$\qquad$

1. If the applied load $P=10 \mathrm{kN}$, determine the axial force in member (1).
$N_{1}=$ $\qquad$ kN

2. If the axial force in member (1) is $38,420 \mathrm{lb}$, determine the resultant force at joint $C$.
$C=$ $\qquad$ lb

3. Determine the normal force in segments (1), (2) and (3), and circle whether they are in tension (T) or compression (C).

(a) $N_{1}=$ $\qquad$ kip ( T or C )
(b) $N_{2}=$ $\qquad$ kip ( T or C )
(c) $N_{3}=$ $\qquad$ kip ( T or C )
4. The single shear connection consists of six 26 -mm-diameter bolts. If the ultimage strength of the bolts is 720 MPa , determine the factor of safety for the connection at an applied load of $P=570 \mathrm{kN}$.
$\mathrm{FS}=$ $\qquad$

5. Member (1) is a steel bar with a cross-sectional area of $1.75 \mathrm{in}^{2}$ and a yield strength of 50 ksi . Member (2) is a pair of aluminum bars having a combined cross-sectional area of 4.50 $i n^{2}$ and a yield strength of 40 ksi . Having already done the statics and knowing that the axial load in the members is $N_{1}=$ $0.7 P$ and $N_{2}=1.22 P$, determine the maximum allowable load $P$ that may be applied to the structure.
$P_{\text {max }}=$ $\qquad$ lb

6. A 8 -mm-diameter wire ( $E=60 \mathrm{GPa}$ ) supports a tension load of $3,500 \mathrm{~N}$. If the total elongation of the wire must not exceed 10 mm , determine the maximum allowable length $L$ of the wire.
$L=$ $\qquad$ m

7. Rigid bar $A B C$ is supported by bronze bar (1) and aluminum rod
(2). A concentrated load $P$ is applied the free end of aluminum rod (3). Bronze rod (1) has a diameter of $d_{1}=0.375$ in. Aluminum rods (2) and (3) have a diameter of $d_{2}=0.625 \mathrm{in}$. and $d_{3}=1.0 \mathrm{in}$. The yield strength of the bronze is 50 ksi , and the yield strength of the aluminum is 36 ksi . Having already done the statics and knowing that the axial load in bars (1) and (2) is $N_{1}=0.375 P$ and $N_{2}=0.625 P$, determine the magnitude of load $P$ that can safely be applied to the structure if a minimum factor of safety of 1.5 is required.
$P_{\text {max }}=$ $\qquad$ lb

8. Fill in the missing pieces of the following derivation. Numbers should be in meters and Newtons. Do NOT solve for $N_{1}$ or simplify your numbers.

A load of $P=170 \mathrm{kN}$ is supported by a structure consisting of rigid bar $A B C$, two identical solid bronze [ $E=100 \mathrm{GPa}$ ] rods, and a solid steel $[E=200 \mathrm{GPa}]$ rod. The bronze rods (1) each have a diameter of 20 mm , and they are symmetrically positioned relative to the center rod (2) and the applied load $P$. Steel rod (2) has a diameter of 24 mm . All bars are unstressed before the load $P$ is applied; however, there is a $3-\mathrm{mm}$ clearance in the bolted connection at $B$. Determine the axial load in the bronze bars.


$$
\begin{aligned}
& N_{1}+N_{2}=170,000 \\
& \delta_{1}=\delta_{2}+.003
\end{aligned}
$$

$\frac{N_{1}}{\left(170,000-N_{1}\right)}+.003$
$N_{1}=\# \# \# \#$
9. Write the statics and compatibility relationships for the following structures.

| Statics: <br> write a formula relating <br> $P, N_{1}$ and $N_{2}$ and state <br> whether $N_{1}$ and $N_{2}$ are in <br> tension or compression | Compatibility: <br> write a formula <br> relating $\delta_{1}$ and $\delta_{2}$ |
| :---: | :---: | :---: | :---: |

10. Fill in the missing pieces of the following derivation. Numbers should be in inches and pounds. Do NOT solve for $N_{1}$ or simplify your numbers.

Rigid bar ABCD is supported by a pin connection at A and by two axial bars (1) and (2). Bar (1) is a 30-in.-long bronze [ $E=$ $15,000 \mathrm{ksi}, \alpha=9.4 \times 10-6 /{ }^{\circ} \mathrm{F}$ ] bar with a cross-sectional area of $1.25 \mathrm{in}^{2}$. Bar (2) is a 40-in.-long aluminum alloy [ $E=10,000$ ksi, $\left.\alpha=12.5 \times 10^{-6} /{ }^{\circ} \mathrm{F}\right]$ bar with a cross-sectional area of $2.00 \mathrm{in} .^{2}$ Both bars are unstressed before the load $P$ is applied. If a concentrated load of $P=27$ kips is applied to the rigid bar at $D$ and the temperature is decreased by $100^{\circ} \mathrm{F}$, determine the axial force in bar (1).


$$
\Sigma M_{A}: 98 P=36 N_{1}+84 N_{2}
$$



$$
\frac{\delta_{1}}{36}=-\frac{\delta_{2}}{84}
$$

$$
\begin{aligned}
& \frac{}{36}+\frac{9.4 \times 10^{-6}(-100)(30)}{36} \\
& =-\left[\frac{\frac{1}{84}\left[98(27000)-36 N_{1}\right](40)}{84\left(10 \times 10^{6}\right)(2)}+\frac{84}{} \quad\left[\begin{array}{l} 
\\
\rightarrow \# \# \# \# \# \#
\end{array}\right]\right.
\end{aligned}
$$

## TRIGONOMETRY



## STATICS

| Symbol | Meaning | Equation | Units |
| :---: | :---: | :---: | :---: |
| $\bar{x}, \bar{y}, \bar{z}$ | centroid position | $\bar{y}=\Sigma \bar{y}_{i} A_{i} / \Sigma A_{i}$ | in, m |
| I | moment of inertia | $\mathrm{I}=\Sigma\left(\mathrm{I}_{\mathrm{i}}+\mathrm{d}_{\mathrm{i}}^{2} \mathrm{~A}_{\mathrm{i}}\right)$ | in ${ }^{4}$, m ${ }^{4}$ |
| J | polar moment of inertia | $\begin{gathered} \mathrm{J}_{\text {solid circular shaft }}=\pi \mathrm{d}^{4} / 32 \\ \mathrm{~J}_{\text {hollow circular shaft }} \\ =\pi\left(\mathrm{d}_{0}^{4}-\mathrm{d}_{\mathrm{i}}^{4}\right) / 32 \end{gathered}$ | in ${ }^{4}$, m ${ }^{4}$ |
| N | normal force |  | lb, N |
| V | shear force | $V=\int-w(x) d x$ | lb, N |
| M | bending moment | $M=\int V(x) d x$ | in-lb, Nm |
| equilibrium |  | $\begin{gathered} \Sigma F=\mathbf{0} \\ \Sigma \mathbf{M}_{(\text {any point })}=0 \end{gathered}$ | $\begin{gathered} \mathrm{lb}, \mathrm{~N} \\ \mathrm{in}-\mathrm{lb}, \mathrm{Nm} \end{gathered}$ |


| SECOND MOMENTS OF PLANE AREAS |  |  |
| :---: | :---: | :---: |
| Rectangular Area $A=b h$ | $I_{x}=\frac{b h^{3}}{12}$ | $I_{x^{\prime}}=\frac{b h^{3}}{3}$ |
| $h$ | $\begin{aligned} & I_{y}=\frac{h b^{3}}{12} \\ & I_{x y}=0 \end{aligned}$ | $\begin{aligned} & I_{y^{\prime}}=\frac{h b^{3}}{3} \\ & I_{x^{\prime} y^{\prime}}=\frac{b^{2} h^{2}}{4} \end{aligned}$ |
| Triangular Area $A=\frac{1}{2} b h$ | $\begin{aligned} & I_{x}=\frac{b h^{3}}{36} \\ & I_{y}=\frac{h b^{3}}{36} \\ & I_{x y}=\frac{b^{2} h^{2}}{72} \end{aligned}$ | $\begin{aligned} & I_{x^{\prime}}=\frac{b h^{3}}{12} \\ & I_{y^{\prime}}=\frac{h b^{3}}{4} \\ & I_{x^{\prime} y^{\prime}}=\frac{b^{2} h^{2}}{8} \end{aligned}$ |
| Circular Area $A=\pi R^{2}$ | $\begin{aligned} & I_{x}=\frac{\pi R^{4}}{4} \\ & I_{y}=\frac{\pi R^{4}}{4} \\ & I_{x y}=0 \end{aligned}$ | $I_{x^{\prime}}=\frac{5 \pi R^{4}}{4}$ |
|  | $\begin{aligned} & I_{x}=\frac{\pi R^{4}}{8}-\frac{8 R^{4}}{9 \pi} \\ & I_{y}=\frac{\pi R^{4}}{8} \\ & I_{x y}=0 \end{aligned}$ | $I_{x^{\prime}}=\frac{\pi R^{4}}{8}$ $I_{x^{\prime} y^{\prime}}=\frac{2 R^{4}}{3}$ |
|  | $I_{x}=\frac{\pi R^{4}}{16}-\frac{4 R^{4}}{9 \pi}$ $I_{x y}=\frac{(9 \pi-32) R^{4}}{72 \pi}$ | $\begin{aligned} & I_{x^{\prime}}=\frac{\pi R^{4}}{16} \\ & I_{y^{\prime}}=\frac{\pi R^{4}}{16} \\ & I_{x^{\prime} y^{\prime}}=\frac{R^{4}}{8} \end{aligned}$ |

## MECHANICS OF MATERIALS

| Topic | Symbol | Meaning | Equation | Units |
| :---: | :---: | :---: | :---: | :---: |
| axial | $\sigma$, sigma | normal stress | $\begin{aligned} \sigma_{\text {axial }} & =\mathrm{N} / \mathrm{A} \\ \tau_{\text {cutting }} & =\mathrm{V} / \mathrm{A} \\ \sigma_{\text {bearing }} & =\mathrm{F}_{\mathrm{b}} / \mathrm{A}_{\mathrm{b}} \end{aligned}$ | psi, Pa |
|  | $\varepsilon$, epsilon | normal strain | $\begin{gathered} \varepsilon_{\text {axial }}=\Delta \mathrm{L} / \mathrm{L}_{\mathrm{o}}=\delta / \mathrm{L}_{\mathrm{o}} \\ \varepsilon_{\text {transverse }}=\Delta \mathrm{d} / \mathrm{d} \end{gathered}$ | in/in, m/m |
|  | $\gamma$, gamma | shear strain | $\gamma=$ change in angle, $\gamma=c \theta$ | rad |
|  | E | Young's modulus, modulus of elasticity | $\sigma=\mathrm{E} \boldsymbol{\varepsilon}$ (one-dimensional only) | psi, Pa |
|  | G | shear modulus, modulus of rigidity | $\mathrm{G}=\tau / \gamma=\mathrm{E} / 2(1+v)$ | psi, Pa |
|  | $v$, nu | Poisson's ratio | $v=-\varepsilon^{\prime} / \varepsilon$ |  |
|  | $\delta$, delta | deformation, elongation, deflection | $N / E A+\alpha \Delta T$ | in, m |
|  | $\alpha$, alpha | coefficient of thermal expansion (CTE) |  | in/inF, m/mC |
|  | F.S. | factor of safety | F.S. = actual strength / design strength |  |


| Topic | Symbol | Meaning | Equation |  | Units |
| :---: | :---: | :---: | :---: | :---: | :---: |
| torsion | $\tau$, tau | shear stress | $\tau_{\text {torsion }}=\mathrm{Tc} / \mathrm{J}$ |  | psi, Pa |
|  | $\phi$, phi | angle of twist | $\phi=\mathrm{TL} / \mathrm{GJ}$ |  | rad, degrees |
|  | $\theta$, theta | angle of twist per unit length, rate of twist | $\theta=\phi / L$ |  | rad/in, rad/m |
|  | P | power | $\mathrm{P}=\mathrm{T} \omega$ | $\begin{gathered} r_{2} T_{1}=r_{1} T_{2} \\ r_{1} \omega_{1}=r_{2} \omega_{2} \end{gathered}$ | $\begin{gathered} \text { watts }=\mathrm{Nm} / \mathrm{s} \\ \mathrm{hp}=6600 \mathrm{in}-\mathrm{lb} / \mathrm{s} \end{gathered}$ |
|  | $\begin{gathered} \omega, \\ \text { omega } \end{gathered}$ | angular speed, speed of rotation |  |  | rad/s |
|  | f | frequency | $\omega=2 \pi \mathrm{f}$ |  | $\mathrm{Hz}=\mathrm{rev} / \mathrm{s}$ |
|  | K | stress concentration factor | $\tau_{\text {max }}=\mathrm{KTc} / \mathrm{J}$ |  | psi, Pa |
| flexure | $\sigma$, sigma | normal stress $\quad 3$ | $\sigma_{\text {beam }}=-\mathrm{My} / \mathrm{l}$ |  | psi, Pa |
|  | $\sigma$, sigma | composite beams, $n=E_{B} / E_{A}$ | $\sigma_{A}=-M y / I^{\top}$ | $\sigma_{B}=-n M y / I^{\top}$ | psi, Pa |
|  | $\tau$, tau | shear stress | $\tau_{\text {beam }}=\mathrm{VQ} / \mathrm{lb}$ where $\mathrm{Q}=\Sigma\left(\mathrm{y}_{\text {bar i }} \mathrm{A}_{\mathrm{i}}\right)$ |  | psi, Pa |
|  | q | shear flow | $\mathrm{q}=\mathrm{V}_{\text {beam }} \mathrm{Q} / \mathrm{I}=\mathrm{n} \mathrm{V}_{\text {fastener }} / \mathrm{s}$ |  |  |
|  | v or y | beam deflection | $v=\iint M(x) d x^{2} / E l$ |  | in, m |
| Topic |  | Equations |  |  | Units |
| stress <br> trans- <br> formation |  | planar rotations $\begin{gathered} \sigma_{\mathrm{u}}=\left(\sigma_{\mathrm{x}}+\sigma_{\mathrm{y}}\right) / 2+\left(\sigma_{\mathrm{x}}-\sigma_{\mathrm{y}}\right) / 2 \cos (2 \theta)+\tau_{\mathrm{xy}} \sin (2 \theta) \\ \sigma_{\mathrm{v}}=\left(\sigma_{\mathrm{x}}+\sigma_{\mathrm{y}}\right) / 2-\left(\sigma_{\mathrm{x}}-\sigma_{\mathrm{y}}\right) / 2 \cos (2 \theta)-\tau_{\mathrm{xy}} \sin (2 \theta) \\ \tau_{\mathrm{uv}}=-\left(\sigma_{\mathrm{x}}-\sigma_{\mathrm{y}}\right) / 2 \sin (2 \theta)+\tau_{\mathrm{xy}} \cos (2 \theta) \end{gathered}$ | principals and max in-plane shear$\begin{gathered} \tan \left(2 \theta_{\mathrm{p}}\right)=2 \tau_{\mathrm{xy}} /\left(\sigma_{\mathrm{x}}-\sigma_{\mathrm{y}}\right), \theta_{\mathrm{s}}=\theta_{\mathrm{p}} \pm 45^{\circ} \\ \sigma_{1,2}=\left(\sigma_{\mathrm{x}}+\sigma_{\mathrm{y}}\right) / 2 \pm \operatorname{sqrt}\left\{\left[\left(\sigma_{\mathrm{x}}-\sigma_{\mathrm{y}}\right) / 2\right]^{2}+\tau_{\mathrm{xy}}{ }^{2}\right\} \\ \tau_{\max }=\operatorname{sqrt}\left\{\left[\left(\sigma_{\mathrm{x}}-\sigma_{\mathrm{y}}\right) / 2\right]^{2}+\tau_{\mathrm{xy}}{ }^{2}\right\}=\left(\sigma_{1}-\sigma_{2}\right) / 2 \\ \sigma_{\mathrm{avg}}=\left(\sigma_{\mathrm{x}}+\sigma_{\mathrm{y}}\right) / 2=\left(\sigma_{1}+\sigma_{2}\right) / 2 \end{gathered}$ |  | psi, Pa |
| strain <br> trans- <br> formation |  | planar rotations $\begin{gathered} \varepsilon_{\mathrm{u}}=\left(\varepsilon_{\mathrm{x}}+\varepsilon_{\mathrm{y}}\right) / 2+\left(\varepsilon_{\mathrm{x}}-\varepsilon_{\mathrm{y}}\right) / 2 \cos (2 \theta)+\gamma_{\mathrm{xy}} / 2 \sin (2 \theta) \\ \varepsilon_{\mathrm{v}}=\left(\varepsilon_{\mathrm{x}}+\varepsilon_{\mathrm{y}}\right) / 2-\left(\varepsilon_{\mathrm{x}}-\varepsilon_{\mathrm{y}}\right) / 2 \cos (2 \theta)-\gamma_{\mathrm{xy}} / 2 \sin (2 \theta) \\ \gamma_{\mathrm{uv}} / 2=-\left(\varepsilon_{\mathrm{x}}-\varepsilon_{\mathrm{y}}\right) / 2 \sin (2 \theta)+\gamma_{\mathrm{xy}} / 2 \cos (2 \theta) \\ \varepsilon_{\mathrm{z}}=-v\left(\varepsilon_{\mathrm{x}}+\varepsilon_{\mathrm{y}}\right) /(1-v) \end{gathered}$ | $\begin{gathered} \text { principals and max in-plane shear } \\ \tan \left(2 \theta_{\mathrm{p}}\right)=\gamma_{\mathrm{xy}} /\left(\varepsilon_{\mathrm{x}}-\varepsilon_{\mathrm{y}}\right), \theta_{\mathrm{s}}=\theta_{\mathrm{p}} \pm 45^{\circ} \\ \varepsilon_{1,2}=\left(\varepsilon_{\mathrm{x}}+\varepsilon_{y}\right) / 2 \pm \operatorname{sqrt}\left\{\left[\left(\varepsilon_{\mathrm{x}}-\varepsilon_{\mathrm{y}}\right) / 2\right]^{2}+\left(\gamma_{\mathrm{xy}} / 2\right)^{2}\right\} \\ \gamma_{\text {max }} / 2=\operatorname{sqrt}\left\{\left[\left(\varepsilon_{\mathrm{x}}-\varepsilon_{\mathrm{y}}\right) / 2\right]^{2}+\left(\gamma_{\mathrm{xy}} / 2\right)^{2}\right\} \\ \varepsilon_{\text {avg }}=\left(\varepsilon_{\mathrm{x}}+\varepsilon_{\mathrm{y}}\right) / 2 \end{gathered}$ |  | psi, Pa <br> in/in, m/m |
| Hooke's law |  | 1D strain to stress $\sigma=E \varepsilon$ <br> 2D strain to stress $\begin{gathered} \sigma_{x}=\mathrm{E}\left(\varepsilon_{\mathrm{x}}+v \varepsilon_{\mathrm{y}}\right) /\left(1-v^{2}\right) \\ \sigma_{\mathrm{y}}=\mathrm{E}\left(\varepsilon_{\mathrm{y}}+v \varepsilon_{\mathrm{x}}\right) /\left(1-v^{2}\right) \\ \tau_{\mathrm{xy}}=\mathrm{G} \gamma_{\mathrm{xy}}=\mathrm{E} \gamma_{\mathrm{xy}} / 2(1+v) \end{gathered}$ | 2D stress to strain$\begin{gathered} \varepsilon_{x}=\left(\sigma_{x}-v \sigma_{y}\right) / E \\ \varepsilon_{y}=\left(\sigma_{y}-v \sigma_{x}\right) / E \\ \varepsilon_{z}=-v\left(\varepsilon_{x}+\varepsilon_{y}\right) /(1-v) \\ \gamma_{x y}=\tau_{x y} / G=2(1+v) \tau_{x y} / E \end{gathered}$ |  | psi, Pa <br> in/in, m/m |
| pressure |  | $\begin{gathered} \sigma_{\text {spherical }}=\mathrm{pr} / 2 \mathrm{t} \\ \sigma_{\text {cylindrical, hoop }}=\mathrm{pr} / \mathrm{t} \\ \sigma_{\text {cylindrical, axial }}=\mathrm{pr} / 2 \mathrm{t} \end{gathered}$ | $\begin{aligned} & \sigma_{\text {radial, outside }}=0 \\ & \sigma_{\text {radial, inside }}=-p \end{aligned}$ |  | psi, Pa |
| failure theories |  | maximum principal stress theory $\sigma_{1,2}<\sigma_{y p}$ | maximum $\tau_{\mathrm{m}}$ | $\begin{aligned} & \text { tress theory } \\ & \sigma_{\mathrm{yp}} \end{aligned}$ | psi, Pa |

