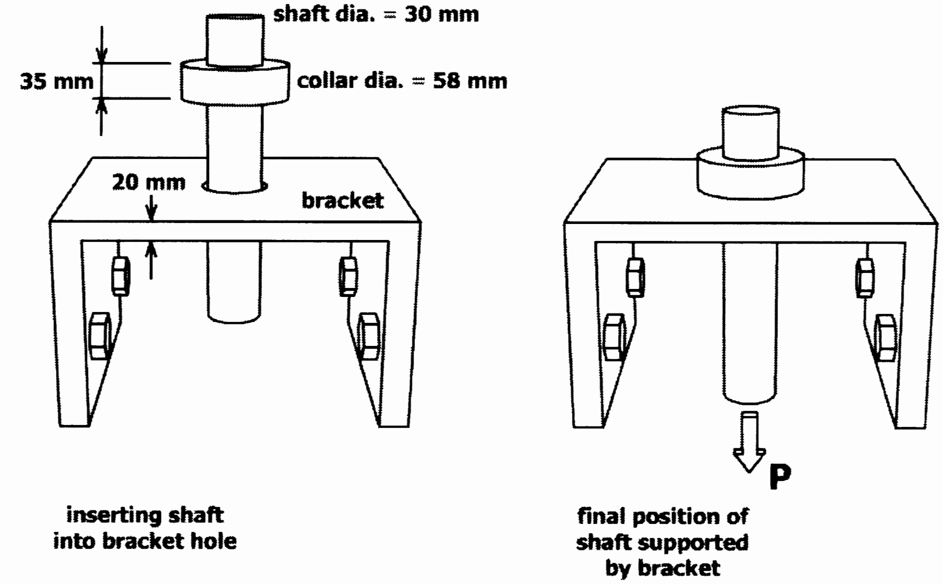
BE 110 – Mechanics of Materials
May 9, 2005

Student Key

1. A shaft with a circular collar is inserted into the 30-mm-diameter hole in the bracket as shown. Determine the maximum axial force P that can be applied to the shaft if:
   (a) The allowable shear stress between the collar and shaft is 200 MPa, and
   (b) The allowable bearing stress between the collar and the bracket is 450 MPa.



$$200\,\text{MPa} = \frac{P}{\pi(.03)(.035)} \Rightarrow \boxed{P = 659.7\,\text{KN}}$$

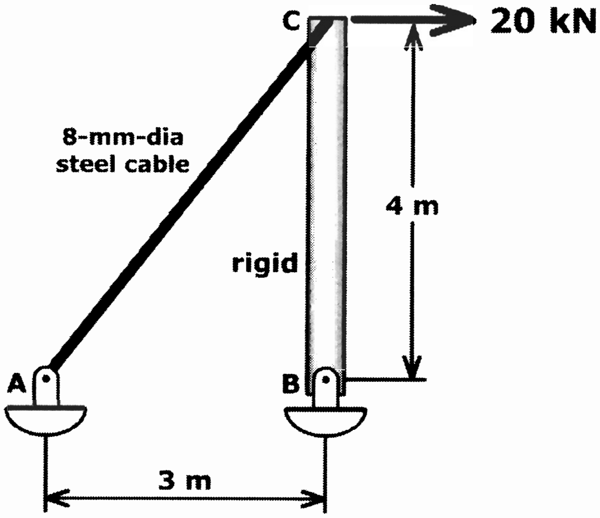
$$450\,\text{MPa} = \frac{P}{\frac{\pi}{4}\left[(.058)^2 - (.03)^2\right]} \Rightarrow P = 870.8\,\text{KN}$$

PRINT CLEARLY
Include proper units
Box-in your answers

BE 110 – Mechanics of Materials
May 9, 2005

Student ______________________________

2. The rigid vertical pipe BC is supported by a pin at B and the 8-mm-diameter steel cable AC. Assume E_S = 200 GPa and $\alpha_S = 12\times10^{-6}$/°C.
   (a) Determine the change in length of member AC when the 20-kN load is applied and the temperature increases from +5°C to +80°C.
   (b) Determine the horizontal displacement of the connection at C.

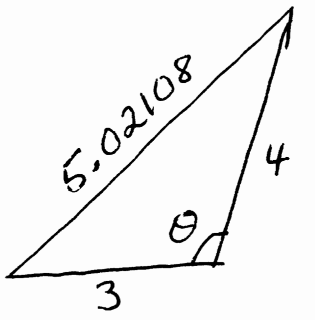


$$\sum M_B^{\circlearrowleft +} = 0 = AC\left(\tfrac{3}{5}\right)(4) - 20\,\text{kN}\,(4)$$

$$AC = 33.33\,\text{kN}\ T$$

a) $$\delta_{AC} = \frac{(33{,}333)(5)}{\frac{\pi}{4}(.008)^2(200\times10^9)} + (5)(12\times10^{-6})(75) = \boxed{0.02108\,\text{m}}$$

b)

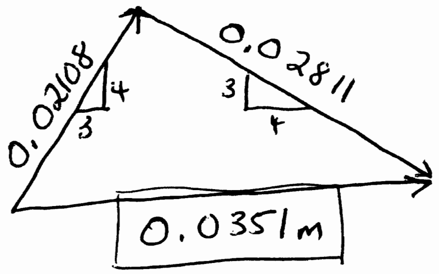


$$5.02108^2 = 3^2 + 4^2 - 2(3)(4)\cos\theta$$

$$\theta = 90.5043^\circ$$

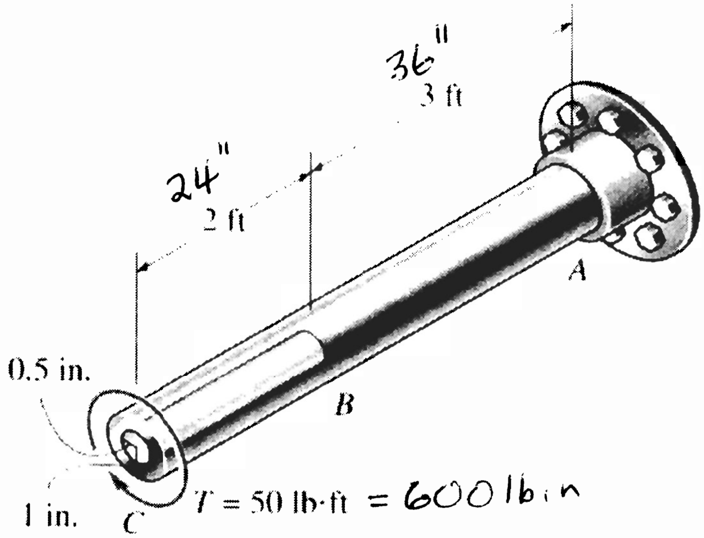
$$\delta_c = 4\sin(0.5043) = \boxed{0.0352\,\text{m} \rightarrow}$$

Small Displacement Approach



PRINT CLEARLY
Include proper units
Box-in your answers

3. Section AB of the shaft is solid steel and section BC is a steel sleeve with a brass core. The entire shaft has a 2-inch outside diameter. The brass core in section BC has a diameter of 1 inch. The shaft is fixed at point A and a torque of 50 lb-ft is applied at end C. Determine the angle of twist at point C. Assume G_S = 11,000 ksi and G_B = 6,000 ksi.



$$\theta_{AB} = \frac{600(36)}{\frac{\pi}{32}(2)^4(11\times10^6)} = \underline{0.00125 \text{ rad}}$$

$$T_s + T_B = 600 \text{ lb·in}$$

$$\theta_s = \frac{T_s(24)}{\frac{\pi}{32}(2^4-1^4)(11\times10^6)} =$$

$$\theta_B = \frac{T_B(24)}{\frac{\pi}{32}(1)^4(6\times10^6)} =$$

$$\theta_s = \theta_B$$

$$T_s = 578.9 \text{ lb·in}$$

$$T_B = 21.05 \text{ lb·in}$$

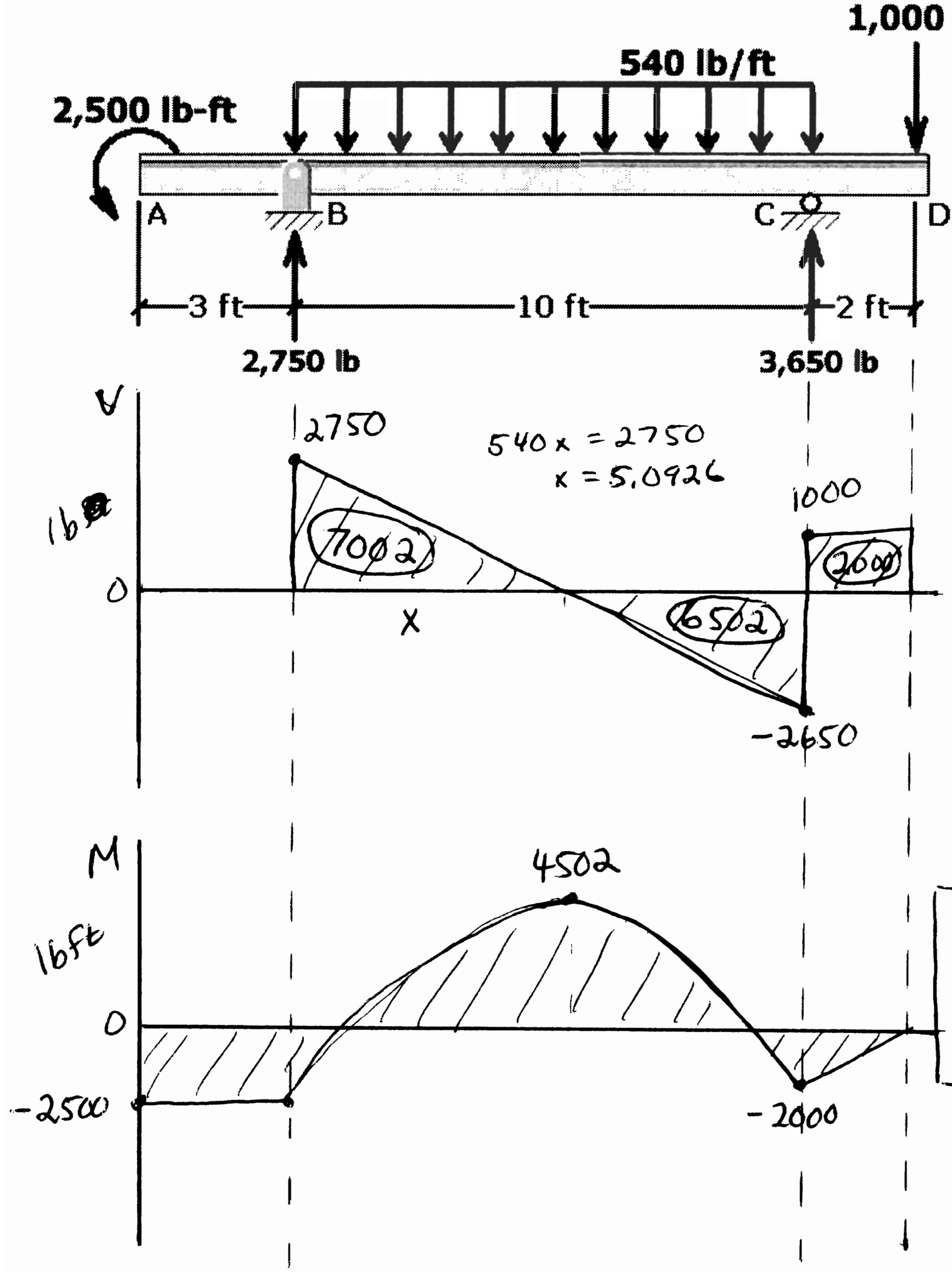
$$\theta_B = \theta_s = \underline{0.0008578 \text{ rad}}$$

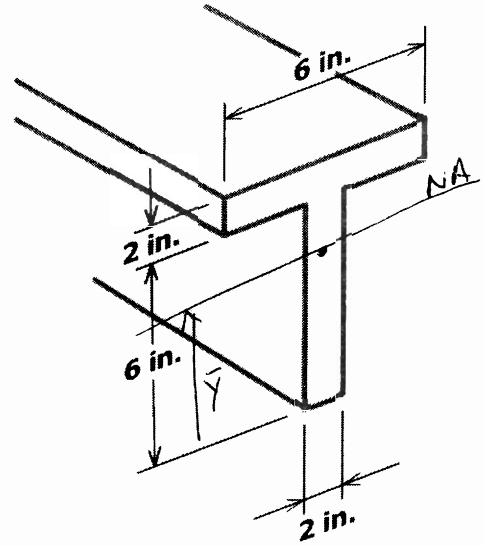
$$\boxed{\theta_c = \theta_{AB} + \theta_s = 0.00211 \text{ rad}}$$

PRINT CLEARLY
Include proper units
Box-in your answers

Student ______________________________

4. Determine the maximum tension and compression bending stresses at any location along the beam. The beam cross section is shown at the right. Support your answer.





$$\bar{y} = \frac{(2)(6)(3)+(2)(6)(7)}{2(2)(6)} = 5 \text{ in.}$$

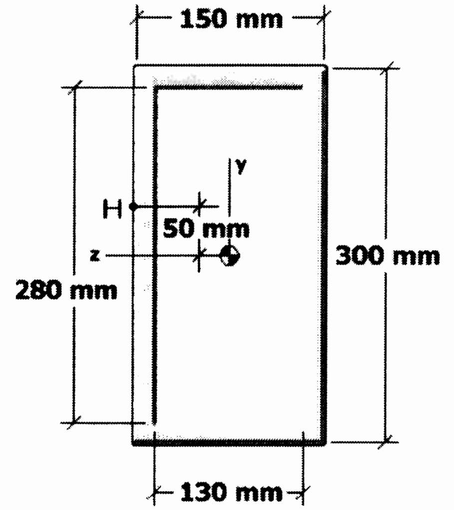
$$I = \frac{(2)(6)^3}{12} + (2)(6)(2)^2 + \frac{(6)(2)^3}{12} + (2)(6)(2)^2 = 136 \text{ in}^4$$

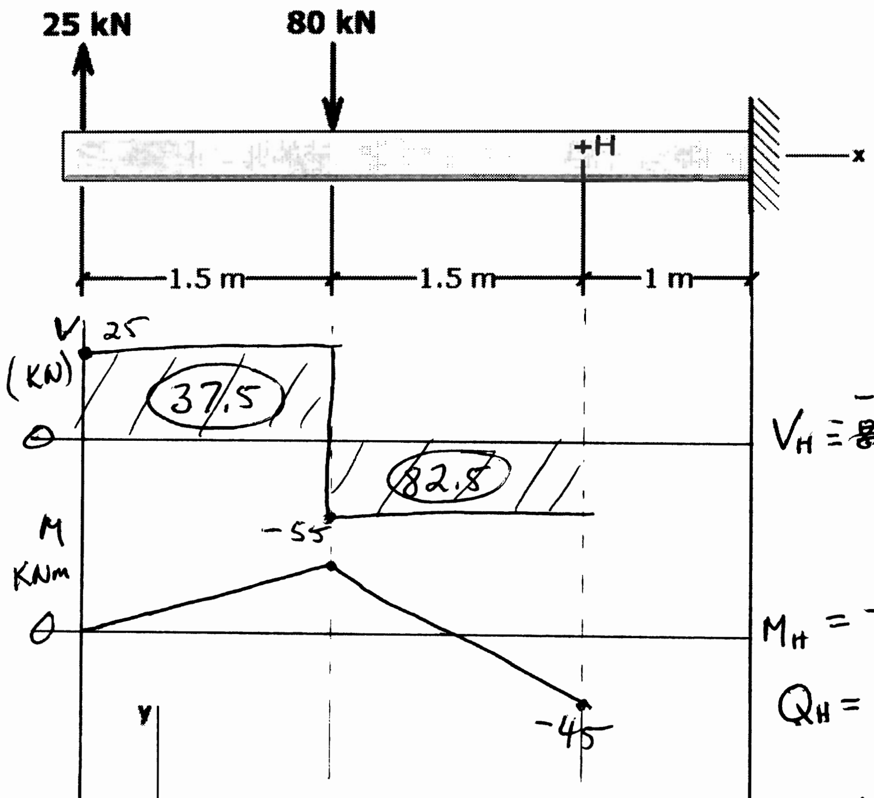
$$4502 \cdot 3'' > 2500 \cdot 5''$$

so:

$$\boxed{\begin{aligned} \text{Max } T &= \frac{4502 \cdot 12 \cdot 5}{136} = 1986 \text{ psi} \\ \text{Max } C &= \frac{4502 \cdot 3 \cdot 12}{136} = 1192 \text{ psi} \end{aligned}}$$

5. Determine the bending stress and the transverse shear stress acting at point H in the beam shown. Point H is located 50 mm above the z centroidal axis of the box cross section. Show your results on the stress element below.





$V_H = -55\text{ KN}$ ~~82.5 KN~~

$$I = \frac{150(300)^3}{12} - \frac{130(280)^3}{12}$$

$$= 99.69 \times 10^6 \text{ mm}^4$$

$$= 99.69 \times 10^{-6} \text{ m}^4$$

$M_H = -45 \text{ KNm}$

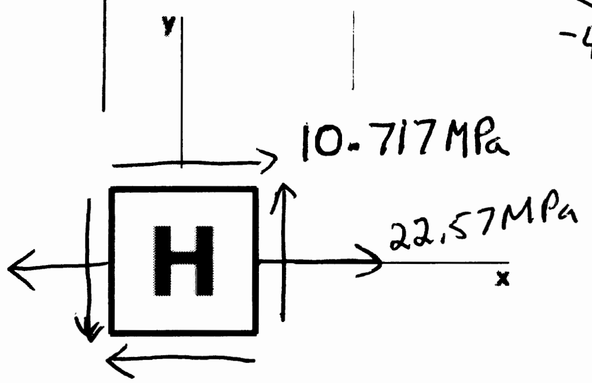
$$Q_H = (100)(150)(100) - (90)(130)(95) = 388{,}500 \text{ mm}^3$$

$$= 388.5 \times 10^{-6} \text{ m}^3$$

$$\sigma_H = \frac{(45{,}000)(.05)}{99.69 \times 10^{-6}} = 22.57 \text{ MPa}$$

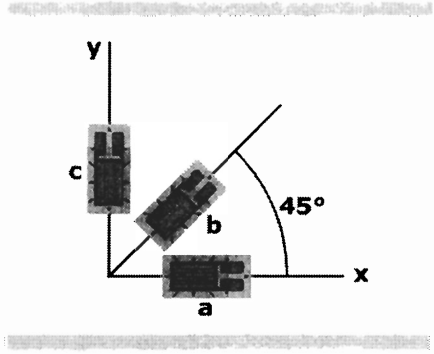
$$\tau_H = \frac{(55{,}000)(388.5 \times 10^{-6})}{(99.69 \times 10^{-6})}$$

$$= 10.717 \text{ MPa}$$



PRINT CLEARLY
Include proper units
Box-in your answers

6. The strain rosette shown was used to obtain the following normal strain data on the free surface of an aluminum plate ($E = 70$ GPa; $\nu = 0.33$). ε_a **= +770** $\mu\varepsilon$; ε_b **= +1,180** $\mu\varepsilon$; ε_c **= -350** $\mu\varepsilon$.



Determine:
(a) The stress components σ_x, σ_y, and τ_{xy} at the point.
(b) The principal stresses at the point.
(c) Show the orientation of the principal stresses on an appropriate sketch.

$\epsilon_x = 770\mu \qquad \epsilon_y = -350\mu$

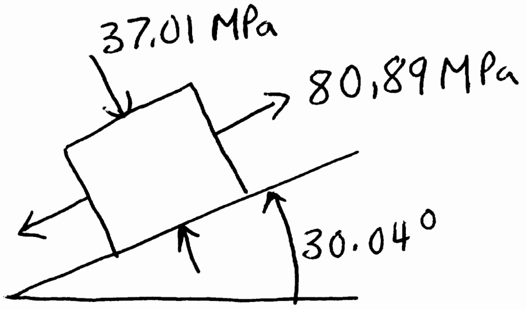
$1180 = 770\cos^2 45 - 350\sin^2 45 + \gamma_{xy}\sin 45\cos 45$

$\gamma_{xy} = 1940\mu$

a)
$$\boxed{\begin{aligned} \sigma_x &= \frac{70\times10^9}{1-.33^2}\left(770\times10^{-6} + (.33)(-350\times10^{-6})\right) = 51.41\text{ MPa} \\ \sigma_y &= \frac{70\times10^9}{1-.33^2}\left(-350\times10^{-6} + (.33)(770\times10^{-6})\right) = -7.53\text{ MPa} \\ \tau_{xy} &= \frac{70\times10^9}{2(1.33)}\left(1940\times10^{-6}\right) = 51.05\text{ MPa} \end{aligned}}$$

b) $$\sigma_{p1,p2} = \frac{51.41-7.53}{2} \pm \sqrt{\left(\frac{51.41+7.53}{2}\right)^2 + 51.05^2} = \{80.89, -37.01\}\text{ MPa}$$

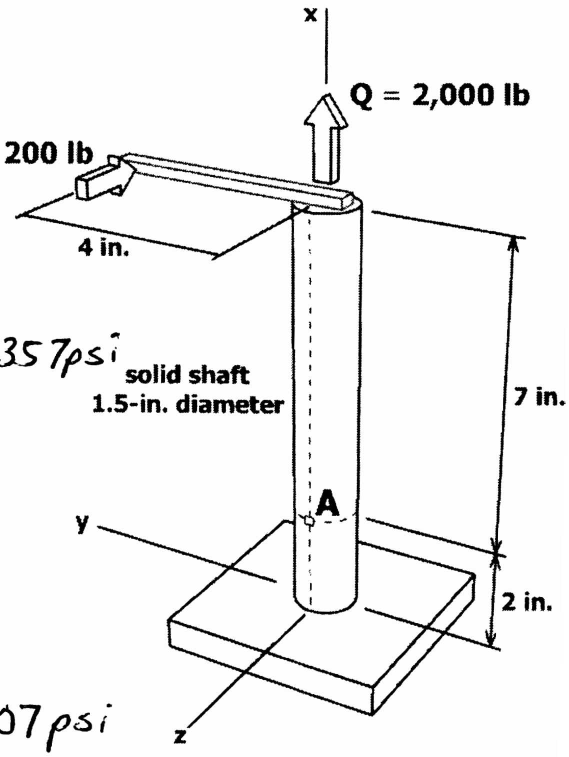
c) $$\tan 2\theta_p = \frac{51.05}{\frac{51.41+7.53}{2}} \qquad \theta_p = 30.04^\circ$$



PRINT CLEARLY
Include proper units
Box-in your answers

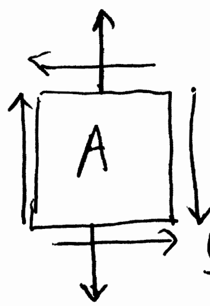
Student ______________________

7. The 1.5-inch-diameter solid shaft is subjected to loads P and Q, as illustrated in the figure. Determine the maximum normal and shear stress at point A on the shaft. [Hint: Please consider the principal and in-plane maximum shearing stresses at A in finding your answer.]



B H
G C
E A
D F

$$(5)\quad \overset{1132}{\frac{2000}{\frac{\pi}{4}(1.5)^2}} + \overset{4225}{\frac{(200)(7)(.75)}{\frac{\pi}{64}(1.5)^4}}\,(6) = 5357\,psi$$



$$(6)\quad \frac{(200)(4)(.75)}{\frac{\pi}{32}(1.5)^4} = 1207\,psi$$

$$(4)\quad \sigma_{max} = \frac{5357}{2} + \sqrt{\left(\frac{5357}{2}\right)^2 + 1207^2} = \boxed{5616\,psi}$$

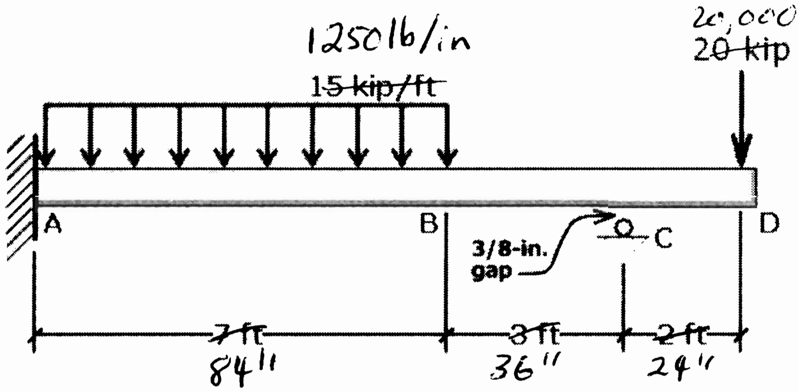
$$(4)\quad \tau_{max} = \sqrt{\left(\frac{5357}{2}\right)^2 + 1207^2} = \boxed{2938\,psi}$$

PRINT CLEARLY
Include proper units
Box-in your answers

BE 110 – Mechanics of Materials
May 9, 2005

Student ______________________________

8. Determine the support reaction at C when there is a gap of 3/8-in. between the bottom of the beam and the top of the support before the beam is loaded. Assume EI = 5×10^9 lb-in^2.



$$\delta_c = -0.375 = \frac{R_c(120)^3}{3EI} - \frac{(20{,}000)(120)^2}{6EI}\left(3(144) - 120\right)$$

$p = 20{,}000$
$x = 120$
$L = 144$

$$-\frac{1250(84)^4}{8EI} - \frac{1250(84)^3}{6EI}(36)$$

$$\boxed{R_c = 43{,}968 \text{ lb}}$$

PRINT CLEARLY
Include proper units
Box-in your answers