\[ Q \approx 1.439 \times 10^9 \text{ miles} \]
\[ \approx 2.31 \times 10^{13} \text{ meters} \]
\[ \text{time} = \frac{Q}{c} = \frac{2.31 \times 10^{13}}{3 \times 10^8} \approx 7.7 \times 10^4 \text{ sec} \]
\[ \approx 21 \text{ hr}, 27 \text{ min} \]

Antenna Gain = \[ \frac{\text{Ant. Area}}{\text{Isotropic Area}} \]

= \[ \frac{\pi r^2}{\left( \frac{\lambda^2}{4\pi} \right)} = \frac{4\pi^2 r^2}{\lambda^2} \]

\[ r \approx \frac{3.7 \text{ m}}{2} \]
\[ \lambda = \frac{c}{f} = \frac{3 \times 10^8}{2.03 \times 10^8} \]
Gain = \frac{4\pi^2 r^2}{\lambda^2} \approx 8,000

10 \log_{10} (8,000) \approx 39 \text{ dBi}

NASA specifies gain is 36 dBi.
So the ant. is 3 dB worse than ideal.

X-band gain

Same calculation, but with

f = 8.4 GHz

Gain \approx 106,000

10 \log_{10} (106,000) \approx 50 \text{ dBi}

NASA Spec: 48 dBi.
Gold stone dish at x-band:

\[ G_{\text{max}} = \frac{4\pi^2 r^2}{\lambda^2} \]

\[ r = \frac{70}{2} \text{ m} \]

\[ \lambda = \frac{c}{f} \]

\[ c = 3 \times 10^8 \]

\[ f = 8.4 \times 10^9 \]

\[ G_{\text{max}} \approx 3.8 \times 10^7 \]

\[ 10 \log_{10} (3.8 \times 10^7) \approx 76 \text{ dBi} \]

\[ \text{NASA spec: 73 dBi} \]
\[ \theta \approx \frac{2}{\sqrt{G_7}} \]

\( \theta \) in radians

\( G \) linear gain.

**S-Band**

\[ G = 8000 \]

\[ \theta = \frac{2}{\sqrt{8000}} \cdot \frac{180}{\pi} \text{ degrees} \]

\[ \theta \approx 1.3^\circ \]

**X-Band**

\[ G = 106,000 \]

\[ \theta \approx 0.35^\circ \]
\[ d = \tan(\theta) \times \frac{30.7}{2} \]

\[ S - Band \]
\[ \theta = 1.3^\circ \]
\[ d = 4.2\text{ cm} \]

\[ X - Band \]
\[ \theta = 0.35^\circ \]
\[ d = 1.01\text{ cm} \]

Dish is 12 feet in diameter.
Edge must be placed accurate to \( \frac{1}{2} \text{ inch} \).
3) Goldstone at X-Band

(Continuation of problem #2)

\[ \text{Gain} = G = 3.8 \times 10^7 \] (problem #1)

\[ \Theta = \frac{2}{159} \frac{180}{\pi} \text{ degrees.} \]

\[ \Theta = 0.0186^\circ \]

about 1 arc minute.

Max pointing error.

\[ d = \tan(\Theta) \frac{70}{2} \]

\[ d = \boxed{6.1 \text{ cm.}} \]

Dish is close to the size of a football field. Edge must be within \( \frac{1}{2} \) inch of ideal location.

Surface imperfections should be less than \( \boxed{\frac{\lambda}{10}} \).
\[ \frac{\lambda}{10} \approx 306 \text{mm} \]
\[ \frac{1}{8} \text{ in. inch} \]

\[ \theta = \tan^{-1}\left(\frac{1}{147}\right) \]
\[ \theta \approx 0.39^\circ \]

If center points at sun, Earth is easily in field of view.
X-Band

Voyager. Earth may be just outside of antenna beam.

5) Linear calculations.

Tx power: 22.4 watts.

Rx power density if Voyager used isotropic transmit antenna,

\[
\frac{22.4}{4\pi d^2} = \frac{22.4}{4\pi (22 \times 10^{12})^2} = 3.07 \times 10^{-27} \frac{W}{m^2}
\]

Rx power density with Voyager and gain,

\[
(3.07 \times 10^{-27})(106,000) = 3.09 \times 10^{-22} \frac{W}{m^2}
\]

Rx power if Goldstone used isotropic receive antenna.
Rx Power:

\[
\left(3.9 \times 10^{-22} \frac{W}{m^2}\right)(\pi \left(\frac{70}{2}\right)^2) = 1.5 \times 10^{-18} W
\]

**Link Budget (dB) Calculations.**

**Tx Power:** 22.4 watts.

\[
10 \log_{10}(22.4) = 13.5 \text{ dBW}
\]

**Path Loss**

\[
20 \log_{10}\left(\frac{4\pi d}{\lambda}\right) = 318 \text{ dB}
\]

\[d = 22 \times 10^{12}
\]

\[\lambda = \frac{c}{f} = \frac{3 \times 10^8}{8.4 \times 10^9}
\]

**Link Budget.**

**Tx Power:** 13.5 dBW

**Tx Ant. Gain:** 50 dBd

**Path Loss:** -318 dB

**Rx Ant. Gain:** 76 dBd

\[-178.5 \text{ dBW}\]
\[-178.5 + 30 = -148 \text{ dBm}\]

6) **S-Band**

Tx Power 13.5 dBW
Tx Ant. Gain 39 dB

Path loss: \[
20 \log_{10} \left( \frac{4\pi d}{\lambda} \right) = 306 \text{ dB}
\]

\[
d = 22 \times 10^{12}
\]

\[
\lambda = \frac{c}{f} = \frac{3 \times 10^8}{2.3 \times 10^9}
\]

Tx Power +13.5 dBW
Tx Ant. +39 dB
Path loss -306 dB
Rx Ant. +64 dB

\[
-190 \text{ dBW}
\]

\[
10^{-19} \text{ Watts}
\]

\[-190 + 30 = -160 \text{ dBm}\]
7) Geosynchronous satellites appear in sky approx where sun and moon do.

The best ant. and orientation, depends on where you are on Earth.

- Equator
  Sat. appear on line, extending from due-East to due-West, and going overhead.

A North-South Horizontal Dipole will be good for all.

E  N  S  W
An East/west horizontal dipole will work OK for those overhead, but not near horizon.

No Good. ———— OK ———— No Good.

Verical ant. will be the opposite. They will work well for the Sat near either horizon—but not overhead.

No Good

OK ———— OK

E ———— W
At North or South Pole, all geosync satellites appear near horizon. Vertical ant. are best here as it catches all of them.

Any type of horizontal dipole will miss some of them.

No Good.
At other latitudes it will be a mix. But for those of us closer to the equator than the poles (most people) a north/south horizontal dipole is probably best.
Angle between Sat, when viewed from Earth should be about 2θ (but if you picked θ, that is OK too)

\[ \theta = \frac{2}{\sqrt{G}} \]

\[ G = \frac{\text{ANT AREA}}{\text{ISO AREA}} \]

\[ G = \frac{\pi r^2}{x^2/4\pi} = \frac{\pi (0.5)^2}{(c/f)^2/4\pi} = \]

\[ G = \frac{\pi (0.5)^2}{4\pi f^2} \]

\[ f = 18 \times 10^9 \text{ Hz} \]

\[ \sqrt{7.15} \]
\[ \theta = \frac{2}{\sqrt{35,000}} = 0.0106 \text{ radians.} \]
\[ \approx 0.6^\circ \]

If we space sat. every 2\(\theta\),
That's about every 10
we can fit about \(\sqrt{360}\) sat
in geo sync. orbit.
12-dBi antenna.

\[
\theta = \frac{2}{\sqrt{\phi}} = \frac{2}{\sqrt{10^{(12/6)}}} = 0.5 \text{ radians.}
\]

\[
\theta \approx 29^\circ
\]

\[
\psi = \tan^{-1}\left(\frac{6400}{22200}\right) = 0.28 \text{ rad.}
\]

\[
\psi \approx 16^\circ
\]

Beam from antenna covers more than just the Earth.
10) \[ T_x \text{ Power} = 44.8 \text{ watts} = 16.05 \text{ dBW} \]

\[ T_x \text{ Ant Gain} = 12 \text{ dBi} \]

\[ R_x \text{ Ant Gain} = 2 \text{ dBi} \]

\[ \text{Path loss} \]

\[ 20 \log_{10} \left( \frac{4\pi d}{\lambda} \right) \]

\[ d = 2 \times 10^7 \text{ m} \]

\[ \lambda = \frac{c}{f} = \frac{3 \times 10^8}{1.5 \times 10^8} = 0.2 \text{ m} \]

\[ \text{Path loss} = 182 \text{ dB} \]

\[ R_x \text{ Power} = 16.05 + 12 - 182 + 2 \]

\[ = -151.5 \text{ dBW} \]

\[ = -121.5 \text{ dBm} \]

\[ = 10^{-15\%} = 7 \times 10^{-16} \text{ watts}. \]

\[ -121 \text{ dBm} \]
Amp.

\[ \text{\(\frac{1}{2}\) wave dipole} \]

73 ohm input resistance for max power transfer.

Power of an A volt sine wave across a 73 \(\Omega\) resistor.

\[ P = \frac{A^2}{2R} \]

\[ 7 \times 10^{-16} = \frac{A^2}{2(73)} \]

\[ A \approx 320 \text{nV} \]

If you did not have to calculate this, I was just curious.
\[(\text{velocity})(t, \text{me}) = \text{distance}\.
(3 \times 10^8)(t) = 0.3 \text{ m}
\]
\[t \approx 1 \text{ nano sec}\]
12) \( 942 \text{ MHz}, \ 22 \text{ dBi Gain} \)

Antenna:

\[
\theta = \frac{\lambda}{4\pi}
\]

\[
\theta \approx \frac{2}{\sqrt{158}} \quad \text{radians, not degrees}
\]

Linear not dBi

\[
G_{\text{dBi}} = 10 \log_{10}(G)
\]

\[
22 = 10 \log_{10}(G)
\]

\[
2.2 = G
\]

\[
158 \approx G
\]

\[
\theta \approx \frac{2}{\sqrt{158}} = 0.016 \text{ radians}
\]

\[
0.016 \cdot \left(\frac{180}{\pi}\right) = 9^0
\]

\[
\text{Receive power proportional to } \frac{1}{d^2}
\]
With isotropic antenna at 50 meters received power P.

\[ \text{constant} \cdot \frac{1}{50^3} \]

When I use the antenna with a gain of 158, at distance d, I get a received power of

\[ \text{constant} \cdot \frac{158}{d^3} \]

Set equal

\[ \text{constant} \cdot \frac{1}{50^3} = \text{constant} \cdot \frac{158}{d^3} \]

\[ d = \sqrt[3]{158 \cdot 50^3} = 270 \]

**Antenna increases range from 50 m to 270 m**
13) A) 

\[ \Rightarrow 0\text{dBm} = 10^{-3}\text{watts.} \]

Amp. Ant

\[ \begin{array}{c}
320\text{mV} \\
\end{array} \]

B) 

\[ \Rightarrow 10^{-3}\text{watts} \]

\[ \text{1 meter.} \]

\[ R_k \text{ power density} = \frac{10^{-3}W}{4\pi(1^2)m^2} \]

\[ = 8 \times 10^{-5} \frac{W}{m^2} \]

\[ R_k \text{ power.} \]

\[ 8 \times 10^{-5} \frac{W}{m^2} \cdot \frac{\lambda^2}{4\pi} \text{m}^2 \]

\[ \lambda = \frac{c}{f} = \frac{3 \times 10^8}{2 \times 10^9} = 0.125 \]
\[ P_{\text{rx}} \text{ Power} \]

\[
\left( 8 \times 10^{-5} \frac{\text{W}}{\text{m}^2} \right) \left( \frac{0.125^2}{4\pi} \right) = 10^{-7} \text{ Watts}
\]

\[ 50 \Omega \]

\[ P_{\text{rx}} = 10^{-7} \text{ Watts} \]

\[ R_x \text{ Antenna, Amp Input.} \]

\[ P_{\text{rx}} = \frac{A^2}{2R} \]

\[ 10^{-7} = \frac{A^2}{2(50)} \]

\[ A = 3.2 \text{ mV} \]
c) \[ 10^{-7} W = 10 \log_{10} (10^{-7}) = -70 \text{ dBW} \]
\[ -70 + 30 = -40 \text{ dBm} \]

d) \[ \frac{A^2}{2(50)} = 2.5 \times 10^{-3} \]
\[ A = 500 \text{ mV} \]

e) \[ \left( 2.5 \times 10^{-3} \text{ W} \right) \left( \frac{1}{4\pi (10^2) \text{ m}^2} \right) \left( \frac{0.125^2}{4\pi} \right) = \]
\[ \uparrow \]
\[ \text{TX Power} \]
\[ \uparrow \]
\[ \text{Ant. \& \ 10 m Sphere.} \]
\[ \uparrow \]
\[ \text{FSO. \ Ant. \ & \ Aim.} \]
\[ 2.5 \times 10^{-9} \text{ W \ Rx Power.} \]
\[ \frac{A^2}{2(50)} = 2.5 \times 10^{-9} \]
\[ A = 500 \mu \text{V} \]
\[ F = 10 \log_{10} (2.5 \times 10^{-9}) = -86 \text{ dBW} \]

\[ -86 + 30 = -56 \text{ dBm} \]