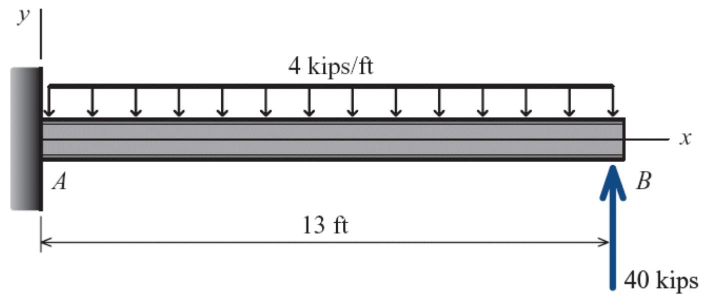
1. Sketch the ground reactions on the diagram and write the following equations *(in units of kips and feet)*. *(8 points)*

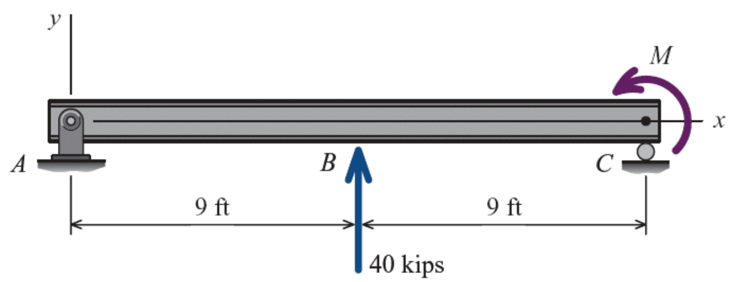



$\rightarrow \Sigma F_x = 0 =$ ____________________________________________

$\uparrow \Sigma F_y = 0 =$ ____________________________________________

$\Sigma M_A = 0 =$ ____________________________________________
(counter-clockwise as positive)

2. Sketch the ground reactions on the diagram and write the following equations *(in units of kips and feet)*. *(8 points)*

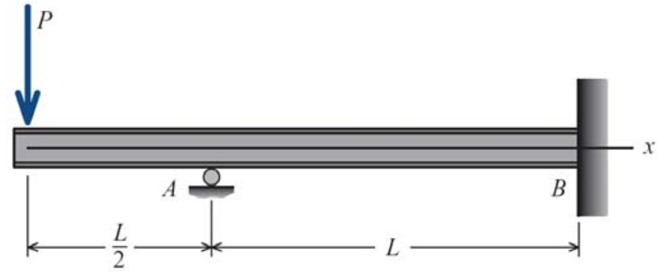



$\Sigma F_x = 0 =$ ____________________________________________

$\Sigma F_y = 0 =$ ____________________________________________

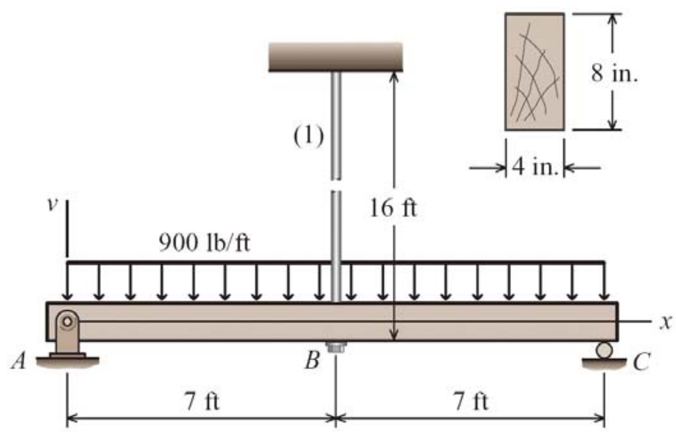
$\Sigma M_A = 0 =$ ____________________________________________
(counter-clockwise as positive)

3. For the beam and loading shown, derive an expression for the reaction at support A. Assume that EI is constant for the beam. *(12 points)*



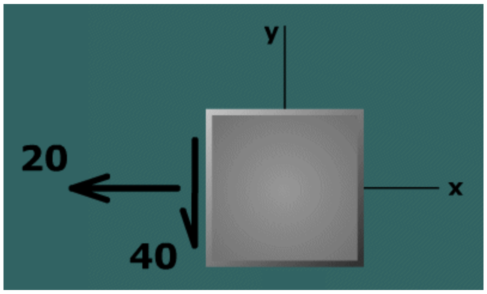

$A_y =$ _________

4. A timber [$E$ = 1,800 ksi] beam is loaded and supported as shown. The cross section of the timber beam is 4 in. wide and 8 in. deep. The beam is supported at $B$ by a 0.5-inch-diameter steel [$E$ = 30,000 ksi] rod, which has no load before the distributed load is applied to the beam. After a distributed load of 900 lb/ft is applied to the beam, determine the force carried by the steel rod. *(12 points)*




$B =$ _________ kips

5. What is the correct value of the normal stress shown on the stress element? *(3 points)*




______ -40

______ -20

______ +40

______ +20

6. What is the correct value of the shear stress shown on the stress element in the previous problem? *(3 points)*

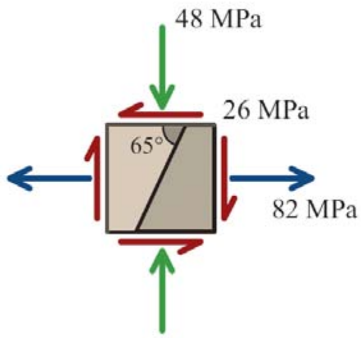
______ -40

______ -20

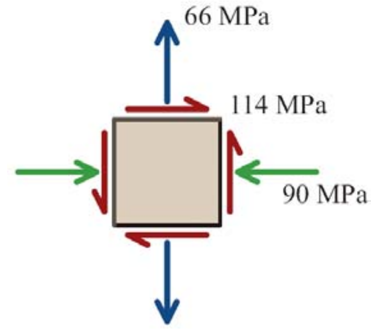
______ +40

______ +20

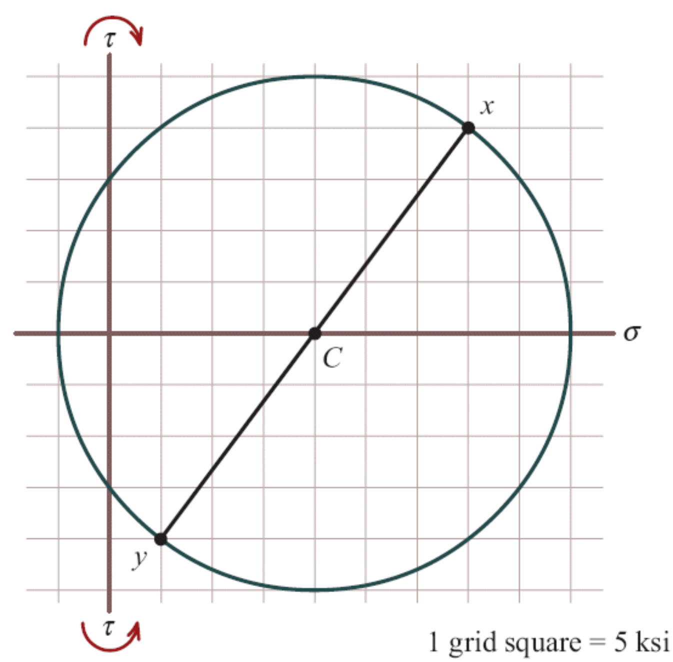
7. The stresses shown act at a point in a stressed body. Determine the normal and shear stresses at this point on the <u>inclined plane</u> shown. Show these on an appropriate **sketch**. *(12 points)*



8. Consider a point in a structural member that is subjected to plane stress. Normal and shear stresses acting on horizontal and vertical planes at the point are shown. Determine the principal stresses acting at the point. Show these stresses on an appropriate **sketch**. *(12 points)*




9. Mohr's circle is shown for a point in a physical object that is subjected to plane stress. Determine the following values. *(16 points)*




$\sigma_x$ = __________ ksi

$\sigma_y$ = __________ ksi

$\tau_{xy}$ = __________ ksi

$\theta_p$ = __________ deg

$\sigma_1$ = __________ ksi

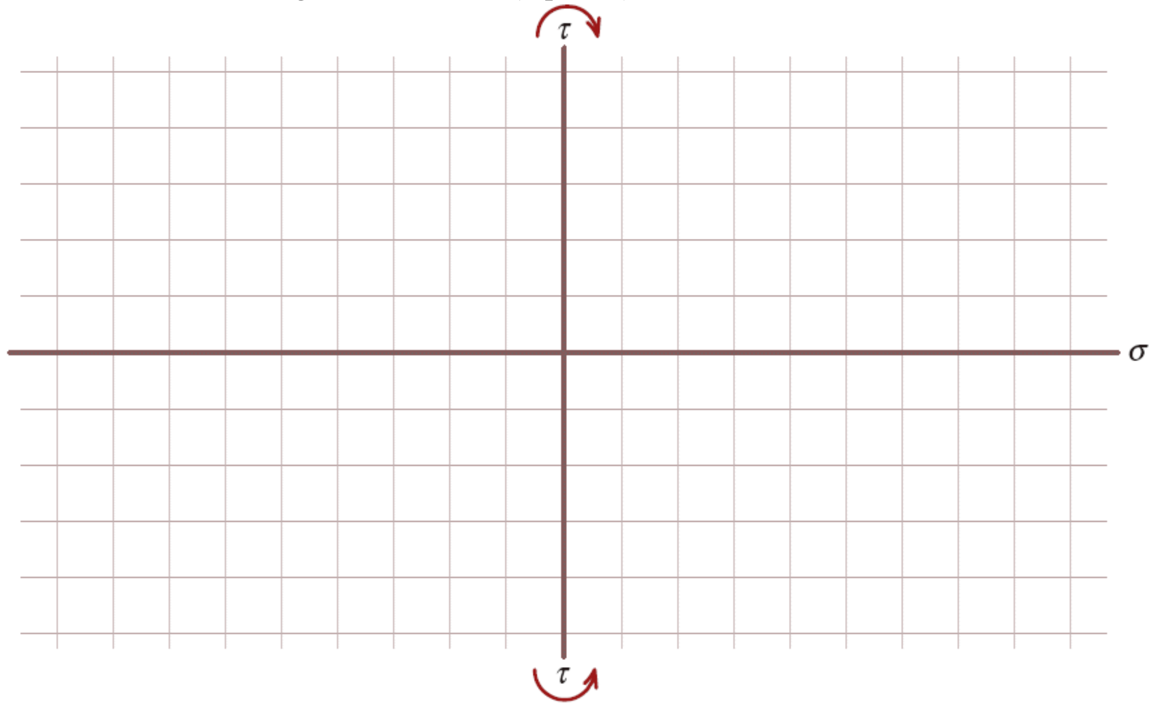
$\sigma_2$ = __________ ksi

$\tau_{\text{in-plane max}}$ = __________ ksi

$\tau_{\text{absolute max}}$ = __________ ksi

10. Sketch Mohr's circle for the following state of stress. *(4 points)*

$\sigma_x = 30$ MPa
$\sigma_y = -50$ MPa
$\tau_{xy} = 45$ MPa




11. Determine the following values from the Mohr's circle in the previous problem. *(10 points)*

$\sigma_{center}$ = __________ MPa

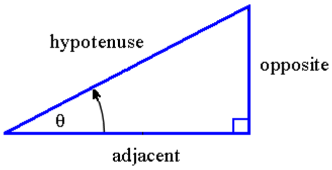
radius = __________ MPa

$\sigma_1$ = __________ MPa

$\sigma_2$ = __________ MPa

$\tau_{in\text{-}plane\ max}$ = __________ MPa

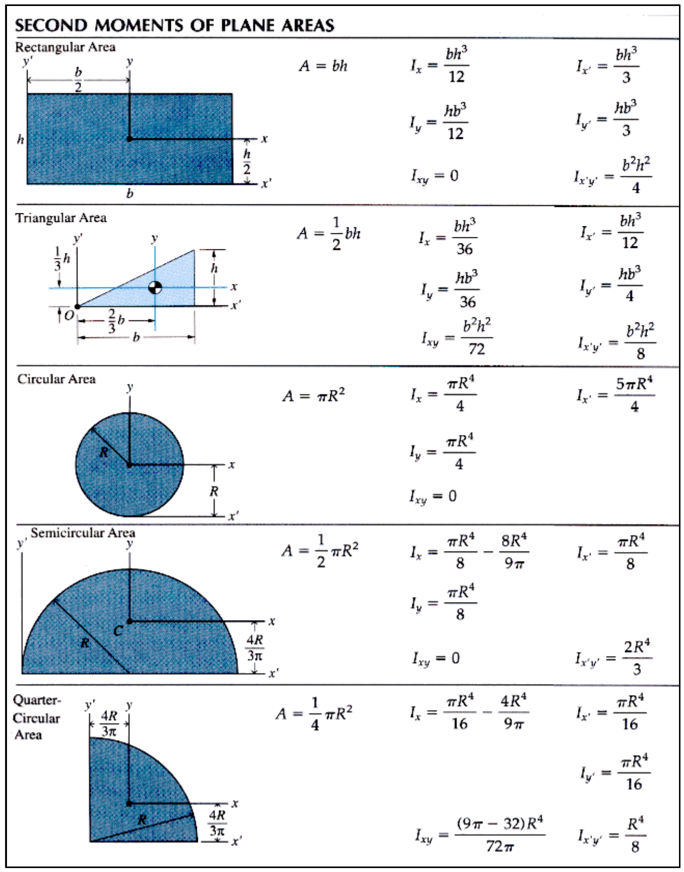
## TRIGONOMETRY




$\sin\theta$ = opp / hyp
$\cos\theta$ = adj / hyp
$\tan\theta$ = opp / adj

## STATICS

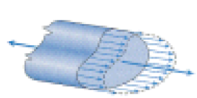
| Symbol | Meaning | Equation | Units |
|---|---|---|---|
| $\bar{x}, \bar{y}, \bar{z}$ | centroid position | $\bar{y} = \Sigma\bar{y}_i A_i / \Sigma A_i$ | in, m |
| I | moment of inertia | $I = \Sigma(I_i + d_i^2 A_i)$ | $in^4$, $m^4$ |
| J | polar moment of inertia | $J_{\text{solid circular shaft}} = \pi d^4/32$<br>$J_{\text{hollow circular shaft}} = \pi(d_o^4 - d_i^4)/32$ | $in^4$, $m^4$ |
| N | normal force | | lb, N |
| V | shear force | $V = \int -w(x)\,dx$ | lb, N |
| M | bending moment | $M = \int V(x)\,dx$ | in-lb, Nm |
| equilibrium | | $\Sigma\mathbf{F} = \mathbf{0}$<br>$\Sigma\mathbf{M}_{(\text{any point})} = \mathbf{0}$ | lb, N<br>in-lb, Nm |

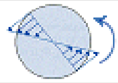
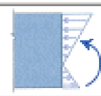
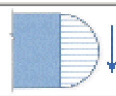


**SECOND MOMENTS OF PLANE AREAS**

| | | | |
|---|---|---|---|
| Rectangular Area | $A = bh$ | $I_x = \frac{bh^3}{12}$, $I_y = \frac{hb^3}{12}$, $I_{xy} = 0$ | $I_{x'} = \frac{bh^3}{3}$, $I_{y'} = \frac{hb^3}{3}$, $I_{x'y'} = \frac{b^2h^2}{4}$ |
| Triangular Area | $A = \frac{1}{2}bh$ | $I_x = \frac{bh^3}{36}$, $I_y = \frac{hb^3}{36}$, $I_{xy} = \frac{b^2h^2}{72}$ | $I_{x'} = \frac{bh^3}{12}$, $I_{y'} = \frac{hb^3}{4}$, $I_{x'y'} = \frac{b^2h^2}{8}$ |
| Circular Area | $A = \pi R^2$ | $I_x = \frac{\pi R^4}{4}$, $I_y = \frac{\pi R^4}{4}$, $I_{xy} = 0$ | $I_{x'} = \frac{5\pi R^4}{4}$ |
| Semicircular Area | $A = \frac{1}{2}\pi R^2$ | $I_x = \frac{\pi R^4}{8} - \frac{8R^4}{9\pi}$, $I_y = \frac{\pi R^4}{8}$, $I_{xy} = 0$ | $I_{x'} = \frac{\pi R^4}{8}$, $I_{x'y'} = \frac{2R^4}{3}$ |
| Quarter-Circular Area | $A = \frac{1}{4}\pi R^2$ | $I_x = \frac{\pi R^4}{16} - \frac{4R^4}{9\pi}$, $I_{xy} = \frac{(9\pi - 32)R^4}{72\pi}$ | $I_{x'} = \frac{\pi R^4}{16}$, $I_{y'} = \frac{\pi R^4}{16}$, $I_{x'y'} = \frac{R^4}{8}$ |

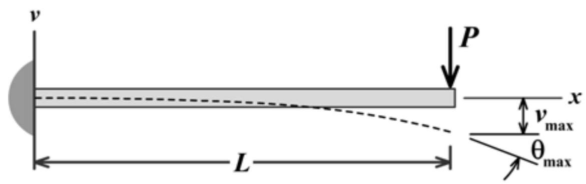
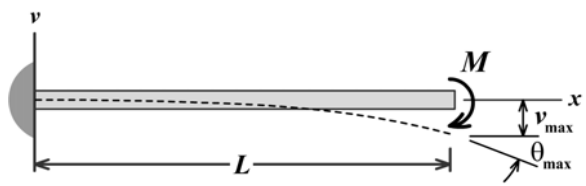
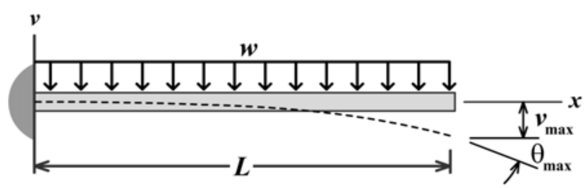
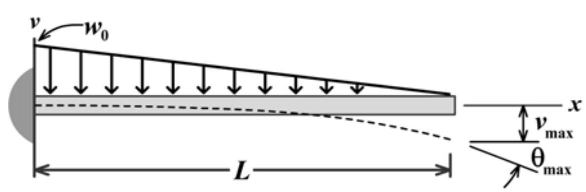
## MECHANICS OF MATERIALS

| Topic | Symbol | Meaning | Equation | Units |
|---|---|---|---|---|
| axial | σ, sigma | normal stress  | $\sigma_{axial} = N/A$<br>$\tau_{cutting} = V/A$<br>$\sigma_{bearing} = F_b/A_b$ | psi, Pa |
| | ε, epsilon | normal strain | $\varepsilon_{axial} = \Delta L/L_o = \delta/L_o$<br>$\varepsilon_{transverse} = \Delta d/d$ | in/in, m/m |
| | γ, gamma | shear strain | $\gamma$ = change in angle, $\gamma = c\theta$ | rad |
| | E | Young's modulus, modulus of elasticity | $\sigma = E\varepsilon$ (one-dimensional only) | psi, Pa |
| | G | shear modulus, modulus of rigidity | $G = \tau/\gamma = E / 2(1+\nu)$ | psi, Pa |
| | ν, nu | Poisson's ratio | $\nu = -\varepsilon'/\varepsilon$ | |
| | δ, delta | deformation, elongation, deflection | $\delta = NL_o/EA + \alpha\Delta T L_o$ | in, m |
| | α, alpha | coefficient of thermal expansion (CTE) | | in/inF, m/mC |
| | F.S. | factor of safety | F.S. = actual strength / design strength | |

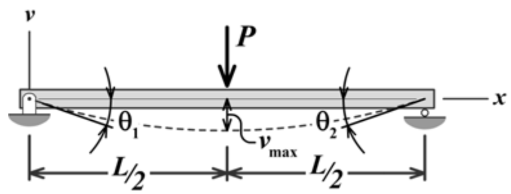
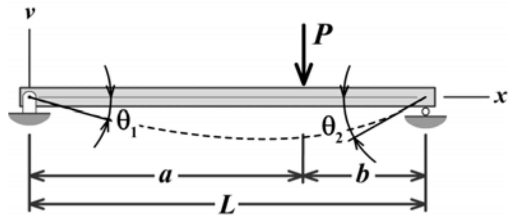
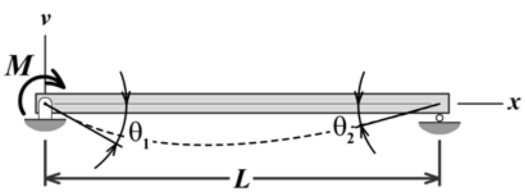
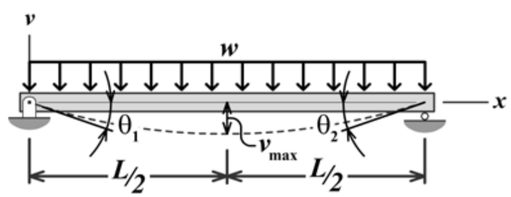
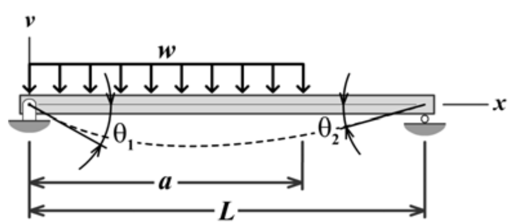
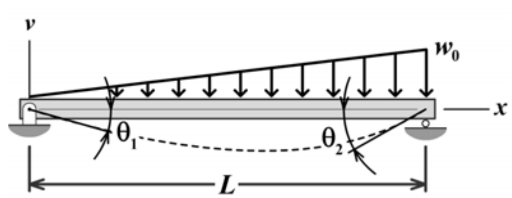
| Topic | Symbol | Meaning | Equation | Units |
|---|---|---|---|---|
| torsion | $\tau$, tau | shear stress  | $\tau_{torsion} = Tc/J$ | psi, Pa |
| | $\phi$, phi | angle of twist | $\phi = TL/GJ$ | rad, degrees |
| | $\theta$, theta | angle of twist per unit length, rate of twist | $\theta = \phi / L$ | rad/in, rad/m |
| | P | power | $P = T\omega$ <br> $r_2T_1 = r_1T_2$ <br> $r_1\omega_1 = r_2\omega_2$ | watts = Nm/s <br> hp=6600 in-lb/s |
| | $\omega$, omega | angular speed, speed of rotation | | rad/s |
| | f | frequency | $\omega = 2\pi f$ | Hz = rev/s |
| | K | stress concentration factor | $\tau_{max} = KTc/J$ | psi, Pa |
| flexure | $\sigma$, sigma | normal stress  | $\sigma_{beam} = -My/I$ | psi, Pa |
| | $\sigma$, sigma | composite beams, $n = E_B/E_A$ | $\sigma_A = -My / I^T$ <br> $\sigma_B = -nMy / I^T$ | psi, Pa |
| | $\tau$, tau | shear stress  | $\tau_{beam} = VQ/Ib$ where $Q = \Sigma(y_{bar\ i} A_i)$ | psi, Pa |
| | q | shear flow | $q = V_{beam}Q/I = nV_{fastener}/s$ | |
| | v or y | beam deflection | $v = \iint M(x)\, dx^2 / EI$ | in, m |

| Topic | Equations | | Units |
|---|---|---|---|
| stress trans-formation | *planar rotations* <br> $\sigma_u = (\sigma_x+\sigma_y)/2 + (\sigma_x-\sigma_y)/2 \cos(2\theta) + \tau_{xy}\sin(2\theta)$ <br> $\sigma_v = (\sigma_x+\sigma_y)/2 - (\sigma_x-\sigma_y)/2 \cos(2\theta) - \tau_{xy}\sin(2\theta)$ <br> $\tau_{uv} = -(\sigma_x-\sigma_y)/2 \sin(2\theta) + \tau_{xy}\cos(2\theta)$ | *principals and max in-plane shear* <br> $\tan(2\theta_p) = 2\tau_{xy} / (\sigma_x-\sigma_y)$, $\theta_s = \theta_p \pm 45^o$ <br> $\sigma_{1,2} = (\sigma_x+\sigma_y)/2 \pm \text{sqrt}\{ [ (\sigma_x-\sigma_y)/2 ]^2 + \tau_{xy}{}^2 \}$ <br> $\tau_{max} = \text{sqrt}\{ [ (\sigma_x-\sigma_y)/2 ]^2 + \tau_{xy}{}^2 \} = (\sigma_1-\sigma_2)/2$ <br> $\sigma_{avg} = (\sigma_x+\sigma_y)/2 = (\sigma_1+\sigma_2)/2$ | psi, Pa |
| strain trans-formation | *planar rotations* <br> $\varepsilon_u = (\varepsilon_x+\varepsilon_y)/2 + (\varepsilon_x-\varepsilon_y)/2 \cos(2\theta) + \gamma_{xy}/2 \sin(2\theta)$ <br> $\varepsilon_v = (\varepsilon_x+\varepsilon_y)/2 - (\varepsilon_x-\varepsilon_y)/2 \cos(2\theta) - \gamma_{xy}/2 \sin(2\theta)$ <br> $\gamma_{uv}/2 = -(\varepsilon_x-\varepsilon_y)/2 \sin(2\theta) + \gamma_{xy}/2 \cos(2\theta)$ <br> $\varepsilon_z = -\nu (\varepsilon_x + \varepsilon_y) / (1-\nu)$ | *principals and max in-plane shear* <br> $\tan(2\theta_p) = \gamma_{xy} / (\varepsilon_x-\varepsilon_y)$, $\theta_s = \theta_p \pm 45^o$ <br> $\varepsilon_{1,2} = (\varepsilon_x+\varepsilon_y)/2 \pm \text{sqrt}\{ [ (\varepsilon_x-\varepsilon_y)/2 ]^2 + (\gamma_{xy}/2)^2 \}$ <br> $\gamma_{max}/2 = \text{sqrt}\{ [ (\varepsilon_x-\varepsilon_y)/2 ]^2 + (\gamma_{xy}/2)^2 \}$ <br> $\varepsilon_{avg} = (\varepsilon_x+\varepsilon_y)/2$ | psi, Pa <br> in/in, m/m |
| Hooke's law | *1D strain to stress* <br> $\sigma = E\varepsilon$ <br> *2D strain to stress* <br> $\sigma_x = E(\varepsilon_x+\nu\varepsilon_y) / (1-\nu^2)$ <br> $\sigma_y = E(\varepsilon_y+\nu\varepsilon_x) / (1-\nu^2)$ <br> $\tau_{xy} = G\gamma_{xy} = E\gamma_{xy} / 2(1+\nu)$ | *2D stress to strain* <br> $\varepsilon_x = (\sigma_x-\nu\sigma_y) / E$ <br> $\varepsilon_y = (\sigma_y-\nu\sigma_x) / E$ <br> $\varepsilon_z = -\nu(\varepsilon_x+\varepsilon_y) / (1-\nu)$ <br> $\gamma_{xy} = \tau_{xy}/G = 2(1+\nu)\tau_{xy} / E$ | psi, Pa <br> in/in, m/m |
| pressure | $\sigma_{spherical} = pr/2t$ <br> $\sigma_{cylindrical,\ hoop} = pr/t$ <br> $\sigma_{cylindrical,\ axial} = pr/2t$ | $\sigma_{radial,\ outside} = 0$ <br> $\sigma_{radial,\ inside} = -p$ | psi, Pa |
| failure theories | *maximum principal stress theory* <br> $\sigma_{1,2} < \sigma_{yp}$ | *maximum shear stress theory* <br> $\tau_{max} < 0.5\ \sigma_{yp}$ | psi, Pa |

# CANTILEVER BEAMS

| Beam | Slope | Deflection | Elastic Curve |
|---|---|---|---|
|  | $\theta_{max} = -\dfrac{PL^2}{2EI}$ | $v_{max} = -\dfrac{PL^3}{3EI}$ | $v = -\dfrac{Px^2}{6EI}(3L - x)$ |
|  | $\theta_{max} = -\dfrac{ML}{EI}$ | $v_{max} = -\dfrac{ML^2}{2EI}$ | $v = -\dfrac{Mx^2}{2EI}$ |
|  | $\theta_{max} = -\dfrac{wL^3}{6EI}$ | $v_{max} = -\dfrac{wL^4}{8EI}$ | $v = -\dfrac{wx^2}{24EI}(6L^2 - 4Lx + x^2)$ |
|  | $\theta_{max} = -\dfrac{w_0 L^3}{24EI}$ | $v_{max} = -\dfrac{w_0 L^4}{30EI}$ | $v = -\dfrac{w_0 x^2}{120LEI}(10L^3 - 10L^2 x + 5Lx^2 - x^3)$ |

# SIMPLY SUPPORTED BEAMS

| Beam | Slope | Deflection | Elastic Curve |
|---|---|---|---|
|   | $\theta_1 = -\theta_2 = -\dfrac{PL^2}{16EI}$ | $v_{\max} = -\dfrac{PL^3}{48EI}$ | $v = -\dfrac{Px}{48EI}(3L^2 - 4x^2)$ <br> $for\ 0 \le x \le L/2$ |
|   | $\theta_1 = -\dfrac{Pb(L^2 - b^2)}{6LEI}$ <br> $\theta_2 = +\dfrac{Pa(L^2 - a^2)}{6LEI}$ | $v\rvert_{x=a} = -\dfrac{Pba}{6LEI}(L^2 - b^2 - a^2)$ | $v = -\dfrac{Pbx}{6LEI}(L^2 - b^2 - x^2)$ <br> $for\ 0 \le x \le a$ |
|   | $\theta_1 = -\dfrac{ML}{3EI}$ <br> $\theta_2 = +\dfrac{ML}{6EI}$ | $v_{\max} = -\dfrac{ML^2}{9\sqrt{3}EI}$ <br> $@\ x = L\left(1 - \dfrac{\sqrt{3}}{3}\right)$ | $v = -\dfrac{Mx}{6LEI}(2L^2 - 3Lx + x^2)$ |
|   | $\theta_1 = -\theta_2 = -\dfrac{wL^3}{24EI}$ | $v_{\max} = -\dfrac{5wL^4}{384EI}$ | $v = -\dfrac{wx}{24EI}(L^3 - 2Lx^2 + x^3)$ |
|   | $\theta_1 = -\dfrac{wa^2}{24LEI}(2L - a)^2$ <br> $\theta_2 = +\dfrac{wa^2}{24LEI}(2L^2 - a^2)$ | $v\rvert_{x=a} = -\dfrac{wa^3}{24LEI}(3a^2 - 7aL + 4L^2)$ | $v = -\dfrac{wx}{24LEI}(a^4 - 4a^3 L + 4a^2 L^2 + 2a^2 x^2$ <br> $- 4aLx^2 + Lx^3) \quad for\ 0 \le x \le a$ <br> $v = -\dfrac{wa^2}{24LEI}(-a^2 L + 4L^2 x + a^2 x - 6Lx^2 + 2x^3)$ <br> $for\ a \le x \le L$ |
|   | $\theta_1 = -\dfrac{7w_0 L^3}{360EI}$ <br> $\theta_2 = +\dfrac{w_0 L^3}{45EI}$ | $v_{\max} = -0.00652\dfrac{w_0 L^4}{EI}$ <br> $@\ x = 0.5193L$ | $v = -\dfrac{w_0 x}{360LEI}(7L^4 - 10L^2 x^2 + 3x^4)$ |