Pressure Vessel Calculations

by Tessa Russell, October 20, 2005

Thin-Walled Pressure Vessel

After strain and pressure data has been collected for all three strain gages (A, B, and C – Figure 1), use Excel to plot the trends. With transformed axis of *x' and* y', the angles for A, B, and C are $\theta_A = 0°$, $\theta_B = 45°$, and $\theta_C = 90°$. We then determine the slope of each strain versus pressure data set (m_A, m_B, and m_C) as seen in Figure 2.

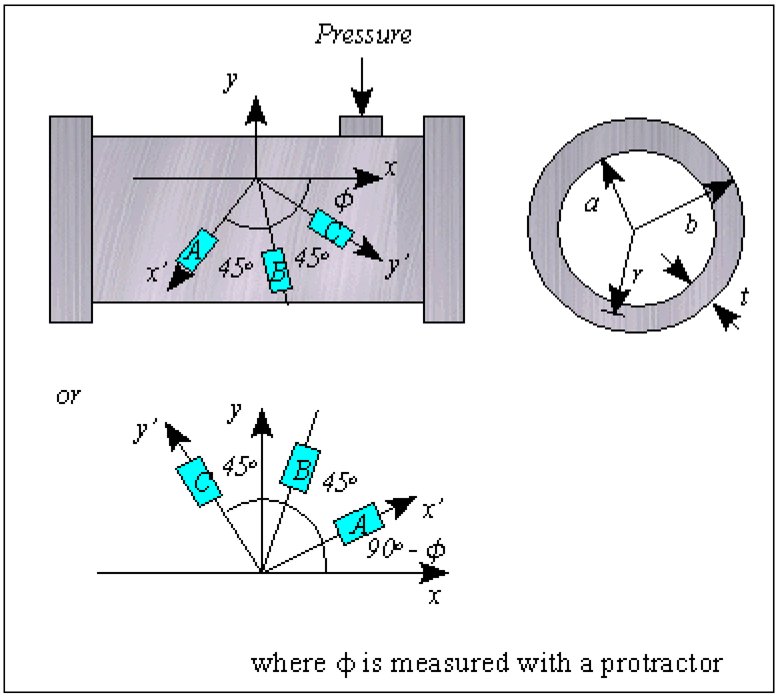

Figure 1: Gage configuration for the Thin-Walled Pressure Vessel

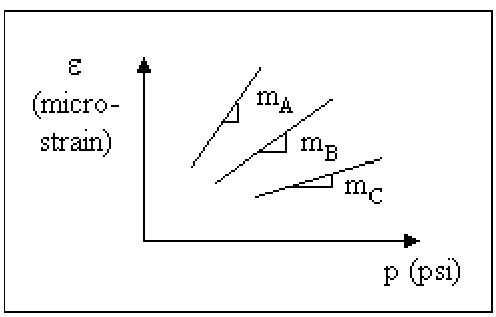

Figure 2: Plot of Strain versus Pressure for the Thin-Walled Pressure Vessel

For this plot, the slope, *m*, is the strain per pressure. Therefore,

$$m_A = \frac{\varepsilon_A}{P} = \frac{\frac{\varepsilon_{X'}}{P} + \frac{\varepsilon_{Y'}}{P}}{2} + \frac{\frac{\varepsilon_{X'}}{P} - \frac{\varepsilon_{Y'}}{P}}{2}\cos(2\theta_A) + \frac{\frac{\gamma_{X'Y'}}{P}}{2}\sin(2\theta_A) = \frac{\varepsilon_{X'}}{P}$$

Equation 1: Calculating Strain from the Slope of Data Set A

and

$$m_C = \frac{\varepsilon_C}{P} = \frac{\frac{\varepsilon_{X'}}{P} + \frac{\varepsilon_{Y'}}{P}}{2} + \frac{\frac{\varepsilon_{X'}}{P} - \frac{\varepsilon_{Y'}}{P}}{2}\cos(2\theta_C) + \frac{\frac{\gamma_{X'Y'}}{P}}{2}\sin(2\theta_C) = \frac{\varepsilon_{Y'}}{P}$$

Equation 2: Calculating Strain from the Slope of Data Set C

The ratio $\frac{\gamma_{X'Y'}}{P}$ can be calculated from the slope of strain gage B.

$$m_B = \frac{\varepsilon_B}{P} = \frac{\frac{\varepsilon_{X'}}{P} + \frac{\varepsilon_{Y'}}{P}}{2} + \frac{\frac{\varepsilon_{X'}}{P} - \frac{\varepsilon_{Y'}}{P}}{2}\cos(2\theta_B) + \frac{\frac{\gamma_{X'Y'}}{P}}{2}\sin(2\theta_B)$$

Equation 3: Calculating Strain form the Slope for Data Set B

$$\rightarrow m_B = \frac{m_A + m_C}{2} + \frac{m_A - m_C}{2}\cos(2\theta_B) + \frac{\frac{\gamma_{X'Y'}}{P}}{2}\sin(2\theta_B)$$

Equation 4: Substituting Slope variables into Equation 3

Rearranging Equation 4 to solve for $\frac{\gamma_{X'Y'}}{P}$, we now have the ratios $\frac{\varepsilon_{X'}}{P}$, $\frac{\varepsilon_{Y'}}{P}$, and $\frac{\gamma_{X'Y'}}{P}$. Equation 5 is used to calculate the principal angle, θ_P. Note: $\theta_P = \phi$ or $\theta_P = 90 - \phi$.

$$\theta_P = \frac{1}{2}\tan^{-1}\left(\frac{\frac{\gamma_{X'Y'}}{P}}{\frac{\varepsilon_{X'}}{P} - \frac{\varepsilon_{Y'}}{P}}\right)$$

Equation 5: Principal Angle Equation

Equation 6 is used to find the principal strain ratios $\frac{\varepsilon_{P1}}{P}$ and $\frac{\varepsilon_{P2}}{P}$. The principal shear strain ratio, $\frac{\gamma_{P12}}{P}$, is equal to zero.

$$\frac{\varepsilon_{P1}}{P}, \frac{\varepsilon_{P2}}{P} = \frac{\frac{\varepsilon_{X'}}{P} + \frac{\varepsilon_{Y'}}{P}}{2} \pm \left[\frac{\frac{\varepsilon_{X'}}{P} - \frac{\varepsilon_{Y'}}{P}}{2}\cos(2\theta_P) + \frac{\frac{\gamma_{X'Y'}}{P}}{2}\sin(2\theta_P)\right]$$

Equation 6: Principal Strain Equation

Hooke's Law is used to calculate the principal stresses in Equations 7 and 8.

$$\frac{\sigma_{Hoop}}{P} = \frac{E}{1-\nu^2}\left(\frac{\varepsilon_{P1}}{P} + \nu\frac{\varepsilon_{P2}}{P}\right)$$

Equation 7: Principal Stress Equation in the Hoop Direction

$$\frac{\sigma_{Axial}}{P} = \frac{E}{1-\nu^2}\left(\frac{\varepsilon_{P2}}{P} + \nu\frac{\varepsilon_{P1}}{P}\right)$$

Equation 8: Principal Stress Equation in the Axial Direction

Thick-Walled Pressure Vessel

For the thick-walled pressure vessel experiment, we will only collect data on strain gages 1 and 2 (Fig. 3). Strain gage 1 is in the hoop direction, and strain gage 2 is in the radial direction. After strain and pressure data has been collected for both strain gages, use Excel to plot the trends. Then, find the slope of each data set (m_1 and m_2) as seen in Fig. 4.

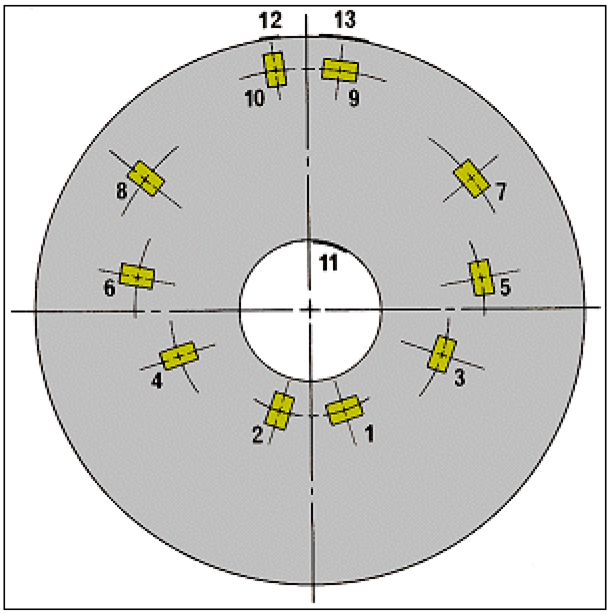

Figure 3: Gage Configuration of the Thick-Walled Pressure Vessel

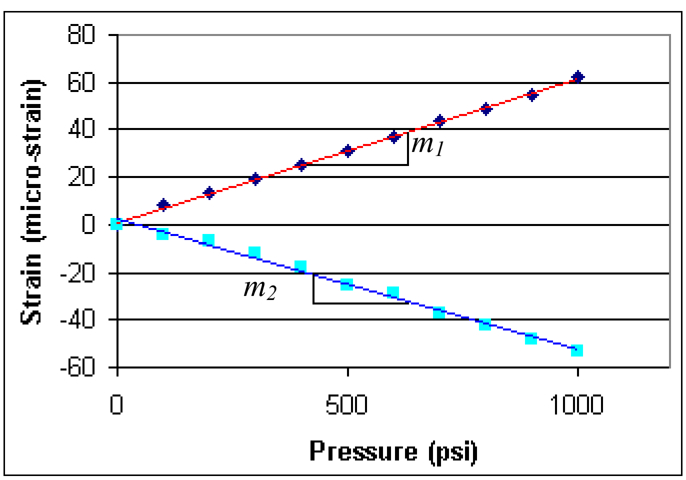

Figure 4: Plot of Strain versus Pressure for the Thin-Walled Pressure Vessel

For this plot, the slope, *m*, is the strain per pressure, where $m_1 = \frac{\varepsilon_1}{P} = \frac{\varepsilon_{Hoop}}{P}$ and $m_2 = \frac{\varepsilon_2}{P} = \frac{\varepsilon_{Radial}}{P}$. With this, we can now calculate the principal stresses using Equations 9 and 10.

$$\frac{\sigma_{Hoop}}{P} = \frac{E}{1-\nu^2}\left(\frac{\varepsilon_{Hoop}}{P} + \nu \frac{\varepsilon_{Radial}}{P}\right)$$

Equation 9: Principal Stress Equation in the Hoop Direction

$$\frac{\sigma_{Radial}}{P} = \frac{E}{1-\nu^2}\left(\frac{\varepsilon_{Radial}}{P} + \nu \frac{\varepsilon_{Hoop}}{P}\right)$$

Equation 10: Principal Stress Equation in the Radial Direction

Comparison to Theories

Now that we have calculated the principal stresses for both experiments, we will compare our experimental values to theoretical values from both the thin-walled and thick-walled pressure vessel theories. The equations for the thin-walled and thick-walled pressure vessel theories can be seen in the following table. Note that variables *a*, *b*, and *t* are defined in Figure 1. Variable *r* is equal to *b* for the thin-walled vessel and 1.102" for the thick-walled vessel.

Direction	**Thin-Wall Theory**	**Thick-Wall Theory**
Hoop	$\frac{\sigma_{Hoop}}{P} = \frac{a}{t}$	$\frac{\sigma_{Hoop}}{P} = a^2 \frac{\left(1+\frac{b^2}{r^2}\right)}{\left(b^2-a^2\right)}$
Axial	$\frac{\sigma_{Axial}}{P} = \frac{a}{2t}$	$\frac{\sigma_{Axial}}{P} = \frac{a^2}{b^2-a^2}$
Radial	$\frac{\sigma_{Radial}}{P} = -1$	$\frac{\sigma_{Radial}}{P} = a^2 \frac{\left(1-\frac{b^2}{r^2}\right)}{\left(b^2-a^2\right)}$