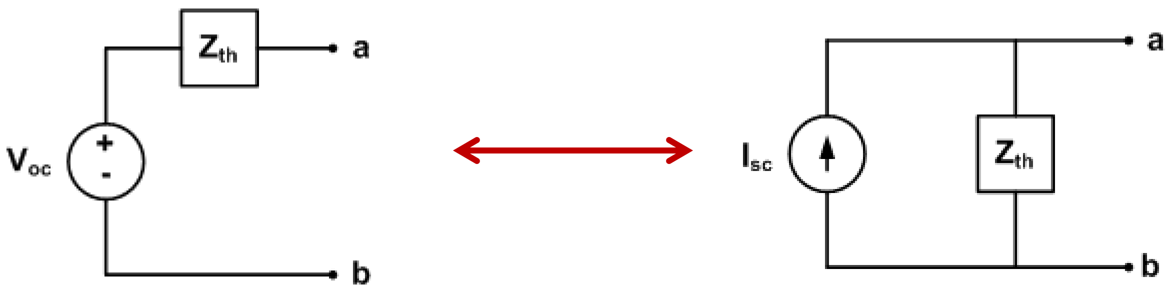
$$v(t) = 97.61\cos(10^3 t - 24.67°)\text{V}$$

$$10\Omega \parallel j4\Omega = 3.71\angle 68.20°\Omega = 1.38 + j3.45\Omega$$





Source Transformations

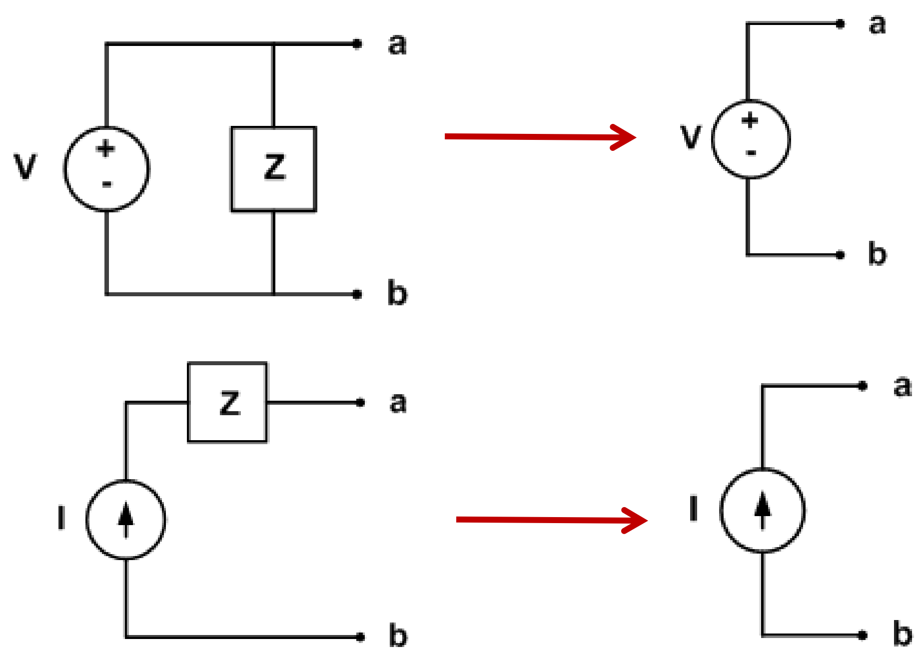


Thévenin Equivalent

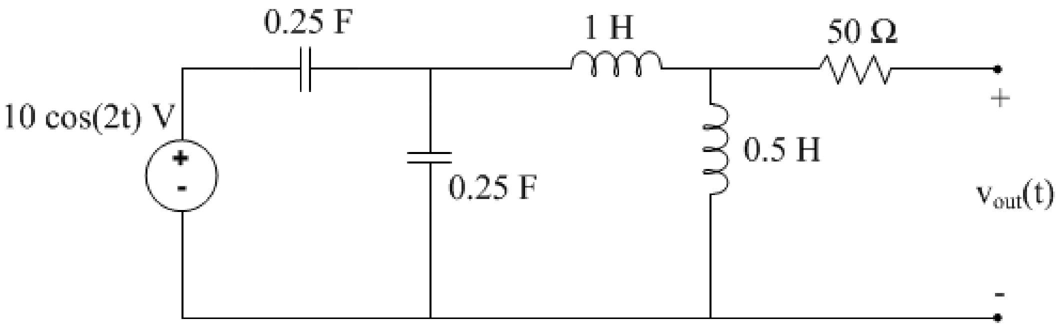
Norton Equivalent

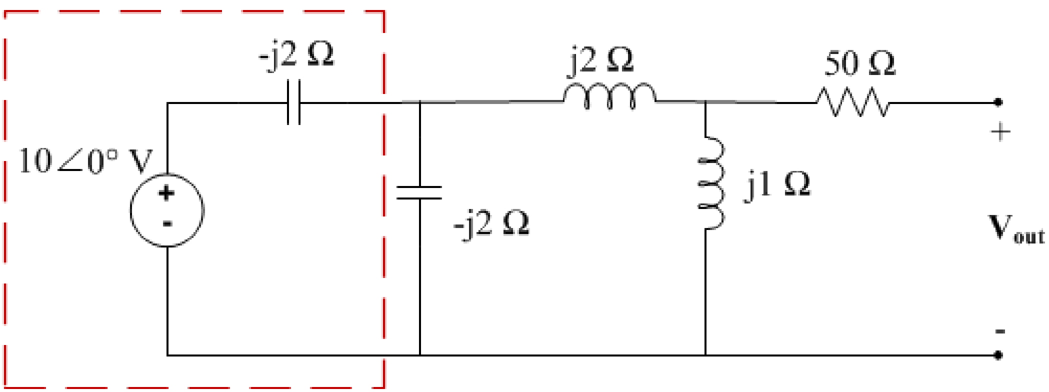
$$\mathbf{V}_{\mathbf{oc}} = \mathbf{I}_{\mathbf{sc}} \cdot Z_{th}$$

Two special cases

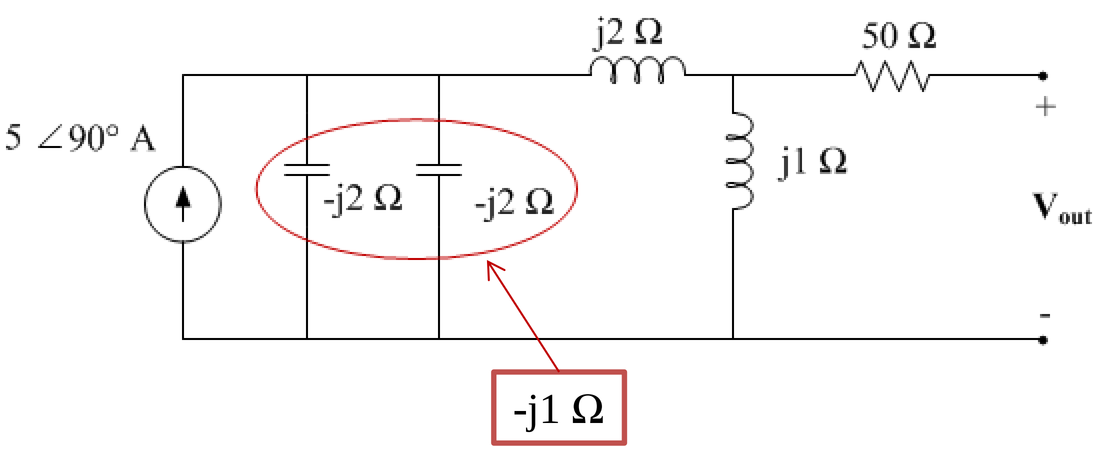


Example: Find the steady-state voltage, $v_{out}(t)$

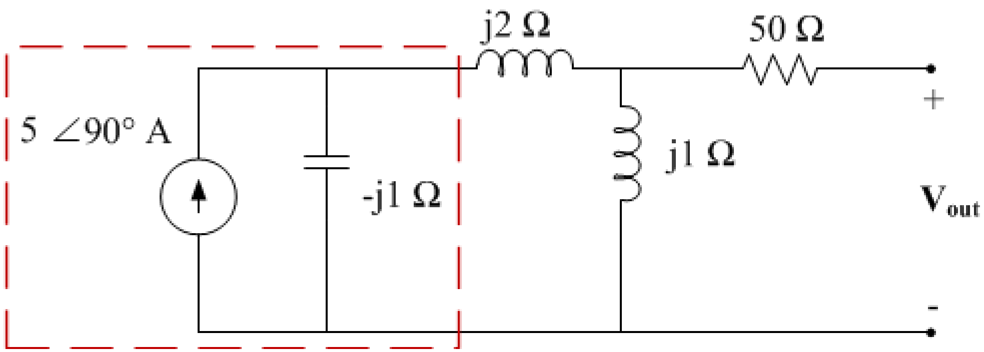




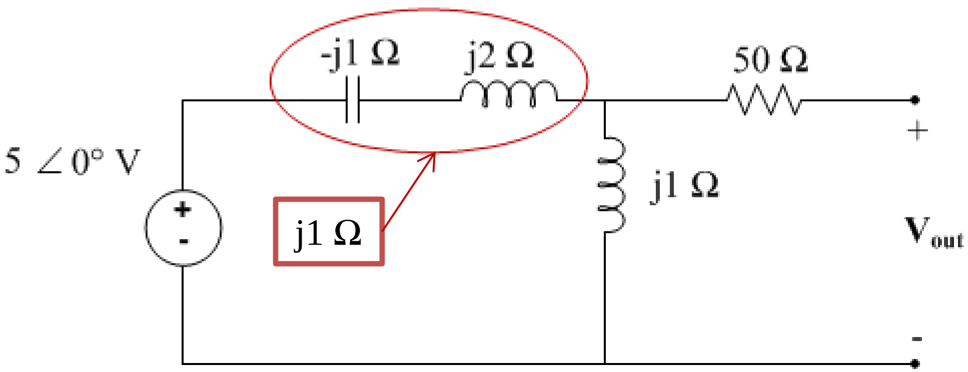
$$I = \frac{10\angle 0^\circ}{-j2} = 5\angle 90^\circ A$$



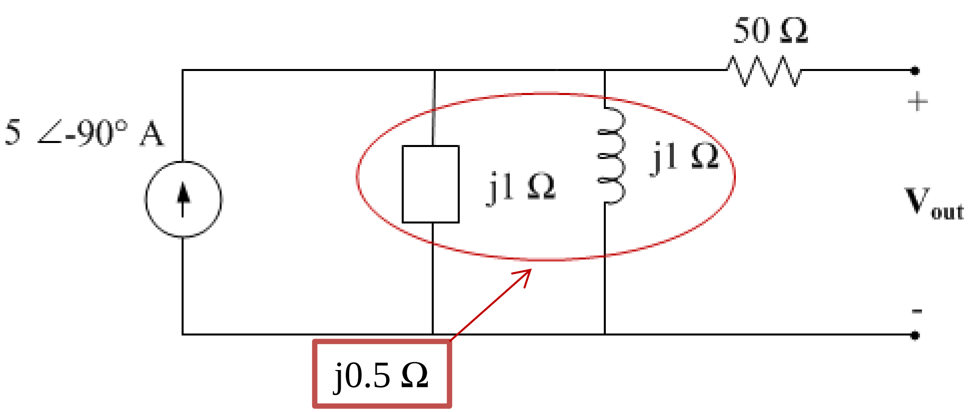
$$Z = \frac{1}{\frac{1}{-j2} + \frac{1}{-j2}} = -j1\Omega$$



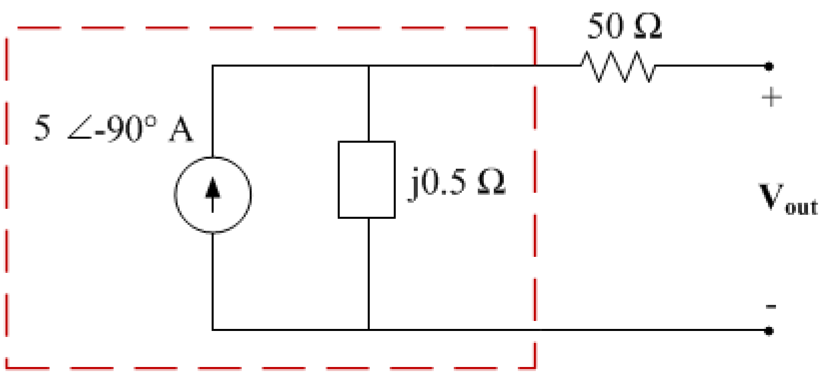
$$V = (5\angle 90^\circ)(-j1) = 5\angle 0^\circ \text{V}$$



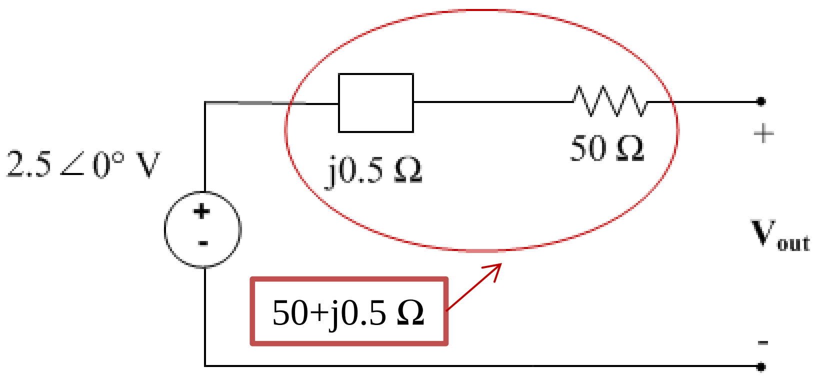
$$I = \frac{5\angle 0^\circ}{j1} = 5\angle -90^\circ \text{A}$$

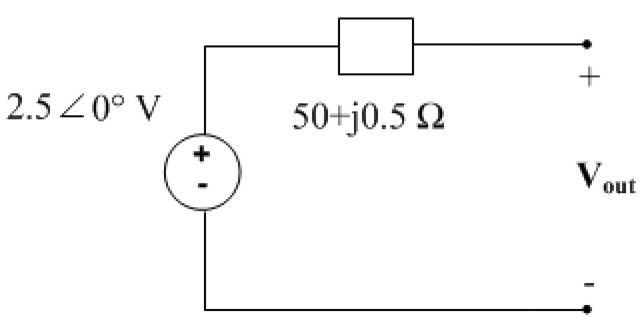


$$Z = \frac{1}{\frac{1}{j1}+\frac{1}{j1}} = j0.5\Omega$$



$$V = (5\angle -90°)(j0.5) = 2.5\angle 0°V$$





Thévenin Equivalent

No current flows through the impedance

$$\mathbf{V_{out}} = 2.5\angle 0°V$$

$$v_{out}(t) = 2.5\cos(2t)V$$

AC Power

Complex Power

$$S = \frac{1}{2}\mathbf{V}\mathbf{I}^* = P + jQ$$

units = VA (volt-amperes)

Average Power

$$P = \frac{1}{2}V_m I_m \cos\theta$$

units = W (watts)

a.k.a. “Active” or “Real” Power

Reactive Power

$$Q = \frac{1}{2}V_m I_m \sin\theta$$

units = VAR
(volt-ampere reactive)

Power Factor

$$PF = \cos\theta$$

θ = impedance angle

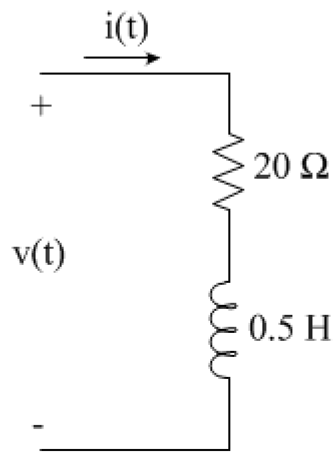
leading or lagging

current is leading the voltage
$\theta<0$

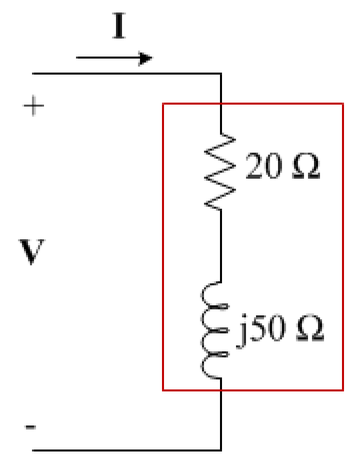
current is lagging the voltage
$\theta>0$

Example:

v(t) = 2000 cos(100t) V



$$\mathbf{V} = 2000\angle 0^\circ \text{ V}$$



$$Z = 20 + j50\ \Omega$$
$$= 53.85\angle 68.20^\circ\ \Omega$$

Current, **I**

$$I = \frac{V}{Z} = \frac{2000\angle 0^\circ}{20 + j50} = 37.14\angle -68.20^\circ \text{A}$$

Power Factor

$$PF = \cos\theta = \cos(68.20^\circ) = 0.371 \text{ lagging}$$

Complex Power Absorbed

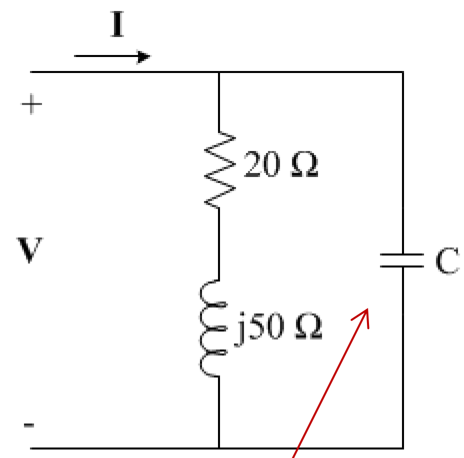
$$S = \frac{1}{2}\mathbf{V}\mathbf{I}^* = \frac{1}{2}(2000\angle 0^\circ)(37.14\angle +68.20^\circ)$$
$$= 37139\angle 68.20^\circ \text{ VA}$$
$$= 13793 + j34483 \text{ VA}$$

Average Power Absorbed

P=13793 W

Power Factor Correction

We want the power factor close to 1 to reduce the current.



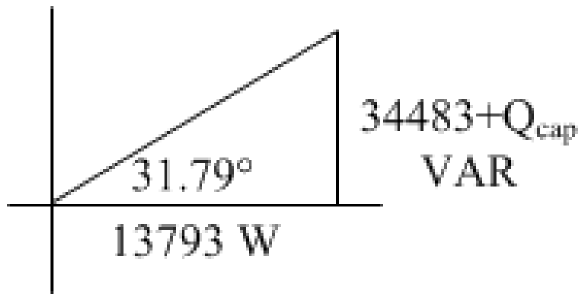
Correct the power factor to 0.85 lagging

$$\theta_{new} = \cos^{-1} 0.85 = +31.79°$$

Add a capacitor in parallel with the load.

Total Complex Power

$$S_{total} = S + S_{cap} = (13793 + j34483) + (0 + jQ_{cap})$$



$$Q_{cap} = P \tan\theta_{new} - Q = 13793 \tan(31.79°) - 34483 = -24934 \text{ VAR}$$

$$Q_{cap} = -25934 \text{ VAR} \rightarrow S_{cap} = -j25934 \text{ VA}$$

S for an ideal capacitor

$$S = -j\frac{1}{2}\omega C V_m^2$$

v(t) = 2000 cos(100t) V

$$-j25934 = -j\frac{1}{2}(100)C(2000)^2$$

$$\boxed{C = 0.13 \text{ mF}}$$

$$S_{total} = S + S_{cap} = (13793 + j34483) + (0 - j25934)$$

$$= 13793 + j8549 \text{ VA} = 16227.5\angle 31.79° \text{ VA}$$

Without the capacitor

$$I = 37.14\angle -68.20° \text{A}$$

Current after capacitor added

$$S = \frac{1}{2}\mathbf{V}\mathbf{I}^* \rightarrow \mathbf{I} = \left(\frac{2S}{\mathbf{V}}\right)^* = \left(\frac{2(13793 + j8549)}{2000\angle 0°}\right)^* = 16.23\angle -31.79° \text{ A}$$

RMS Current & Voltage

a.k.a. "Effective" current or voltage

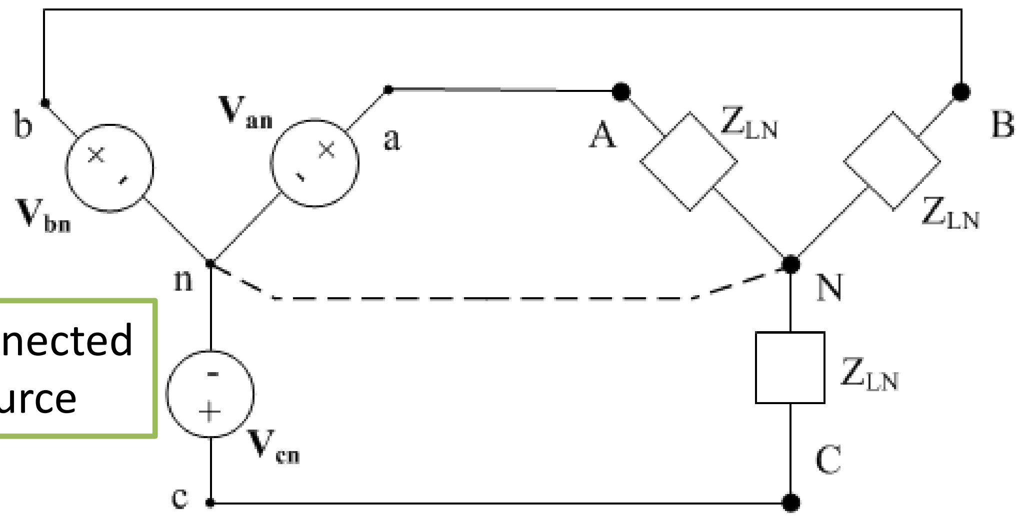
$$V_{rms} = \left(\frac{1}{T} \int_0^T v^2 dt \right)^{1/2} \qquad I_{rms} = \left(\frac{1}{T} \int_0^T i^2 dt \right)^{1/2}$$

RMS value of a sinusoid

$$A \cos(\omega t + \varphi) \quad \rightarrow \quad \frac{A}{\sqrt{2}}$$

$$\boxed{P = V_{rms} \cdot I_{rms} \cos\theta}$$

Balanced Three-Phase Systems



Y-connected source

Y-connected load

Phase Voltages

$$\mathbf{V}_{\mathbf{an}} = V_m \angle 0°$$

$$\mathbf{V}_{\mathbf{bn}} = V_m \angle -120°$$

$$\mathbf{V}_{\mathbf{cn}} = V_m \angle 120°$$

Line Currents

$$\mathbf{I}_{\mathbf{aA}} = \mathbf{I}_{\mathbf{AN}} = \frac{\mathbf{V}_{\mathbf{AN}}}{Z_{LN}} = I_L \angle \varphi$$

$$\mathbf{I}_{\mathbf{bB}} = \mathbf{I}_{\mathbf{BN}} = \frac{\mathbf{V}_{\mathbf{BN}}}{Z_{LN}} = I_L \angle \varphi - 120°$$

$$\mathbf{I}_{\mathbf{cC}} = \mathbf{I}_{\mathbf{CN}} = \frac{\mathbf{V}_{\mathbf{CN}}}{Z_{LN}} = I_L \angle \varphi + 120°$$

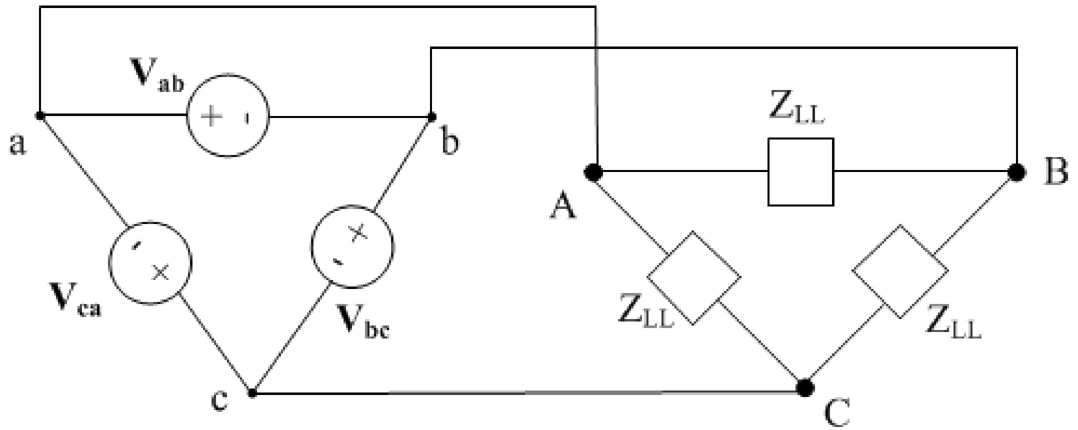
Line Voltages

$$\mathbf{V}_{\mathbf{ab}} = \mathbf{V}_{\mathbf{an}} - \mathbf{V}_{\mathbf{bn}} = \left(\sqrt{3}\angle 30°\right)\mathbf{V}_{\mathbf{an}} = V_L \angle \psi$$

$$\mathbf{V}_{\mathbf{bc}} = V_L \angle \psi - 120°$$

$$\mathbf{V}_{\mathbf{ca}} = V_L \angle \psi + 120°$$

Balanced Three-Phase Systems



Δ-connected source

Δ-connected load

Line Voltages

$$\mathbf{V}_{\mathbf{ab}} = V_m \angle 0^\circ$$

$$\mathbf{V}_{\mathbf{bc}} = V_m \angle -120^\circ$$

$$\mathbf{V}_{\mathbf{ca}} = V_m \angle 120^\circ$$

Phase Currents

$$\mathbf{I}_{\mathbf{AB}} = \frac{\mathbf{V}_{\mathbf{AB}}}{Z_{LL}} = I_L \angle \varphi$$

$$\mathbf{I}_{\mathbf{BC}} = \frac{\mathbf{V}_{\mathbf{BC}}}{Z_{LL}} = I_L \angle \varphi - 120^\circ$$

$$\mathbf{I}_{\mathbf{CA}} = \frac{\mathbf{V}_{\mathbf{CA}}}{Z_{LL}} = I_L \angle \varphi + 120^\circ$$

Line Currents

$$\mathbf{I}_{\mathbf{aA}} = \mathbf{I}_{\mathbf{AB}} - \mathbf{I}_{\mathbf{CA}} = \left(\sqrt{3}\angle -30^\circ\right)\mathbf{I}_{\mathbf{AB}} = I_L \angle \psi$$

$$\mathbf{I}_{\mathbf{bB}} = I_L \angle \psi - 120^\circ$$

$$\mathbf{I}_{\mathbf{cC}} = I_L \angle \psi + 120^\circ$$

Currents and Voltages are specified in RMS

$$S = \frac{1}{2}\mathbf{V}\mathbf{I}^* \quad \rightarrow \quad S = \mathbf{V}\mathbf{I}^* \quad \rightarrow \quad S = 3\mathbf{V}\mathbf{I}^*$$

S for peak voltage & current

S for RMS voltage & current

S for 3-phase voltage & current

$$V_L = \text{line voltage} = |\mathbf{V}_{\mathbf{ab}}|$$

$$I_L = \text{line current} = |\mathbf{I}_{\mathbf{aA}}|$$

$$\theta = \text{impedance angle} = \angle Z$$

Average Power

$$P = \sqrt{3} V_L I_L \cos\theta$$

Complex Power for Y-connected load

$$S = 3\mathbf{V}_{\mathbf{AN}}\mathbf{I}_{\mathbf{AN}}^* = \sqrt{3} V_L I_L \angle\theta$$

Complex Power for Δ-connected load

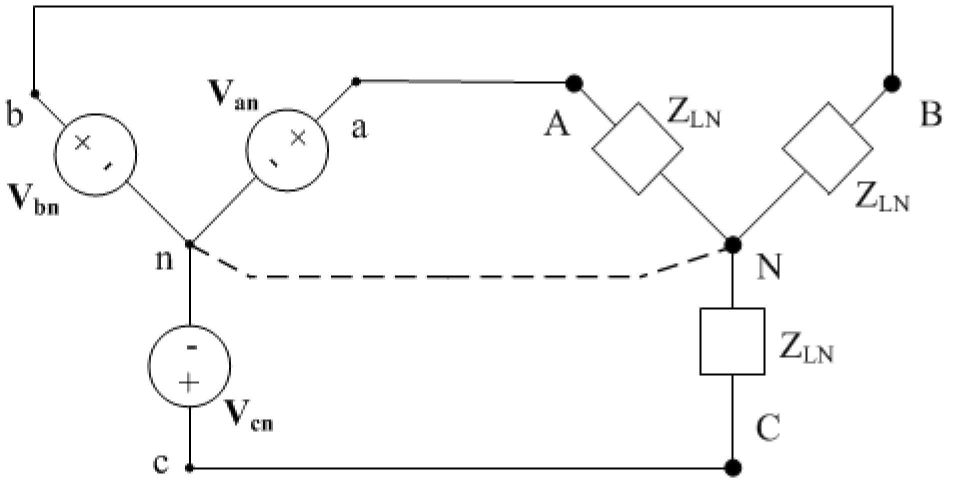
$$S = 3\mathbf{V}_{\mathbf{AB}}\mathbf{I}_{\mathbf{AB}}^* = \sqrt{3} V_L I_L \angle\theta$$

Power Factor

$$PF = \cos\theta \quad \text{(leading or lagging)}$$

Example:

Find the total real power supplied by the source in the balanced wye-connected circuit



Given:

$$\mathbf{V}_{\mathbf{an}} = 540\angle 0°\ \mathrm{V}$$

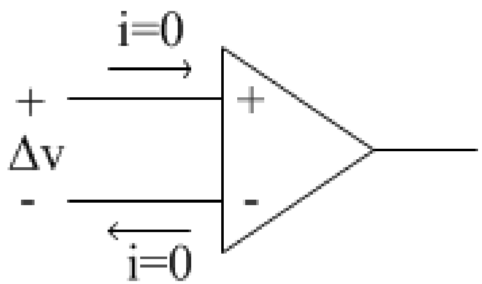
$$Z_{LN} = 270 + j270\ \Omega$$

$$\mathbf{I}_{\mathbf{aA}} = \mathbf{I}_{\mathbf{AN}} = \frac{\mathbf{V}_{\mathbf{AN}}}{Z_{LN}} = \frac{540\angle 0°}{270 + j270} = 1.41\angle -45°\mathrm{A}$$

$$S = 3\mathbf{V}_{\mathbf{AN}}\mathbf{I}_{\mathbf{AN}}^{*} = 3\left(540\angle 0°\right)\left(1.41\angle 45°\right) = 2291\angle 45°\mathrm{VA} = 1620 + j1620\mathrm{VA}$$

P=1620 W

Ideal Operational Amplifier (Op Amp)

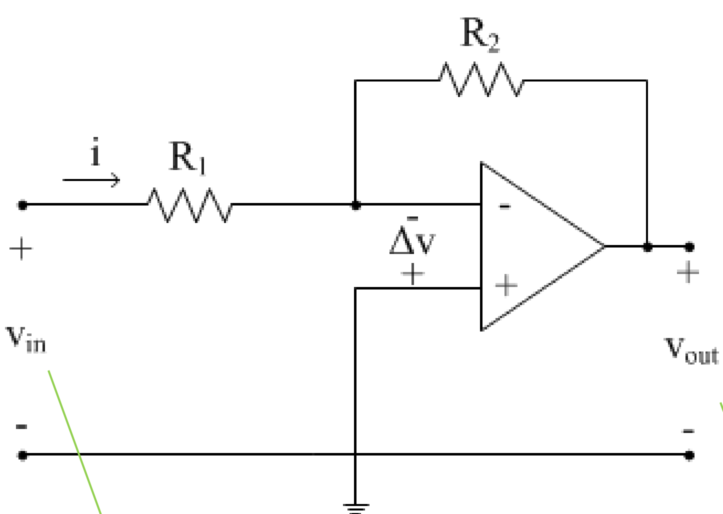


With negative feedback

i=0

Δv=0

Linear Amplifier



$$\text{KVL}: \; -v_{in} + R_1 i - \cancelto{0}{\Delta v} = 0 \quad \rightarrow \quad i = \frac{v_{in}}{R_1}$$

$$\text{KVL}: \; -v_{in} + R_1 i + R_2 i + v_{out} = 0$$

$$-v_{in} + R_1\left(\frac{v_{in}}{R_1}\right) + R_2\left(\frac{v_{in}}{R_1}\right) + v_{out} = 0$$

$$\boxed{v_{out} = -\frac{R_2}{R_1} v_{in}}$$

mV or µV reading from sensor

0-5 V output to A/D converter

Magnetic Fields

$$\oint_S \mathbf{B} \bullet d\mathbf{s} = 0 \qquad \oint_l \mathbf{H} \bullet d\boldsymbol{l} = \oint_S \mathbf{J} \bullet d\mathbf{S} = I$$

Net magnetic flux through a closed surface is zero.

B – magnetic flux density (tesla)
H – magnetic field strength (A/m)
J - current density

$$\mathbf{B} = \mu \mathbf{H}$$

Magnetic Flux ϕ passing through a surface

$$\phi = \int_S \mathbf{B} \bullet d\mathbf{S}$$

Energy stored in the magnetic field

$$w = \frac{1}{2} \iiint_V \mu \left| H \right|^2 dV$$

Enclosing a surface with N turns of wire produces a voltage across the terminals

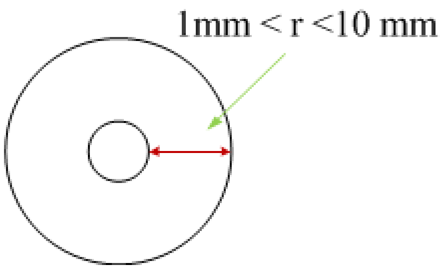
$$v = -N \frac{d\phi}{dt}$$

Magnetic field produces a force perpendicular to the current direction and the magnetic field direction

$$\mathbf{F} = I\mathbf{L} \times \mathbf{B}$$

Example:

A coaxial cable with an inner wire of radius 1 mm carries 10-A current. The outer cylindrical conductor has a diameter of 10 mm and carries a 10-A uniformly distributed current in the opposite direction. Determine the approximate magnetic energy stored per unit length in this cable. Use μ_0 for the permeablility of the material between the wire and conductor.



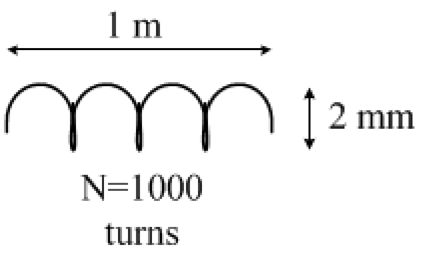
$$\oint \mathbf{H} \bullet d\boldsymbol{l} = H_\varphi 2\pi r = I_0 = 10 \text{ A}$$

$$H_\varphi = \frac{10}{2\pi r} \quad \text{for} \quad 10^{-3}\text{m} < r < 10^{-2}\text{m}$$

$$w = \frac{1}{2}\iiint_V \mu |H|^2 dV = \frac{1}{2}\int_0^1 \int_0^{2\pi} \int_{0.001}^{0.01} \mu_0 \left(\frac{10}{2\pi r}\right)^2 r dr d\varphi dz = 18.3\mu_0 \text{ J}$$

Example:

A cylindrical coil of wire has an air core and 1000 turns. It is 1 m long with a diameter of 2 mm so has a relatively uniform field. Find the current necessary to achieve a magnetic flux density of 2 T.



$$\int \mathbf{H} \bullet d\boldsymbol{l} = NI_0$$

$$HL = NI_0$$

$$\left(\frac{B}{\mu_0}\right)L = NI_0 \quad \rightarrow \quad I = \frac{BL}{N\mu_0} = \frac{(2T)(1m)}{1000\left(4\pi \times 10^{-7}\right)} = 1590\ A$$

Questions?