Fundamental Mechanics of Materials Equations

Basic definitions

σ sigma
ε epsilon
τ tau
γ gamma
ν nu
δ Δ delta
α alpha
φ phi
ω omega
θ theta

Average normal stress in an axial member

$$\sigma_{avg} = \frac{F}{A}$$

Average direct shear stress

$$\tau_{avg} = \frac{V}{A_V}$$

Average bearing stress

$$\sigma_b = \frac{F}{A_b}$$

Average normal strain in an axial member

$$\varepsilon_{avg} = \frac{\delta}{L} \qquad \varepsilon_{transverse} = \frac{\Delta d}{d} \text{ or } \frac{\Delta w}{w} \text{ or } \frac{\Delta t}{t}$$

$$\gamma = \text{change in angle from } 90°$$

Average normal strain caused by temperature change

$$\varepsilon_T = \alpha \Delta T$$

Hooke's Law (one-dimensional)

$$\sigma = E\varepsilon \quad \text{and} \quad \tau = G\gamma$$

Poisson's ratio

$$\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}}$$

Relationship between E, G, and ν

$$G = \frac{E}{2(1+\nu)}$$

Definition of allowable stress

$$\sigma_{allow} = \frac{\sigma_{failure}}{FS} \quad \text{or} \quad \tau_{allow} = \frac{\tau_{failure}}{FS}$$

Factor of safety

$$FS = \frac{\sigma_{failure}}{\sigma_{actual}} \quad \text{or} \quad FS = \frac{\tau_{failure}}{\tau_{actual}}$$

Axial deformation

Deformation in axial members

$$\delta = \frac{FL}{AE} \quad \text{or} \quad \delta = \sum_i \frac{F_i L_i}{A_i E_i}$$

Force-temperature-deformation relationship

$$\delta = \frac{FL}{AE} + \alpha \Delta T L$$

Torsion

Maximum torsion shear stress in a circular shaft

$$\tau_{max} = \frac{Tc}{J}$$

where the polar moment of inertia J is defined as

$$J = \frac{\pi}{2}[R^4 - r^4] = \frac{\pi}{32}[D^4 - d^4]$$

Angle of twist in a circular shaft

$$\phi = \frac{TL}{JG} \quad \text{or} \quad \phi = \sum_i \frac{T_i L_i}{J_i G_i}$$

gears

$$r_2 T_1 = r_1 T_2$$
$$r_1 \omega_1 = r_2 \omega_2$$

Power transmission in a shaft

$$P = T\omega$$

$$watts = Nm/s$$
$$hp = 6600 \; in \cdot lb/s$$

Six rules for constructing shear-force and bending-moment diagrams

Rule 1: $\Delta V = P_0$

Rule 2: $\Delta V = V_2 - V_1 = \int_{x_1}^{x_2} w(x)\,dx$

Rule 3: $\frac{dV}{dx} = w(x)$

Rule 4: $\Delta M = M_2 - M_1 = \int_{x_1}^{x_2} V\,dx$

Rule 5: $\frac{dM}{dx} = V$

Rule 6: $\Delta M = -M_0$

Flexure

Flexure formula

$$\sigma_x = -\frac{My}{I} \quad \text{or} \quad \sigma_{max} = \frac{Mc}{I} = \frac{M}{S} \quad \text{where} \quad S = \frac{I}{c}$$

Unsymmetric bending of arbitrary cross sections

$$\sigma_x = \left[\frac{I_z z - I_{yz} y}{I_y I_z - I_{yz}^2}\right] M_y + \left[\frac{-I_y y + I_{yz} z}{I_y I_z - I_{yz}^2}\right] M_z$$

composite beams

$$n = \frac{E_B}{E_A}$$
$$\sigma_A = \frac{-My}{I^T}$$
$$\sigma_B = \frac{-nMy}{I^T}$$

Unsymmetric bending of symmetric cross sections

$$\sigma_x = \frac{M_y z}{I_y} - \frac{M_z y}{I_z} \qquad \tan\beta = \frac{M_y I_z}{M_z I_y}$$

Horizontal shear stress associated with bending

$$\tau_H = \frac{VQ}{It} \qquad \text{where } Q = \sum \bar{y}_i A_i$$

Shear flow formula

$$q = \frac{VQ}{I}$$

Shear flow, fastener spacing, and fastener shear relationship

$$qs \le n_f V_f = n_f \tau_f A_f \quad \text{or} \quad q = \frac{V_{beam} Q}{I} = \frac{n V_{fastener}}{s}$$

For circular cross sections,

$$Q = \frac{1}{12} d^3 \quad \text{(solid sections)}$$

$$Q = \frac{2}{3}[R^3 - r^3] = \frac{1}{12}[D^3 - d^3] \quad \text{(hollow sections)}$$

Beam deflections

Elastic curve relations between w, V, M, θ, and v for constant EI

Deflection $= v$

Slope $= \frac{dv}{dx} = \theta$

Moment $M = EI \frac{d^2 v}{dx^2}$

Shear $V = \frac{dM}{dx} = EI \frac{d^3 v}{dx^3}$

Load $w = \frac{dV}{dx} = EI \frac{d^4 v}{dx^4}$

Fundamental Mechanics of Materials Equations

Plane stress transformations

Normal and shear stresses on an arbitrary plane

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$
$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

or

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$\sigma_t = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$
$$\tau_{nt} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Principal stress magnitudes

$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Orientation of principal planes

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

Maximum in-plane shear stress magnitude

$$\tau_{\max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \text{or} \quad \tau_{\max} = \frac{\sigma_{p1} - \sigma_{p2}}{2}$$

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2}$$

Absolute maximum shear stress magnitude

$$\tau_{\text{abs max}} = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$

Normal, stress invariance

$$\sigma_x + \sigma_y = \sigma_n + \sigma_t = \sigma_{p1} + \sigma_{p2}$$

Plane strain transformations

Normal and shear strain in arbitrary directions

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$
$$\gamma_{nt} = -2(\varepsilon_x - \varepsilon_y)\sin\theta\cos\theta + \gamma_{xy}(\cos^2\theta - \sin^2\theta)$$

or

$$\varepsilon_n = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$
$$\varepsilon_t = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$
$$\frac{\gamma_{nt}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\varepsilon_z = \frac{-\nu}{1-\nu}(\varepsilon_x + \varepsilon_y)$$

Principal strain magnitudes

$$\varepsilon_{p1,p2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

Orientation of principal strains

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

Maximum in-plane shear strain

$$\frac{\gamma_{\max}}{2} = \pm\sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} \quad \text{or} \quad \gamma_{\max} = \varepsilon_{p1} - \varepsilon_{p2}$$

$$\varepsilon_{\text{avg}} = \frac{\varepsilon_x + \varepsilon_y}{2}$$

Normal strain invariance

$$\varepsilon_x + \varepsilon_y = \varepsilon_n + \varepsilon_t = \varepsilon_{p1} + \varepsilon_{p2}$$

Generalized Hooke's Law

Normal stress/normal strain relationships

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$
$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$
$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

Shear stress/shear strain relationships

$$\gamma_{xy} = \frac{1}{G}\tau_{xy} \qquad \gamma_{yz} = \frac{1}{G}\tau_{yz} \qquad \gamma_{zx} = \frac{1}{G}\tau_{zx}$$

where

$$G = \frac{E}{2(1+\nu)}$$

Normal stress/normal strain relationships for plane stress

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) \qquad \sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y)$$
$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) \quad \text{or} \quad \sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x)$$
$$\varepsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y)$$

Shear stress/shear strain relationships for plane stress

$$\gamma_{xy} = \frac{1}{G}\tau_{xy} \quad \text{or} \quad \tau_{xy} = G\gamma_{xy}$$

Pressure vessels

Axial stress in spherical pressure vessel

$$\sigma_a = \frac{pr}{2t} = \frac{pd}{4t}$$

Longitudinal and hoop stresses in cylindrical pressure vessels

$$\sigma_{radial-outside} = 0$$
$$\sigma_{radial-inside} = -p$$

$$\sigma_{\text{long}} = \frac{pr}{2t} = \frac{pd}{4t} \qquad \sigma_{\text{hoop}} = \frac{pr}{t} = \frac{pd}{2t}$$

Failure theories

Mises equivalent stress for plane stress

$$\sigma_M = \left[\sigma_{p1}^2 - \sigma_{p1}\sigma_{p2} + \sigma_{p2}^2\right]^{1/2} = \left[\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right]^{1/2}$$

Column buckling

Euler buckling load

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

Euler buckling stress

$$\sigma_{cr} = \frac{\pi^2 E}{(KL/r)^2}$$

Radius of gyration

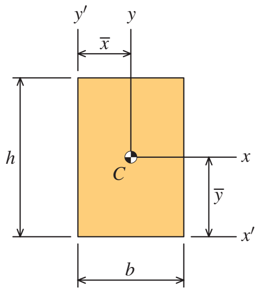
$$r^2 = \frac{I}{A}$$

$$\bar{x} = \frac{\Sigma x_i A_i}{\Sigma A_i} \qquad \bar{y} = \frac{\Sigma y_i A_i}{\Sigma A_i}$$

$$I = \Sigma(I_c + d^2 A)$$

Table A.1 Properties of Plane Figures

1. Rectangle



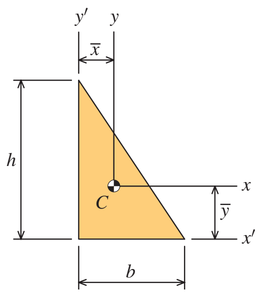
$$A = bh$$

$$\bar{y} = \frac{h}{2} \qquad I_x = \frac{bh^3}{12}$$

$$\bar{x} = \frac{b}{2} \qquad I_y = \frac{hb^3}{12}$$

$$I_{x'} = \frac{bh^3}{3} \qquad I_{y'} = \frac{hb^3}{3}$$

2. Right Triangle



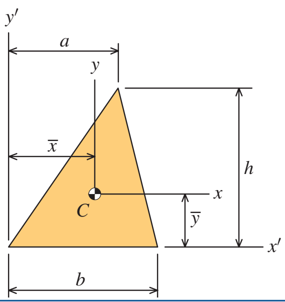
$$A = \frac{bh}{2}$$

$$\bar{y} = \frac{h}{3} \qquad I_x = \frac{bh^3}{36}$$

$$\bar{x} = \frac{b}{3} \qquad I_y = \frac{hb^3}{36}$$

$$I_{x'} = \frac{bh^3}{12} \qquad I_{y'} = \frac{hb^3}{12}$$

3. Triangle



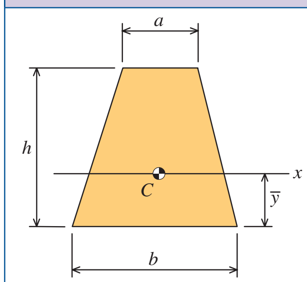
$$A = \frac{bh}{2}$$

$$\bar{y} = \frac{h}{3} \qquad I_x = \frac{bh^3}{36}$$

$$\bar{x} = \frac{(a+b)}{3} \qquad I_y = \frac{bh}{36}(a^2 - ab + b^2)$$

$$I_{x'} = \frac{bh^3}{12}$$

4. Trapezoid

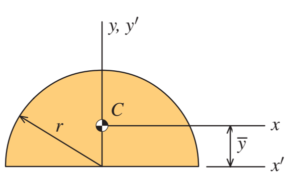


$$A = \frac{(a+b)h}{2}$$

$$\bar{y} = \frac{1}{3}\left(\frac{2a+b}{a+b}\right)h$$

$$I_x = \frac{h^3}{36(a+b)}(a^2 + 4ab + b^2)$$

5. Semicircle

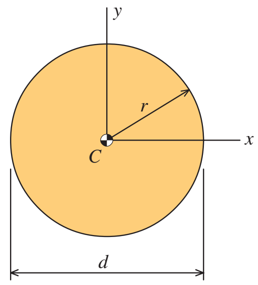


$$A = \frac{\pi r^2}{2}$$

$$\bar{y} = \frac{4r}{3\pi} \qquad I_x = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)r^4$$

$$I_{x'} = I_{y'} = \frac{\pi r^4}{8}$$

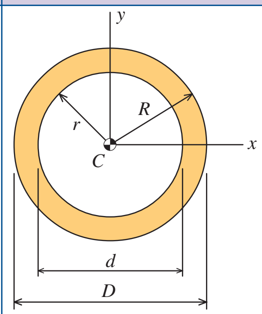
6. Circle



$$A = \pi r^2 = \frac{\pi d^2}{4}$$

$$I_x = I_y = \frac{\pi r^4}{4} = \frac{\pi d^4}{64}$$

7. Hollow Circle

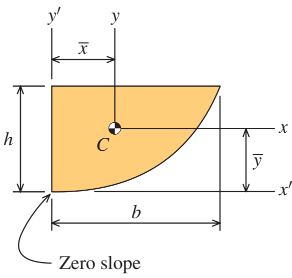


$$A = \pi(R^2 - r^2) = \frac{\pi}{4}(D^2 - d^2)$$

$$I_x = I_y = \frac{\pi}{4}(R^4 - r^4)$$

$$= \frac{\pi}{64}(D^4 - d^4)$$

8. Parabola

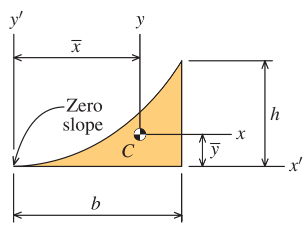


$$y' = \frac{h}{b^2}x'^2$$

$$A = \frac{2bh}{3}$$

$$\bar{x} = \frac{3b}{8} \qquad \bar{y} = \frac{3h}{5}$$

9. Parabolic Spandrel

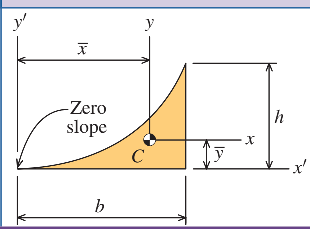


$$y' = \frac{h}{b^2}x'^2$$

$$A = \frac{bh}{3}$$

$$\bar{x} = \frac{3b}{4} \qquad \bar{y} = \frac{3h}{10}$$

10. General Spandrel

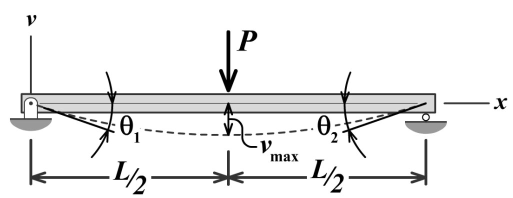
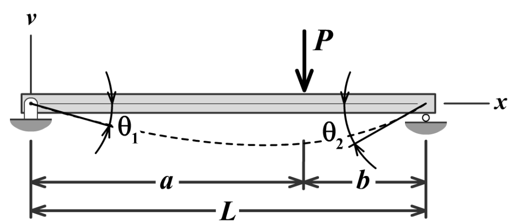
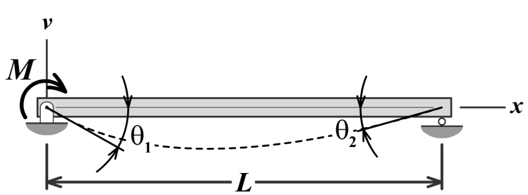
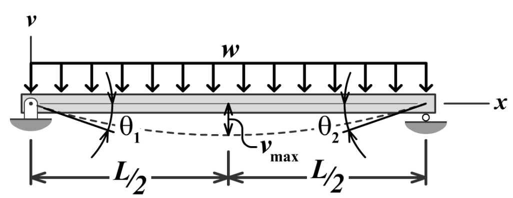
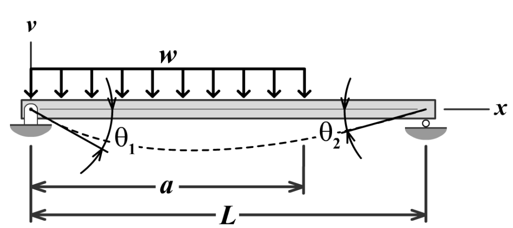
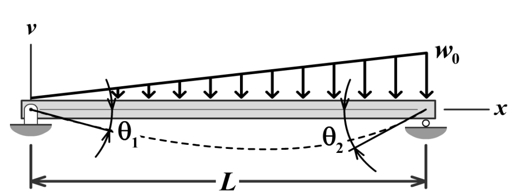


$$y' = \frac{h}{b^n}x'^n$$

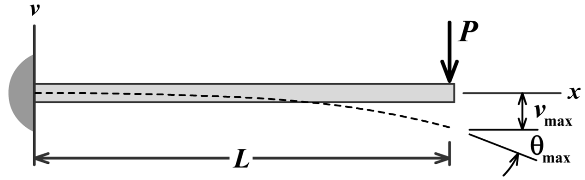
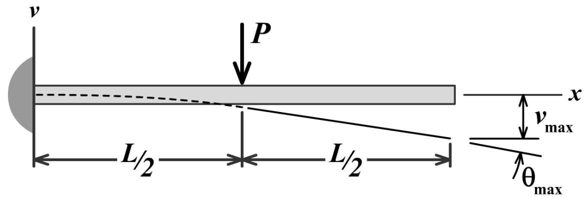
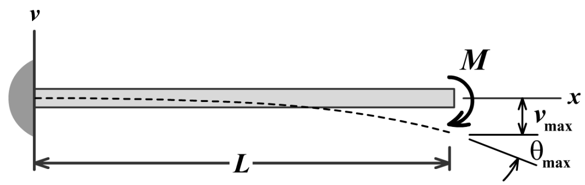
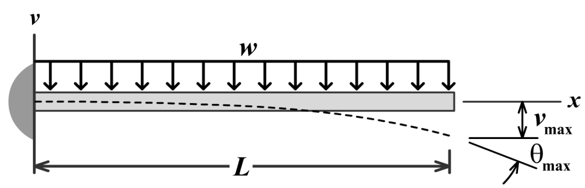
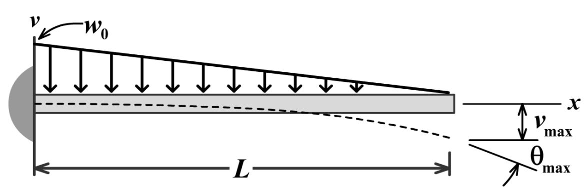
$$A = \frac{bh}{n+1}$$

$$\bar{x} = \frac{n+1}{n+2}b \qquad \bar{y} = \frac{n+1}{4n+2}h$$

SIMPLY SUPPORTED BEAMS

Beam	Slope	Deflection	Elastic Curve
	1 $\theta_1 = -\theta_2 = -\dfrac{PL^2}{16EI}$	2 $v_{\max} = -\dfrac{PL^3}{48EI}$	3 $v = -\dfrac{Px}{48EI}(3L^2 - 4x^2)$ for $0 \le x \le L/2$
	4 $\theta_1 = -\dfrac{Pb(L^2 - b^2)}{6LEI}$ $\theta_2 = +\dfrac{Pa(L^2 - a^2)}{6LEI}$	5 $v = -\dfrac{Pa^2b^2}{3LEI}$ at $x = a$	6 $v = -\dfrac{Pbx}{6LEI}(L^2 - b^2 - x^2)$ for $0 \le x \le a$
	7 $\theta_1 = -\dfrac{ML}{3EI}$ $\theta_2 = +\dfrac{ML}{6EI}$	8 $v_{\max} = -\dfrac{ML^2}{9\sqrt{3}\,EI}$ at $x = L\left(1 - \dfrac{\sqrt{3}}{3}\right)$	9 $v = -\dfrac{Mx}{6LEI}(2L^2 - 3Lx + x^2)$
	10 $\theta_1 = -\theta_2 = -\dfrac{wL^3}{24EI}$	11 $v_{\max} = -\dfrac{5wL^4}{384EI}$	12 $v = -\dfrac{wx}{24EI}(L^3 - 2Lx^2 + x^3)$
	13 $\theta_1 = -\dfrac{wa^2}{24LEI}(2L - a)^2$ $\theta_2 = +\dfrac{wa^2}{24LEI}(2L^2 - a^2)$	14 $v = -\dfrac{wa^3}{24LEI}(4L^2 - 7aL + 3a^2)$ at $x = a$	$v = -\dfrac{wx}{24LEI}(Lx^3 - 4aLx^2 + 2a^2x^2 + 4a^2L^2 - 4a^3L + a^4)$ for $0 \le x \le a$ $v = -\dfrac{wa^2}{24LEI}(2x^3 - 6Lx^2 + a^2x + 4L^2x - a^2L)$ 15 for $a \le x \le L$
	16 $\theta_1 = -\dfrac{7w_0L^3}{360EI}$ $\theta_2 = +\dfrac{w_0L^3}{45EI}$	17 $v_{\max} = -0.00652\dfrac{w_0L^4}{EI}$ at $x = 0.5193L$	18 $v = -\dfrac{w_0x}{360LEI}(7L^4 - 10L^2x^2 + 3x^4)$

CANTILEVER BEAMS

Beam	Slope	Deflection	Elastic Curve
	19 $\theta_{max} = -\dfrac{PL^2}{2EI}$	20 $v_{max} = -\dfrac{PL^3}{3EI}$	21 $v = -\dfrac{Px^2}{6EI}(3L - x)$
	22 $\theta_{max} = -\dfrac{PL^2}{8EI}$	23 $v_{max} = -\dfrac{5PL^3}{48EI}$	24 $v = -\dfrac{Px^2}{12EI}(3L - 2x)$ for $0 \le x \le L/2$ $v = -\dfrac{PL^2}{48EI}(6x - L)$ for $L/2 \le x \le L$
	25 $\theta_{max} = -\dfrac{ML}{EI}$	26 $v_{max} = -\dfrac{ML^2}{2EI}$	27 $v = -\dfrac{Mx^2}{2EI}$
	28 $\theta_{max} = -\dfrac{wL^3}{6EI}$	29 $v_{max} = -\dfrac{wL^4}{8EI}$	30 $v = -\dfrac{wx^2}{24EI}(6L^2 - 4Lx + x^2)$
	31 $\theta_{max} = -\dfrac{w_0 L^3}{24EI}$	32 $v_{max} = -\dfrac{w_0 L^4}{30EI}$	33 $v = -\dfrac{w_0 x^2}{120LEI}(10L^3 - 10L^2 x + 5Lx^2 - x^3)$