$\qquad$

1. In calculating the shear flow associated with the nail shown, which areas should be included in the calculation of $Q$ ? (3 points)
$\qquad$ Areas (1) and (5)
$\qquad$ Areas (1) through (5)
$\qquad$ Areas (2), (3) and (4)
$\qquad$ Areas (1), (2), (4) and (5)

2. In calculating the shear flow associated with the two nails shown, which areas should be included in the calculation of $Q$ ? (3 points)
$\qquad$ Areas (2) through (6)
$\qquad$ Areas (2), (3), (5) and (6)
$\qquad$ Area (4)
$\qquad$ Areas (3), (4) and (5)
3. What is the value of $Q$ needed to determine the shear force acting on nail $B$ ? (6 points)
$\qquad$ $390,625 \mathrm{~mm}^{3}$
$\qquad$ $1,140,625 \mathrm{~mm}^{3}$
$\qquad$ $875,000 \mathrm{~mm}^{3}$
$\qquad$ $750,000 \mathrm{~mm}^{3}$

4. What is the value of $Q$ needed to determine the shear force acting on the two nails shown? (6 points)
$\qquad$ $480,000 \mathrm{~mm}^{3}$
$\qquad$ $3,200 \mathrm{~mm}^{3}$
$\qquad$ $96,000 \mathrm{~mm}^{3}$
$\qquad$ $144,000 \mathrm{~mm}^{3}$

5. If boards (1) and (2) were glued to board (3) instead of nailed, what would be the shear stress in the glue if $V=1,125 \mathrm{lb}$ ? (6 points)
$\qquad$ 16.5 psi
$\qquad$ 49.6 psi
$\qquad$ 99.3 psi

$\qquad$ 198.5 psi
6. The allowable load for each weld is $2.4 \mathrm{kip} / \mathrm{inch}$ in the longitudinal direction. What is the maximum allowable shear force $V$ ? (8 points)
$\qquad$ 10.5 kips
$\qquad$ 12.8 kips
$\qquad$ 20.9 kips
$\qquad$ 25.6 kips

7. A 1.00 -inch-diameter solid steel shaft supports loads $P_{\mathrm{A}}=300 \mathrm{lb}$ and $P_{\mathrm{D}}=500$ lb. Assume $L_{1}=5 \mathrm{in}$., $L_{2}=16 \mathrm{in}$., and $L_{3}$ $=8 \mathrm{in}$. Determine the values for $V, Q, I$, and $b$ that would be used in the maximum horizontal shear stress anywhere in the shaft. (20 points)


$$
V=\ldots \mathrm{lb}
$$

$Q=$ $\qquad$ in. ${ }^{3}$
$I=$ $\qquad$ in. ${ }^{4}$
$b=$ $\qquad$ in.
8. For the beam and loading shown, use the integration method to determine the equation of the elastic curve for segment $A B$ of the beam. Do NOT solve for the integration constants. (8 points)

9. List the boundary, continuity, and/or symmetry conditions that could be used to solve the integration constants in the previous problem. (6 points)
10. Determine the beam deflection at point $H$.

Assume that $E I=40,000 \mathrm{kN-m}{ }^{2}$ is constant for the beam. ( 6 points)

11. Determine the beam deflection at point $H$. Assume that $E I=40,000 \mathrm{kN}-\mathrm{m}^{2}$ is constant for the beam. (10 points)
$v_{\mathrm{H}}=$ $\qquad$ mm

12. Determine the beam deflection at point $H$. Assume that $E I=40,000 \mathrm{kN}^{2} \mathrm{~m}^{2}$ is constant for the beam. (8 points)
$v_{\mathrm{H}}=$ $\qquad$ mm

13. Determine the beam deflection at point $C$.

Assume that $E I=40,000 \mathrm{kN}-\mathrm{m}^{2}$ is constant for the beam. (10 points)
$v_{\mathrm{C}}=$ $\qquad$ mm


## TRIGONOMETRY



## STATICS

| Symbol | Meaning | Equation | Units |
| :---: | :---: | :---: | :---: |
| $\bar{x}, \bar{y}, \bar{z}$ | centroid position | $\bar{y}=\Sigma \bar{y}_{i} A_{i} / \Sigma A_{i}$ | in, m |
| I | moment of inertia | $\mathrm{I}=\Sigma\left(\mathrm{I}_{\mathrm{i}}+\mathrm{d}_{\mathrm{i}}^{2} \mathrm{~A}_{\mathrm{i}}\right)$ | in ${ }^{4}$, m ${ }^{4}$ |
| J | polar moment of inertia | $\begin{gathered} \mathrm{J}_{\text {solid circular shaft }}=\pi \mathrm{d}^{4} / 32 \\ \mathrm{~J}_{\text {hollow circular shaft }} \\ =\pi\left(\mathrm{d}_{0}^{4}-\mathrm{d}_{\mathrm{i}}^{4}\right) / 32 \end{gathered}$ | in ${ }^{4}$, m ${ }^{4}$ |
| N | normal force |  | lb, N |
| V | shear force | $V=\int-w(x) d x$ | lb, N |
| M | bending moment | $M=\int V(x) d x$ | in-lb, Nm |
| equilibrium |  | $\begin{gathered} \Sigma F=\mathbf{0} \\ \Sigma \mathbf{M}_{(\text {any point })}=0 \end{gathered}$ | $\begin{gathered} \mathrm{lb}, \mathrm{~N} \\ \mathrm{in}-\mathrm{lb}, \mathrm{Nm} \end{gathered}$ |


| SECOND MOMENTS OF PLANE AREAS |  |  |
| :---: | :---: | :---: |
| Rectangular Area $A=b h$ | $I_{x}=\frac{b h^{3}}{12}$ | $I_{x^{\prime}}=\frac{b h^{3}}{3}$ |
| $h$ | $\begin{aligned} & I_{y}=\frac{h b^{3}}{12} \\ & I_{x y}=0 \end{aligned}$ | $\begin{aligned} & I_{y^{\prime}}=\frac{h b^{3}}{3} \\ & I_{x^{\prime} y^{\prime}}=\frac{b^{2} h^{2}}{4} \end{aligned}$ |
| Triangular Area $A=\frac{1}{2} b h$ | $\begin{aligned} & I_{x}=\frac{b h^{3}}{36} \\ & I_{y}=\frac{h b^{3}}{36} \\ & I_{x y}=\frac{b^{2} h^{2}}{72} \end{aligned}$ | $\begin{aligned} & I_{x^{\prime}}=\frac{b h^{3}}{12} \\ & I_{y^{\prime}}=\frac{h b^{3}}{4} \\ & I_{x^{\prime} y^{\prime}}=\frac{b^{2} h^{2}}{8} \end{aligned}$ |
| Circular Area $A=\pi R^{2}$ | $\begin{aligned} & I_{x}=\frac{\pi R^{4}}{4} \\ & I_{y}=\frac{\pi R^{4}}{4} \\ & I_{x y}=0 \end{aligned}$ | $I_{x^{\prime}}=\frac{5 \pi R^{4}}{4}$ |
|  | $\begin{aligned} & I_{x}=\frac{\pi R^{4}}{8}-\frac{8 R^{4}}{9 \pi} \\ & I_{y}=\frac{\pi R^{4}}{8} \\ & I_{x y}=0 \end{aligned}$ | $I_{x^{\prime}}=\frac{\pi R^{4}}{8}$ $I_{x^{\prime} y^{\prime}}=\frac{2 R^{4}}{3}$ |
|  | $I_{x}=\frac{\pi R^{4}}{16}-\frac{4 R^{4}}{9 \pi}$ $I_{x y}=\frac{(9 \pi-32) R^{4}}{72 \pi}$ | $\begin{aligned} & I_{x^{\prime}}=\frac{\pi R^{4}}{16} \\ & I_{y^{\prime}}=\frac{\pi R^{4}}{16} \\ & I_{x^{\prime} y^{\prime}}=\frac{R^{4}}{8} \end{aligned}$ |

## MECHANICS OF MATERIALS

| Topic | Symbol | Meaning | Equation | Units |
| :---: | :---: | :---: | :---: | :---: |
| axial | $\sigma$, sigma | normal stress | $\begin{aligned} \sigma_{\text {axial }} & =\mathrm{N} / \mathrm{A} \\ \tau_{\text {cutting }} & =\mathrm{V} / \mathrm{A} \\ \sigma_{\text {bearing }} & =\mathrm{F}_{\mathrm{b}} / \mathrm{A}_{\mathrm{b}} \end{aligned}$ | psi, Pa |
|  | $\varepsilon$, epsilon | normal strain | $\begin{gathered} \varepsilon_{\text {axial }}=\Delta \mathrm{L} / \mathrm{L}_{\mathrm{o}}=\delta / \mathrm{L}_{\mathrm{o}} \\ \varepsilon_{\text {transverse }}=\Delta \mathrm{d} / \mathrm{d} \end{gathered}$ | in/in, m/m |
|  | $\gamma$, gamma | shear strain | $\gamma=$ change in angle, $\gamma=c \theta$ | rad |
|  | E | Young's modulus, modulus of elasticity | $\sigma=\mathrm{E} \boldsymbol{\varepsilon}$ (one-dimensional only) | psi, Pa |
|  | G | shear modulus, modulus of rigidity | $\mathrm{G}=\tau / \gamma=\mathrm{E} / 2(1+v)$ | psi, Pa |
|  | $v$, nu | Poisson's ratio | $v=-\varepsilon^{\prime} / \varepsilon$ |  |
|  | $\delta$, delta | deformation, elongation, deflection | $N / E A+\alpha \Delta T$ | in, m |
|  | $\alpha$, alpha | coefficient of thermal expansion (CTE) |  | in/inF, m/mC |
|  | F.S. | factor of safety | F.S. = actual strength / design strength |  |


| Topic | Symbol | Meaning | Equation |  | Units |
| :---: | :---: | :---: | :---: | :---: | :---: |
| torsion | $\tau$, tau | shear stress | $\tau_{\text {torsion }}=\mathrm{Tc} / \mathrm{J}$ |  | psi, Pa |
|  | $\phi$, phi | angle of twist | $\phi=\mathrm{TL} / \mathrm{GJ}$ |  | rad, degrees |
|  | $\theta$, theta | angle of twist per unit length, rate of twist | $\theta=\phi / L$ |  | rad/in, rad/m |
|  | P | power | $\mathrm{P}=\mathrm{T} \omega$ | $\begin{gathered} r_{2} T_{1}=r_{1} T_{2} \\ r_{1} \omega_{1}=r_{2} \omega_{2} \end{gathered}$ | $\begin{gathered} \text { watts }=\mathrm{Nm} / \mathrm{s} \\ \mathrm{hp}=6600 \mathrm{in}-\mathrm{lb} / \mathrm{s} \end{gathered}$ |
|  | $\begin{gathered} \omega, \\ \text { omega } \end{gathered}$ | angular speed, speed of rotation |  |  | rad/s |
|  | f | frequency | $\omega=2 \pi \mathrm{f}$ |  | $\mathrm{Hz}=\mathrm{rev} / \mathrm{s}$ |
|  | K | stress concentration factor | $\tau_{\text {max }}=\mathrm{KTc} / \mathrm{J}$ |  | psi, Pa |
| flexure | $\sigma$, sigma | normal stress $\quad 3$ | $\sigma_{\text {beam }}=-\mathrm{My} / \mathrm{l}$ |  | psi, Pa |
|  | $\sigma$, sigma | composite beams, $n=E_{B} / E_{A}$ | $\sigma_{A}=-M y / I^{\top}$ | $\sigma_{B}=-n M y / I^{\top}$ | psi, Pa |
|  | $\tau$, tau | shear stress | $\tau_{\text {beam }}=\mathrm{VQ} / \mathrm{lb}$ where $\mathrm{Q}=\Sigma\left(\mathrm{y}_{\text {bar i }} \mathrm{A}_{\mathrm{i}}\right)$ |  | psi, Pa |
|  | q | shear flow | $\mathrm{q}=\mathrm{V}_{\text {beam }} \mathrm{Q} / \mathrm{I}=\mathrm{n} \mathrm{V}_{\text {fastener }} / \mathrm{s}$ |  |  |
|  | v or y | beam deflection | $v=\iint M(x) d x^{2} / E l$ |  | in, m |
| Topic |  | Equations |  |  | Units |
| stress <br> trans- <br> formation |  | planar rotations $\begin{gathered} \sigma_{\mathrm{u}}=\left(\sigma_{\mathrm{x}}+\sigma_{\mathrm{y}}\right) / 2+\left(\sigma_{\mathrm{x}}-\sigma_{\mathrm{y}}\right) / 2 \cos (2 \theta)+\tau_{\mathrm{xy}} \sin (2 \theta) \\ \sigma_{\mathrm{v}}=\left(\sigma_{\mathrm{x}}+\sigma_{\mathrm{y}}\right) / 2-\left(\sigma_{\mathrm{x}}-\sigma_{\mathrm{y}}\right) / 2 \cos (2 \theta)-\tau_{\mathrm{xy}} \sin (2 \theta) \\ \tau_{\mathrm{uv}}=-\left(\sigma_{\mathrm{x}}-\sigma_{\mathrm{y}}\right) / 2 \sin (2 \theta)+\tau_{\mathrm{xy}} \cos (2 \theta) \end{gathered}$ | principals and max in-plane shear$\begin{gathered} \tan \left(2 \theta_{\mathrm{p}}\right)=2 \tau_{\mathrm{xy}} /\left(\sigma_{\mathrm{x}}-\sigma_{\mathrm{y}}\right), \theta_{\mathrm{s}}=\theta_{\mathrm{p}} \pm 45^{\circ} \\ \sigma_{1,2}=\left(\sigma_{\mathrm{x}}+\sigma_{\mathrm{y}}\right) / 2 \pm \operatorname{sqrt}\left\{\left[\left(\sigma_{\mathrm{x}}-\sigma_{\mathrm{y}}\right) / 2\right]^{2}+\tau_{\mathrm{xy}}{ }^{2}\right\} \\ \tau_{\max }=\operatorname{sqrt}\left\{\left[\left(\sigma_{\mathrm{x}}-\sigma_{\mathrm{y}}\right) / 2\right]^{2}+\tau_{\mathrm{xy}}{ }^{2}\right\}=\left(\sigma_{1}-\sigma_{2}\right) / 2 \\ \sigma_{\mathrm{avg}}=\left(\sigma_{\mathrm{x}}+\sigma_{\mathrm{y}}\right) / 2=\left(\sigma_{1}+\sigma_{2}\right) / 2 \end{gathered}$ |  | psi, Pa |
| strain <br> trans- <br> formation |  | planar rotations $\begin{gathered} \varepsilon_{\mathrm{u}}=\left(\varepsilon_{\mathrm{x}}+\varepsilon_{\mathrm{y}}\right) / 2+\left(\varepsilon_{\mathrm{x}}-\varepsilon_{\mathrm{y}}\right) / 2 \cos (2 \theta)+\gamma_{\mathrm{xy}} / 2 \sin (2 \theta) \\ \varepsilon_{\mathrm{v}}=\left(\varepsilon_{\mathrm{x}}+\varepsilon_{\mathrm{y}}\right) / 2-\left(\varepsilon_{\mathrm{x}}-\varepsilon_{\mathrm{y}}\right) / 2 \cos (2 \theta)-\gamma_{\mathrm{xy}} / 2 \sin (2 \theta) \\ \gamma_{\mathrm{uv}} / 2=-\left(\varepsilon_{\mathrm{x}}-\varepsilon_{\mathrm{y}}\right) / 2 \sin (2 \theta)+\gamma_{\mathrm{xy}} / 2 \cos (2 \theta) \\ \varepsilon_{\mathrm{z}}=-v\left(\varepsilon_{\mathrm{x}}+\varepsilon_{\mathrm{y}}\right) /(1-v) \end{gathered}$ | $\begin{gathered} \text { principals and max in-plane shear } \\ \tan \left(2 \theta_{\mathrm{p}}\right)=\gamma_{\mathrm{xy}} /\left(\varepsilon_{\mathrm{x}}-\varepsilon_{\mathrm{y}}\right), \theta_{\mathrm{s}}=\theta_{\mathrm{p}} \pm 45^{\circ} \\ \varepsilon_{1,2}=\left(\varepsilon_{\mathrm{x}}+\varepsilon_{y}\right) / 2 \pm \operatorname{sqrt}\left\{\left[\left(\varepsilon_{\mathrm{x}}-\varepsilon_{\mathrm{y}}\right) / 2\right]^{2}+\left(\gamma_{\mathrm{xy}} / 2\right)^{2}\right\} \\ \gamma_{\text {max }} / 2=\operatorname{sqrt}\left\{\left[\left(\varepsilon_{\mathrm{x}}-\varepsilon_{\mathrm{y}}\right) / 2\right]^{2}+\left(\gamma_{\mathrm{xy}} / 2\right)^{2}\right\} \\ \varepsilon_{\text {avg }}=\left(\varepsilon_{\mathrm{x}}+\varepsilon_{\mathrm{y}}\right) / 2 \end{gathered}$ |  | psi, Pa <br> in/in, m/m |
| Hooke's law |  | 1D strain to stress $\sigma=E \varepsilon$ <br> 2D strain to stress $\begin{gathered} \sigma_{x}=\mathrm{E}\left(\varepsilon_{\mathrm{x}}+v \varepsilon_{\mathrm{y}}\right) /\left(1-v^{2}\right) \\ \sigma_{\mathrm{y}}=\mathrm{E}\left(\varepsilon_{\mathrm{y}}+v \varepsilon_{\mathrm{x}}\right) /\left(1-v^{2}\right) \\ \tau_{\mathrm{xy}}=\mathrm{G} \gamma_{\mathrm{xy}}=\mathrm{E} \gamma_{\mathrm{xy}} / 2(1+v) \end{gathered}$ | $2 D$ stress to strain$\begin{gathered} \varepsilon_{x}=\left(\sigma_{x}-v \sigma_{y}\right) / E \\ \varepsilon_{y}=\left(\sigma_{y}-v \sigma_{x}\right) / E \\ \varepsilon_{z}=-v\left(\varepsilon_{x}+\varepsilon_{y}\right) /(1-v) \\ \gamma_{x y}=\tau_{x y} / G=2(1+v) \tau_{x y} / E \end{gathered}$ |  | psi, Pa <br> in/in, m/m |
| pressure |  | $\begin{gathered} \sigma_{\text {spherical }}=\mathrm{pr} / 2 \mathrm{t} \\ \sigma_{\text {cylindrical, hoop }}=\mathrm{pr} / \mathrm{t} \\ \sigma_{\text {cylindrical, axial }}=\mathrm{pr} / 2 \mathrm{t} \end{gathered}$ | $\begin{aligned} & \sigma_{\text {radial, outside }}=0 \\ & \sigma_{\text {radial, inside }}=-p \end{aligned}$ |  | psi, Pa |
| failure theories |  | maximum principal stress theory $\sigma_{1,2}<\sigma_{y p}$ | maximum $\tau_{\mathrm{m}}$ | $\begin{aligned} & \text { tress theory } \\ & \sigma_{\mathrm{yp}} \end{aligned}$ | psi, Pa |


| Cantilever Beams |  |  |  |
| :---: | :---: | :---: | :---: |
| Beam | Slope | Deflection | Elastic Curve |
|  | $\theta_{\text {max }}=-\frac{P L^{2}}{2 E I}$ | $v_{\text {max }}=-\frac{P L^{3}}{3 E I}$ | $v=-\frac{P x^{2}}{6 E I}(3 L-x)$ |
|  | $\theta_{\text {max }}=-\frac{M L}{E I}$ | $v_{\text {max }}=-\frac{M L^{2}}{2 E I}$ | $v=-\frac{M x^{2}}{2 E I}$ |
|  | $\theta_{\text {max }}=-\frac{w L^{3}}{6 E I}$ | $v_{\text {max }}=-\frac{w L^{4}}{8 E I}$ | $v=-\frac{w x^{2}}{24 E I}\left(6 L^{2}-4 L x+x^{2}\right)$ |
|  | $\theta_{\text {max }}=-\frac{w_{0} L^{3}}{24 E I}$ | $v_{\text {max }}=-\frac{w_{0} L^{4}}{30 E I}$ | $v=-\frac{w_{0} x^{2}}{120 L E I}\left(10 L^{3}-10 L^{2} x+5 L x^{2}-x^{3}\right)$ |


| SIMPLY SUPPORTED BEAMS |  |  |  |
| :---: | :---: | :---: | :---: |
| Beam | Slope | Deflection | Elastic Curve |
|  | $\theta_{1}=-\theta_{2}=-\frac{P L^{2}}{16 E I}$ | $v_{\max }=-\frac{P L^{3}}{48 E I}$ | $\begin{aligned} & v=-\frac{P x}{48 E I}\left(3 L^{2}-4 x^{2}\right) \\ & \\ & \quad \text { for } 0 \leq x \leq L / 2 \end{aligned}$ |
|  | $\begin{aligned} & \theta_{1}=-\frac{P b\left(L^{2}-b^{2}\right)}{6 L E I} \\ & \theta_{2}=+\frac{P a\left(L^{2}-a^{2}\right)}{6 L E I} \end{aligned}$ | $\left.v\right\|_{x=a}=-\frac{P b a}{6 L E I}\left(L^{2}-b^{2}-a^{2}\right)$ | $\begin{array}{r} v=-\frac{P b x}{6 L E I}\left(L^{2}-b^{2}-x^{2}\right) \\ \text { for } 0 \leq x \leq a \end{array}$ |
|  | $\begin{aligned} & \theta_{1}=-\frac{M L}{3 E I} \\ & \theta_{2}=+\frac{M L}{6 E I} \end{aligned}$ | $\begin{aligned} & v_{\max }=-\frac{M L^{2}}{9 \sqrt{3} E I} \\ & @ x=L\left(1-\frac{\sqrt{3}}{3}\right) \end{aligned}$ | $v=-\frac{M x}{6 L E I}\left(2 L^{2}-3 L x+x^{2}\right)$ |
|  | $\theta_{1}=-\theta_{2}=-\frac{w L^{3}}{24 E I}$ | $v_{\max }=-\frac{5 w L^{4}}{384 E I}$ | $v=-\frac{w x}{24 E I}\left(L^{3}-2 L x^{2}+x^{3}\right)$ |
|  | $\begin{aligned} & \theta_{1}=-\frac{w a^{2}}{24 L E I}(2 L-a)^{2} \\ & \theta_{2}=+\frac{w a^{2}}{24 L E I}\left(2 L^{2}-a^{2}\right) \end{aligned}$ | $\left.v\right\|_{x=a}=-\frac{w a^{3}}{24 L E I}\left(3 a^{2}-7 a L+4 L^{2}\right)$ | $\begin{gathered} v=-\frac{w x}{24 L E I}\left(a^{4}-4 a^{3} L+4 a^{2} L^{2}+2 a^{2} x^{2}\right. \\ \left.-4 a L x^{2}+L x^{3}\right) \quad \text { for } 0 \leq x \leq a \\ v=-\frac{w a^{2}}{24 L E I}\left(-a^{2} L+4 L^{2} x+a^{2} x-6 L x^{2}+2 x^{3}\right) \\ \text { for } a \leq x \leq L \end{gathered}$ |
|  | $\begin{aligned} & \theta_{1}=-\frac{7 w_{0} L^{3}}{360 E I} \\ & \theta_{2}=+\frac{w_{0} L^{3}}{45 E I} \end{aligned}$ | $v_{\max }=-0.00652 \frac{w_{0} L^{4}}{E I}$ <br> @ $x=0.5193 L$ | $v=-\frac{w_{0} x}{360 L E I}\left(7 L^{4}-10 L^{2} x^{2}+3 x^{4}\right)$ |

