$\qquad$

1. The strain components $\varepsilon_{x}=946 \mu \varepsilon, \varepsilon_{y}=-294 \mu \varepsilon$ and $\gamma_{\mathrm{xy}}=-362 \mu \varepsilon$ are given for a point in a body subjected to plane strain. Determine the strain components $\varepsilon_{\mathrm{n}}$, $\varepsilon_{\mathrm{t}}$, and $\gamma_{\mathrm{nt}}$ at the point if the $\mathrm{n}-\mathrm{t}$ axes are rotated with respect to the $x-y$ axes by the amount and in the direction indicated by the angle $\theta=12^{\circ}$. ( 9 points)

$\varepsilon_{\mathrm{n}}=$ $\qquad$ $\mu \varepsilon$
$\varepsilon_{\mathrm{t}}=$ $\qquad$ $\mu \varepsilon$
$\gamma_{\mathrm{nt}}=$ $\qquad$ $\mu \mathrm{rad}$

Sketch the deformed shape of the element. (3 points)
2. The strain components $\varepsilon_{x}=-800 \mu \varepsilon, \varepsilon_{\mathrm{y}}=400 \mu \varepsilon$ and $\gamma_{\mathrm{xy}}=-1350 \mu \varepsilon$ are given for a point in a body subjected to plane strain. Determine $\theta_{p}$ and the normal and shear strains at the $\theta_{\mathrm{p}}$ orientation (i.e. the "principals"). (12 points)
$\theta_{\mathrm{p}}=$ $\qquad$ deg
$\varepsilon_{\mathrm{n}}=$ $\qquad$ $\mu \varepsilon$
$\varepsilon_{\mathrm{t}}=$ $\qquad$ $\mu \varepsilon$
$\gamma_{\mathrm{nt}}=$ $\qquad$ $\mu \mathrm{rad}$

Determine $\theta_{\mathrm{s}}$ and the normal and shear strains at the $\theta_{\mathrm{s}}$ orientation (i.e. the "average normal strain" and "in-plane maximum shear strain"). (12 points)
$\theta_{\mathrm{s}}=$ $\qquad$ deg
$\varepsilon_{\mathrm{n}}=$ $\qquad$ $\mu \varepsilon$
$\varepsilon_{\mathrm{t}}=$ $\qquad$ $\mu \varepsilon$
$\gamma_{\mathrm{nt}}=$ $\qquad$ $\mu \mathrm{rad}$

Show the angle $\theta_{p}$, the principal strain deformations, and the maximum in-plane shear strain distortion on a sketch. (4 points)
3. The strain components $\varepsilon_{x}=-200 \mu \varepsilon, \varepsilon_{y}=-700 \mu \varepsilon$ and $\gamma_{x y}=600 \mu \varepsilon$ are given for a point in a body subjected to plane strain. Draw Mohr's circle. (5 points)


Using the circle, determine the principal strains, the maximum in-plane shear strain, and the absolute maximum shear strain at the point. (18 points)
$\varepsilon_{\text {center }}=$ $\qquad$ $\mu \varepsilon$
radius $=$ $\qquad$ $\mu \varepsilon$
$\varepsilon_{1}=$ $\qquad$ $\mu \varepsilon$
$\varepsilon_{2}=$ $\qquad$ $\mu \varepsilon$
$\gamma_{\text {in-plane max }}=$ $\qquad$ $\mu \mathrm{rad}$
$\gamma_{\mathrm{absolute} \text { max }}=$ $\qquad$ $\mu \mathrm{rad}$
4. The strain rosette shown in the figures was used to obtain normal strain data at a point on the free surface of a machine part. Determine the strain components $\varepsilon_{x}, \varepsilon_{y}$ at the point. To save time, $\gamma_{\mathrm{xy}}$ is not needed. (15 points)
$\varepsilon_{\mathrm{a}}=-600 \mu \varepsilon$
$\varepsilon_{\mathrm{b}}=250 \mu \varepsilon$
$\varepsilon_{\mathrm{c}}=800 \mu \varepsilon$

$\varepsilon_{\mathrm{x}}=$ $\qquad$ $\mu \varepsilon$
$\varepsilon_{y}=$ $\qquad$ $\mu \varepsilon$
5. A strain gage is used to monitor the strain in a spherical steel tank ( $\mathrm{E}=$ $210 \mathrm{GPa} ; \mathrm{v}=0.32$ ), which contains a fluid under pressure. The tank has an inside diameter of 2.50 m and a wall thickness of 100 mm . Determine the internal pressure in the tank when the strain gage reads $120 \mu \varepsilon$. (10 points)
$\mathrm{p}=$ $\qquad$ MPa

6. A closed cylindrical vessel contains a fluid at a pressure of 640 psi. The cylinder, which has an outside diameter of 72 in. and a wall thickness of 1.000 in., is fabricated from stainless steel [ $\mathrm{E}=28,000 \mathrm{ksi} ; v=0.32$ ]. Determine the axial and hoop stresses and principal strains. (12 points)
$\sigma_{\text {axial }}=$ $\qquad$ psi

$\sigma_{\text {hoop }}=$ $\qquad$ psi
$\varepsilon_{1}=$ $\qquad$ $\mu \varepsilon$
$\varepsilon_{2}=$ $\qquad$ $\mu \varepsilon$

## TRIGONOMETRY



## STATICS

| Symbol | Meaning | Equation | Units |
| :---: | :---: | :---: | :---: |
| $\bar{x}, \bar{y}, \bar{z}$ | centroid position | $\bar{y}=\Sigma \bar{y}_{i} A_{i} / \Sigma A_{i}$ | in, m |
| I | moment of inertia | $\mathrm{I}=\Sigma\left(\mathrm{I}_{\mathrm{i}}+\mathrm{d}_{\mathrm{i}}^{2} \mathrm{~A}_{\mathrm{i}}\right)$ | in ${ }^{4}$, m ${ }^{4}$ |
| J | polar moment of inertia | $\begin{gathered} \mathrm{J}_{\text {solid circular shaft }}=\pi \mathrm{d}^{4} / 32 \\ \mathrm{~J}_{\text {hollow circular shaft }} \\ =\pi\left(\mathrm{d}_{0}^{4}-\mathrm{d}_{\mathrm{i}}^{4}\right) / 32 \end{gathered}$ | in ${ }^{4}$, m ${ }^{4}$ |
| N | normal force |  | lb, N |
| V | shear force | $V=\int-w(x) d x$ | lb, N |
| M | bending moment | $M=\int V(x) d x$ | in-lb, Nm |
| equilibrium |  | $\begin{gathered} \Sigma F=\mathbf{0} \\ \Sigma \mathbf{M}_{(\text {any point })}=0 \end{gathered}$ | $\begin{gathered} \mathrm{lb}, \mathrm{~N} \\ \mathrm{in}-\mathrm{lb}, \mathrm{Nm} \end{gathered}$ |


| SECOND MOMENTS OF PLANE AREAS |  |  |
| :---: | :---: | :---: |
| Rectangular Area $A=b h$ | $I_{x}=\frac{b h^{3}}{12}$ | $I_{x^{\prime}}=\frac{b h^{3}}{3}$ |
| $h$ | $\begin{aligned} & I_{y}=\frac{h b^{3}}{12} \\ & I_{x y}=0 \end{aligned}$ | $\begin{aligned} & I_{y^{\prime}}=\frac{h b^{3}}{3} \\ & I_{x^{\prime} y^{\prime}}=\frac{b^{2} h^{2}}{4} \end{aligned}$ |
| Triangular Area $A=\frac{1}{2} b h$ | $\begin{aligned} & I_{x}=\frac{b h^{3}}{36} \\ & I_{y}=\frac{h b^{3}}{36} \\ & I_{x y}=\frac{b^{2} h^{2}}{72} \end{aligned}$ | $\begin{aligned} & I_{x^{\prime}}=\frac{b h^{3}}{12} \\ & I_{y^{\prime}}=\frac{h b^{3}}{4} \\ & I_{x^{\prime} y^{\prime}}=\frac{b^{2} h^{2}}{8} \end{aligned}$ |
| Circular Area $A=\pi R^{2}$ | $\begin{aligned} & I_{x}=\frac{\pi R^{4}}{4} \\ & I_{y}=\frac{\pi R^{4}}{4} \\ & I_{x y}=0 \end{aligned}$ | $I_{x^{\prime}}=\frac{5 \pi R^{4}}{4}$ |
|  | $\begin{aligned} & I_{x}=\frac{\pi R^{4}}{8}-\frac{8 R^{4}}{9 \pi} \\ & I_{y}=\frac{\pi R^{4}}{8} \\ & I_{x y}=0 \end{aligned}$ | $I_{x^{\prime}}=\frac{\pi R^{4}}{8}$ $I_{x^{\prime} y^{\prime}}=\frac{2 R^{4}}{3}$ |
|  | $I_{x}=\frac{\pi R^{4}}{16}-\frac{4 R^{4}}{9 \pi}$ $I_{x y}=\frac{(9 \pi-32) R^{4}}{72 \pi}$ | $\begin{aligned} & I_{x^{\prime}}=\frac{\pi R^{4}}{16} \\ & I_{y^{\prime}}=\frac{\pi R^{4}}{16} \\ & I_{x^{\prime} y^{\prime}}=\frac{R^{4}}{8} \end{aligned}$ |

## MECHANICS OF MATERIALS

| Topic | Symbol | Meaning | Equation | Units |
| :---: | :---: | :---: | :---: | :---: |
| axial | $\sigma$, sigma | normal stress | $\begin{aligned} \sigma_{\text {axial }} & =\mathrm{N} / \mathrm{A} \\ \tau_{\text {cutting }} & =\mathrm{V} / \mathrm{A} \\ \sigma_{\text {bearing }} & =\mathrm{F}_{\mathrm{b}} / \mathrm{A}_{\mathrm{b}} \end{aligned}$ | psi, Pa |
|  | $\varepsilon$, epsilon | normal strain | $\begin{gathered} \varepsilon_{\text {axial }}=\Delta \mathrm{L} / \mathrm{L}_{\mathrm{o}}=\delta / \mathrm{L}_{\mathrm{o}} \\ \varepsilon_{\text {transverse }}=\Delta \mathrm{d} / \mathrm{d} \end{gathered}$ | in/in, m/m |
|  | $\gamma$, gamma | shear strain | $\gamma=$ change in angle, $\gamma=c \theta$ | rad |
|  | E | Young's modulus, modulus of elasticity | $\sigma=\mathrm{E} \boldsymbol{\varepsilon}$ (one-dimensional only) | psi, Pa |
|  | G | shear modulus, modulus of rigidity | $\mathrm{G}=\tau / \gamma=\mathrm{E} / 2(1+v)$ | psi, Pa |
|  | $v$, nu | Poisson's ratio | $v=-\varepsilon^{\prime} / \varepsilon$ |  |
|  | $\delta$, delta | deformation, elongation, deflection | $N / E A+\alpha \Delta T$ | in, m |
|  | $\alpha$, alpha | coefficient of thermal expansion (CTE) |  | in/inF, m/mC |
|  | F.S. | factor of safety | F.S. = actual strength / design strength |  |


| Topic | Symbol | Meaning | Equation |  | Units |
| :---: | :---: | :---: | :---: | :---: | :---: |
| torsion | $\tau$, tau | shear stress | $\tau_{\text {torsion }}=\mathrm{Tc} / \mathrm{J}$ |  | psi, Pa |
|  | $\phi$, phi | angle of twist | $\phi=\mathrm{TL} / \mathrm{GJ}$ |  | rad, degrees |
|  | $\theta$, theta | angle of twist per unit length, rate of twist | $\theta=\phi / L$ |  | rad/in, rad/m |
|  | P | power | $\mathrm{P}=\mathrm{T} \omega$ | $\begin{gathered} r_{2} T_{1}=r_{1} T_{2} \\ r_{1} \omega_{1}=r_{2} \omega_{2} \end{gathered}$ | $\begin{gathered} \text { watts }=\mathrm{Nm} / \mathrm{s} \\ \mathrm{hp}=6600 \mathrm{in}-\mathrm{lb} / \mathrm{s} \end{gathered}$ |
|  | $\begin{gathered} \omega, \\ \text { omega } \end{gathered}$ | angular speed, speed of rotation |  |  | rad/s |
|  | f | frequency | $\omega=2 \pi \mathrm{f}$ |  | $\mathrm{Hz}=\mathrm{rev} / \mathrm{s}$ |
|  | K | stress concentration factor | $\tau_{\text {max }}=\mathrm{KTc} / \mathrm{J}$ |  | psi, Pa |
| flexure | $\sigma$, sigma | normal stress $\quad 3$ | $\sigma_{\text {beam }}=-\mathrm{My} / \mathrm{l}$ |  | psi, Pa |
|  | $\sigma$, sigma | composite beams, $n=E_{B} / E_{A}$ | $\sigma_{A}=-M y / I^{\top}$ | $\sigma_{B}=-n M y / I^{\top}$ | psi, Pa |
|  | $\tau$, tau | shear stress | $\tau_{\text {beam }}=\mathrm{VQ} / \mathrm{lb}$ where $\mathrm{Q}=\Sigma\left(\mathrm{y}_{\text {bar i }} \mathrm{A}_{\mathrm{i}}\right)$ |  | psi, Pa |
|  | q | shear flow | $\mathrm{q}=\mathrm{V}_{\text {beam }} \mathrm{Q} / \mathrm{I}=\mathrm{n} \mathrm{V}_{\text {fastener }} / \mathrm{s}$ |  |  |
|  | v or y | beam deflection | $v=\iint M(x) d x^{2} / E l$ |  | in, m |
| Topic |  | Equations |  |  | Units |
| stress <br> trans- <br> formation |  | planar rotations $\begin{gathered} \sigma_{\mathrm{u}}=\left(\sigma_{\mathrm{x}}+\sigma_{\mathrm{y}}\right) / 2+\left(\sigma_{\mathrm{x}}-\sigma_{\mathrm{y}}\right) / 2 \cos (2 \theta)+\tau_{\mathrm{xy}} \sin (2 \theta) \\ \sigma_{\mathrm{v}}=\left(\sigma_{\mathrm{x}}+\sigma_{\mathrm{y}}\right) / 2-\left(\sigma_{\mathrm{x}}-\sigma_{\mathrm{y}}\right) / 2 \cos (2 \theta)-\tau_{\mathrm{xy}} \sin (2 \theta) \\ \tau_{\mathrm{uv}}=-\left(\sigma_{\mathrm{x}}-\sigma_{\mathrm{y}}\right) / 2 \sin (2 \theta)+\tau_{\mathrm{xy}} \cos (2 \theta) \end{gathered}$ | principals and max in-plane shear$\begin{gathered} \tan \left(2 \theta_{\mathrm{p}}\right)=2 \tau_{\mathrm{xy}} /\left(\sigma_{\mathrm{x}}-\sigma_{\mathrm{y}}\right), \theta_{\mathrm{s}}=\theta_{\mathrm{p}} \pm 45^{\circ} \\ \sigma_{1,2}=\left(\sigma_{\mathrm{x}}+\sigma_{\mathrm{y}}\right) / 2 \pm \operatorname{sqrt}\left\{\left[\left(\sigma_{\mathrm{x}}-\sigma_{\mathrm{y}}\right) / 2\right]^{2}+\tau_{\mathrm{xy}}{ }^{2}\right\} \\ \tau_{\max }=\operatorname{sqrt}\left\{\left[\left(\sigma_{\mathrm{x}}-\sigma_{\mathrm{y}}\right) / 2\right]^{2}+\tau_{\mathrm{xy}}{ }^{2}\right\}=\left(\sigma_{1}-\sigma_{2}\right) / 2 \\ \sigma_{\mathrm{avg}}=\left(\sigma_{\mathrm{x}}+\sigma_{\mathrm{y}}\right) / 2=\left(\sigma_{1}+\sigma_{2}\right) / 2 \end{gathered}$ |  | psi, Pa |
| strain <br> trans- <br> formation |  | planar rotations $\begin{gathered} \varepsilon_{\mathrm{u}}=\left(\varepsilon_{\mathrm{x}}+\varepsilon_{\mathrm{y}}\right) / 2+\left(\varepsilon_{\mathrm{x}}-\varepsilon_{\mathrm{y}}\right) / 2 \cos (2 \theta)+\gamma_{\mathrm{xy}} / 2 \sin (2 \theta) \\ \varepsilon_{\mathrm{v}}=\left(\varepsilon_{\mathrm{x}}+\varepsilon_{\mathrm{y}}\right) / 2-\left(\varepsilon_{\mathrm{x}}-\varepsilon_{\mathrm{y}}\right) / 2 \cos (2 \theta)-\gamma_{\mathrm{xy}} / 2 \sin (2 \theta) \\ \gamma_{\mathrm{uv}} / 2=-\left(\varepsilon_{\mathrm{x}}-\varepsilon_{\mathrm{y}}\right) / 2 \sin (2 \theta)+\gamma_{\mathrm{xy}} / 2 \cos (2 \theta) \\ \varepsilon_{\mathrm{z}}=-v\left(\varepsilon_{\mathrm{x}}+\varepsilon_{\mathrm{y}}\right) /(1-v) \end{gathered}$ | $\begin{gathered} \text { principals and max in-plane shear } \\ \tan \left(2 \theta_{\mathrm{p}}\right)=\gamma_{\mathrm{xy}} /\left(\varepsilon_{\mathrm{x}}-\varepsilon_{\mathrm{y}}\right), \theta_{\mathrm{s}}=\theta_{\mathrm{p}} \pm 45^{\circ} \\ \varepsilon_{1,2}=\left(\varepsilon_{\mathrm{x}}+\varepsilon_{y}\right) / 2 \pm \operatorname{sqrt}\left\{\left[\left(\varepsilon_{\mathrm{x}}-\varepsilon_{\mathrm{y}}\right) / 2\right]^{2}+\left(\gamma_{\mathrm{xy}} / 2\right)^{2}\right\} \\ \gamma_{\text {max }} / 2=\operatorname{sqrt}\left\{\left[\left(\varepsilon_{\mathrm{x}}-\varepsilon_{\mathrm{y}}\right) / 2\right]^{2}+\left(\gamma_{\mathrm{xy}} / 2\right)^{2}\right\} \\ \varepsilon_{\text {avg }}=\left(\varepsilon_{\mathrm{x}}+\varepsilon_{\mathrm{y}}\right) / 2 \end{gathered}$ |  | psi, Pa <br> in/in, m/m |
| Hooke's law |  | 1D strain to stress $\sigma=E \varepsilon$ <br> 2D strain to stress $\begin{gathered} \sigma_{x}=\mathrm{E}\left(\varepsilon_{\mathrm{x}}+v \varepsilon_{\mathrm{y}}\right) /\left(1-v^{2}\right) \\ \sigma_{\mathrm{y}}=\mathrm{E}\left(\varepsilon_{\mathrm{y}}+v \varepsilon_{\mathrm{x}}\right) /\left(1-v^{2}\right) \\ \tau_{\mathrm{xy}}=\mathrm{G} \gamma_{\mathrm{xy}}=\mathrm{E} \gamma_{\mathrm{xy}} / 2(1+v) \end{gathered}$ | 2D stress to strain$\begin{gathered} \varepsilon_{x}=\left(\sigma_{x}-v \sigma_{y}\right) / E \\ \varepsilon_{y}=\left(\sigma_{y}-v \sigma_{x}\right) / E \\ \varepsilon_{z}=-v\left(\varepsilon_{x}+\varepsilon_{y}\right) /(1-v) \\ \gamma_{x y}=\tau_{x y} / G=2(1+v) \tau_{x y} / E \end{gathered}$ |  | psi, Pa <br> in/in, m/m |
| pressure |  | $\begin{gathered} \sigma_{\text {spherical }}=\mathrm{pr} / 2 \mathrm{t} \\ \sigma_{\text {cylindrical, hoop }}=\mathrm{pr} / \mathrm{t} \\ \sigma_{\text {cylindrical, axial }}=\mathrm{pr} / 2 \mathrm{t} \end{gathered}$ | $\begin{aligned} & \sigma_{\text {radial, outside }}=0 \\ & \sigma_{\text {radial, inside }}=-p \end{aligned}$ |  | psi, Pa |
| failure theories |  | maximum principal stress theory $\sigma_{1,2}<\sigma_{y p}$ | maximum $\tau_{\mathrm{m}}$ | $\begin{aligned} & \text { tress theory } \\ & \sigma_{\mathrm{yp}} \end{aligned}$ | psi, Pa |

