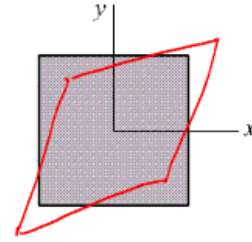
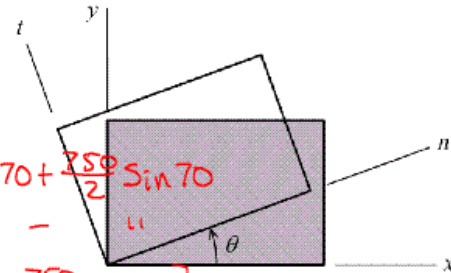


1. The strain components $\epsilon_x = 520 \mu\epsilon$, $\epsilon_y = -650 \mu\epsilon$, and $\gamma_{xy} = 750 \mu\text{rad}$ are given for a point in a body subjected to plane strain.

a. (4 points) Sketch the deformed shape on the element to the right.



b. (9 points) Determine the strain components ϵ_n , ϵ_t , and γ_{nt} at the point if the nt -axes are rotated with respect to the xy -axes by $\theta = 35^\circ$, as illustrated in the figure. Do not sketch the deformed shape of the element.

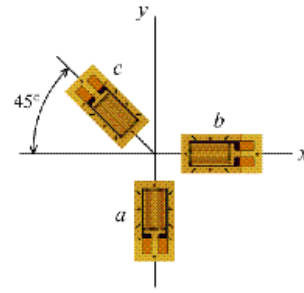


$$\begin{aligned} \epsilon_n &= \underline{487.5} \mu\epsilon = \frac{520-650}{2} + \frac{520+650}{2} \cos 70 + \frac{750}{2} \sin 70 \\ \epsilon_t &= \underline{-617.5} \mu\epsilon = \text{''} - \text{''} - \text{''} \\ \gamma_{nt} &= \underline{-842.9} \mu\text{rad} = 2 \left[-\frac{520+650}{2} \sin 70 + \frac{750}{2} \cos 70 \right] \end{aligned}$$

c. (14 points) Determine the angles θ_p and θ_s , principal strains, and maximum in-plane shear strain at the point. Do not sketch the deformed shapes of the element.

$$\begin{aligned} \theta_p &= \underline{16.33} \text{ deg} = \frac{1}{2} \tan^{-1} \left(\frac{750}{520+650} \right) \\ \theta_s &= \underline{-28.67} \text{ deg} = \theta_p \pm 45^\circ \\ \left. \begin{aligned} \epsilon_1 &= \underline{629.9} \mu\epsilon \\ \epsilon_2 &= \underline{-759.9} \mu\epsilon \end{aligned} \right\} &= \frac{520-650}{2} \pm \sqrt{\left(\frac{520+650}{2} \right)^2 + \left(\frac{750}{2} \right)^2} \\ \gamma_{max} &= \underline{1390} \mu\text{rad} = 2 \sqrt{\left(\frac{520+650}{2} \right)^2 + \left(\frac{750}{2} \right)^2} \end{aligned}$$

2. (10 points) The strain rosette shown in the figure was used to obtain normal strain data at a point on the free surface of a machine part. $\epsilon_a = 550\mu$, $\epsilon_b = -730\mu$, and $\epsilon_c = -375\mu$. Determine the strain components ϵ_x , ϵ_y , and γ_{xy} at the point.



$$\epsilon_x = \underline{-730} \mu\epsilon$$

$$\epsilon_y = \underline{550} \mu\epsilon$$

$$\gamma_{xy} = \underline{570} \mu\text{rad}$$

$$-375 = \frac{-730 + 550}{2} - \frac{730 - 550}{2} \cos^2 -90 + \frac{\gamma_{xy}}{2} \sin^2 -90$$

3. (8 points) The strain components $\epsilon_x = 390\mu$, $\epsilon_y = 820\mu$, and $\gamma_{xy} = -560\mu$ are given for a point on the free surface of a machine component. The modulus of elasticity for the material is $E = 73 \text{ GPa}$ and the Poisson's ratio is $\nu = 0.30$. Determine the stresses σ_x and τ_{xy} at the point.

$$\sigma_x = \underline{51.02} \text{ MPa} = \frac{73 \text{ E}9}{1 - 0.3^2} (390 \text{ E} -6 + 0.3(820 \text{ E} -6))$$

$$\tau_{xy} = \underline{-15.72} \text{ MPa} = \frac{73 \text{ E}9}{2(1 + 0.3)} (-560 \text{ E} -6)$$

4. (5 points) A spherical gas-storage tank with an inside diameter of 12 m is being constructed to store gas under an internal pressure of 1.75 MPa. The tank will be constructed from structural steel that has a yield strength of 250 MPa. If a factor of safety of 3.0 with respect to the yield strength is required, determine the minimum wall thickness required for the spherical tank.

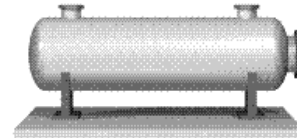
$$t_{\min} = \underline{63} \text{ mm}$$

$$\sigma = \frac{1.75 \text{E}6 (6)}{2t} \leq \frac{250 \text{E}6}{3}$$

5. (5 points) A cylindrical boiler with an outside diameter of 3.60 m and a wall thickness of 40 mm is made of a steel alloy that has a yield stress of 415 MPa. Determine the maximum normal stress produced by an internal pressure of 2 MPa.

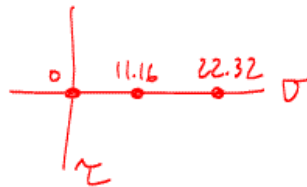
$$\sigma_{\max} = \underline{88} \text{ MPa} = \frac{2 \text{E}6 (1.76)}{.04}$$

6. A closed cylindrical vessel contains a fluid at a pressure of 720 psi. Assume $\sigma_{hoop} = 22.32$ ksi and $\sigma_{axial} = 11.16$ ksi. Determine:



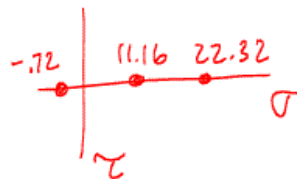
- a. (4 points) the absolute maximum shear stress on the outer surface of the cylinder.

$$\tau_{\text{abs max}} = \underline{11.16} \text{ ksi}$$



- b. (4 points) the absolute maximum shear stress on the inner surface of the cylinder.

$$\tau_{\text{abs max}} = \underline{11.52} \text{ ksi}$$



7. (9 points) Three loads are applied to the short rectangular post. The cross-sectional dimensions of the post are shown.

a. Using the positive sign convention shown, determine the following internal forces and moments acting on a x - z plane through points H and K .

$$F_x = -65 \text{ kN}$$

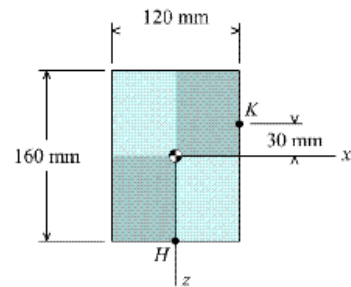
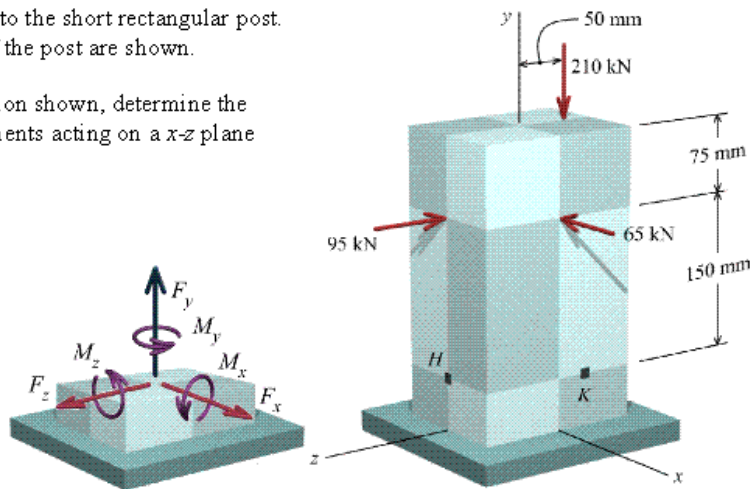
$$F_y = -210 \text{ kN}$$

$$F_z = -95 \text{ kN}$$

$$M_x = -24.75 \text{ kN-m}$$

$$M_y = 0 \text{ kN-m}$$

$$M_z = 9.75 \text{ kN-m}$$



b. (14 points) For the following loadings, determine the normal and shear stresses at point H .

$$F_x = 48 \text{ kN}$$

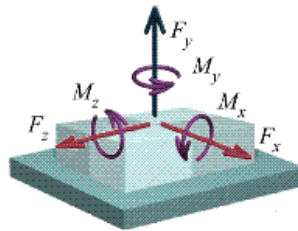
$$F_y = 0 \text{ kN}$$

$$F_z = 73 \text{ kN}$$

$$M_x = 3 \text{ kN-m}$$

$$M_y = 0 \text{ kN-m}$$

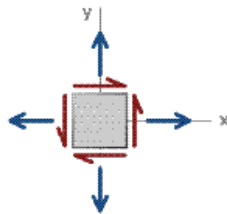
$$M_z = -2.5 \text{ kN-m}$$



$$\sigma_x = 0 \text{ MPa}$$

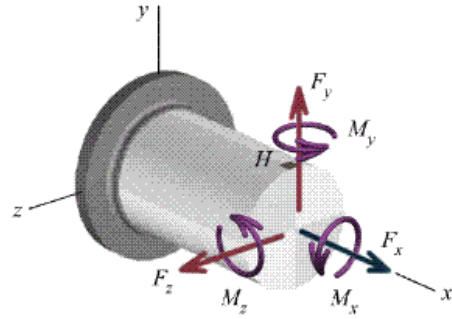
$$\sigma_y = -5.859 \text{ MPa} = \frac{-3000(.08)}{.12(.16)^3/12}$$

$$\tau_{xy} = 3.75 \text{ MPa} = \frac{48,000(.03)(.06)(.16)}{-16(.12)^3(.16)/12}$$



9. (14 points) A solid steel crank has an outside diameter of 30 mm. For the following loadings, determine the normal and shear stresses on the top surface of the crank at point H .

$$\begin{aligned} F_x &= 2350 \text{ N} \\ F_y &= -1275 \text{ N} \\ F_z &= 0 \text{ N} \\ M_x &= 204 \text{ Nm} \\ M_y &= 376 \text{ Nm} \\ M_z &= 0 \text{ Nm} \end{aligned}$$



$$\sigma_x = \underline{3.325} \text{ MPa} = \frac{2350}{\frac{\pi}{4} (.03)^2}$$

$$\sigma_z = \underline{0} \text{ MPa}$$

$$\tau_{xz} = \underline{-38.48} \text{ MPa} = \frac{-204(.015)}{\frac{\pi}{32} (.03)^4}$$

