An Optimization-based Composite Indicators Approach to Performance Assessment

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Abstract. Composite indicators (CIs) approach has been widely accepted as a useful tool for assessing system performance at macro level. Several recent studies have shown that the weighted product (WP) method, a multiple criteria decision analysis method, may be a good choice in constructing CIs. However, a problem in its application is the subjectivity in determining the weights for sub-indicators. This paper extends the WP method and proposes an optimization-based approach to constructing CIs. The proposed approach requires no prior knowledge of the weights for sub-indicators. The weights used can be generated by solving a series of multiplicative DEA type models that can be transformed into equivalent linear programs. Additional information on the weights can be easily incorporated into the proposed models. A case study on assessing the performance of APEC economies towards sustainable energy development is finally presented to illustrate the use of the approach.

Keywords. Performance assessment; Composite indicators; Multiple criteria decision analysis

Introduction

Indicators refer to quantitative measures that represent the state of an individual object such as a product, a process or a complex system (Zhou, and Ang 2008). According to the OECD Glossary of Statistical Terms, a composite indicator (CI) is formed when individual indicators are compiled into a single index on the basis of an underlying model of the multi-dimensional concept that is being measured. Owing to its ability in providing an analytical foundation for systems performance analysis, public communication and decision making, CI has been increasingly accepted as a useful tool for systems performance assessment at macro level, e.g. economy, environment, technology/innovation, and society (OECD 2008). Several well-known CI examples are the OECD’s Composite Leading Indicators, the United Nations’ Human Development Index, and the Environmental Performance/Sustainability Index produced by a joint effort from Yale, Columbia, World Economic Forum and the Joint Research Center of European Commission. The information server (http://farmweb.jrc.cec.eu.int/ci/) maintained by the Joint Research Center of European Commission provides a list of CIs that are classified by their specific application areas.
The popularity of CIs in practice is likely due to what has been pointed out by Saisana, Saltelli, and Tarantola (2005): “the temptation of stakeholders and practitioners to summarize complex and sometime elusive process (e.g. sustainability or a single-market policy) into a single figure to benchmark country performance for policy consumption seems irresistible”. Nevertheless, since a CI is essentially a mathematical aggregation of a set of sub-indicators with different measurement units, its quality and reliability heavily depends on the underlying weighting and aggregation schemes (Saisana, Saltelli, and Tarantola 2005). As a result, the study on data weighting and aggregation has always been an interesting but controversial topic in the area of CI construction (Esty et al. 2006). In recent years, the applicability of two major systems analysis techniques, namely data envelopment analysis (DEA) and multiple criteria decision analysis (MCDA), have been widely explored in the field of CI construction.

Roughly speaking, the application of DEA to CI construction follows two different routes. One need to identify inputs and outputs first and then use the DEA models in envelopment form to construct a composite efficiency index. Examples of such studies include Lovell, Pastor, and Turner (1995) and Ramanathan (2006). In the other line, all the sub-indicators are firstly transformed into the same type of variables (benefit or cost type) and then aggregated into a CI by the variants of some traditional DEA models. In recent years, much attention has been focused on this line of research, e.g. Lau and Lam (2002), Despotis (2005a, b), Zhou et al. (2007a) and Cherchye et al. (2008).

Within the MCDA group, Ebert and Welsch (2004) showed that the weighted product (WP) method is theoretically superior to the simple additive weighting method in CI construction when only the ordinal information on CIs is expected. Munda (2005) highlighted the advantages of the non-compensatory MCDA approach in constructing CIs over the compensatory MCDA methods. Zhou, Ang, and Poh (2006) showed that the WP method seems to have better properties than several other MCDA methods, provided that the cardinality characteristic of CIs is concerned. More recently, Zhou, Ang, Poh (2007b), and Zhou, and Ang (in press) gave further evidences on the superiority of the WP method based on the “minimum information loss” concept.

Despite the many advantages of the WP method, a major problem in applying the WP method to construct CIs is the determination of weights for sub-indicators. Theoretically, there exist a number of weighting methods which can be used to derive the weights for sub-indicators. OECD (2008) has recently given a discussion on the alternative weighting methods that have their specific strength and weakness. Obviously, the existence of many weighting methods brings difficulty in the choice of an appropriate one. To avoid the subjectivity in determining the weights for sub-indicators, in this paper we extend the WP method and present an optimization approach to constructing CIs. A key feature of the proposed approach is that it considers data weighting and aggregation simultaneously but still reserves the feature of the WP method in aggregation manner. Since the proposed approach uses two sets of weights that are most and least favourable for each entity, it may provide a more reasonable and encompassing CI.

The remainder of this paper is organized as follows. In Section 2 we give a brief introduction to the problem of CI construction and the WP method. Section 3 presents our optimization-based CI construction approach. In Section 4, we present a case study on assessing the performance of APEC economies towards sustainable energy development. Section 5 concludes this study.
The Weighted Product Method

Assume that there are $m$ entities, e.g. countries, whose CIs are to be calculated based on $n$ sub-indicators. Let $I_{ij}$ denote the value of entity $i$ with respect to sub-indicator $j$. Without loss of generality, we further assume that all the sub-indicators are positive and of the benefit type, i.e. they satisfy the property of “the larger the better”. Our purpose is to aggregate $I_{ij} (j = 1, 2, \ldots, n)$ into a composite indicator $CI_i$ for entity $i$, which is described as Figure 1.

$$
\begin{bmatrix}
I_{11} & I_{12} & \cdots & I_{1n} \\
I_{21} & I_{22} & \cdots & I_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
I_{m1} & I_{m2} & \cdots & I_{mn}
\end{bmatrix} \rightarrow
\begin{bmatrix}
CI_1 \\
CI_2 \\
\vdots \\
CI_m
\end{bmatrix}
$$

Figure 1. Graphical representation of CI construction.

In the context of MCDA, the simple additive weighting (SAW) method is one of the most popular aggregation methods for constructing CIs. Its use requires the pre-determination of the weights for all the sub-indicators. In some cases, it also requires the normalization of the sub-indicators before aggregation. Suppose that the weight for sub-indicator $j$ is $w_j$. The SAW method can be formulated as

$$
CI_i = \sum_{j=1}^{n} w_j I_{ij} \quad i = 1, 2, \ldots, m
$$

It should be pointed out that an assumption implied by the SAW method is that the sub-indicators need to be mutually preferentially independent, which may be difficult to satisfy. Another problem in the SAW method is that the weights carry the meaning of trade-off ratios, which is inconsistent with its original meaning as importance coefficients (Munda 2005). Despite of these issues, the SAW method has been widely adopted in practice due to its transparency and ease of understanding for non-experts.

Compared to the SAW method, the WP method seems to be a better choice in CI construction since it possesses some desirable properties as discussed by Ebert and Welsch (2004), Zhou, Ang, and Poh (2006), and Zhou, and Ang (in press). Mathematically, the WP method can be written as

$$
CI_i = \prod_{j=1}^{n} I_{ij}^{w_j} \quad i = 1, 2, \ldots, m
$$

The difference between the WP method and the SAW method is that the former aggregates sub-indicators in a multiplicative rather than additive manner. Theoretically, the WP method represents a concept that lies between the MCDA approach with full compensability, e.g. the SAW method, and the non-compensatory MCDA approach (OECD 2008). However, a problem in applying the WP method to construct CIs is the determination of the weights for sub-indicators. To avoid the subjectivity in assigning the weights, we extend the WP method and propose a optimization-based CI construction approach, which will be introduced in the next section.

Optimization-based Composite Indicator

As mentioned earlier, a critical issue in using the WP method to construct CIs is the subjectivity in
assigning weights to sub-indicators. Since different weight combinations may lead to different CI values and therefore different ranking results, it is unlikely that all the entities would easily reach a consensus in determining an appropriate set of weights. In addition, obtaining the expert information for deriving the weights is not an easy task. To avoid these issues, we propose an optimization-based CI construction approach which consists of several models. Our first model can be formulated as

\[
g_{l,i} = \max \prod_{j=1}^{n} I_{ij}^w
\]

subject to

\[
\prod_{j=1}^{n} I_{ij}^w \leq e, \quad i = 1,2,\ldots,m
\]

\[
w_j \geq 0, \quad j = 1,2,\ldots,n
\]

Model (3) provides an aggregated performance score for entity \(i\) in terms of all the underlying sub-indicators. By solving (3) repeatedly for each entity, we can obtain a set of performance scores \(g_{l,i}, g_{l,2}, \ldots, g_{l,m}\) for these entities. Note that the objective function in Model (3) is externally similar to the WP method as described in Section 2. The difference is that in Model (3) the weights for sub-indicators are endogenous and changeable while in the WP method they are exogenous and fixed. Externally, (3) is similar to the multiplicitive DEA model for efficiency analysis proposed in Charnes et al. (1982). It also shares a common feature with previous additive DEA type models in the sense that they all attempt to help each entity select the best set of weights for use (Zhou, and Fan 2007). It should be pointed out that Model (3) is not new as it has recently been proposed and applied to the Technology Achievement Index case in Zhou, Ang, Poh, and Fan (2007).

Since Model (3) can help each entity select the “best” set of weights for use, it avoids the subjectivity in determining the weights and therefore provides a relatively objective performance score for each entity. However, if an entity has a value dominating other entities in terms of a certain sub-indicator, this entity would always obtain a score of \(e\) even if it has severely bad values in other more important sub-indicators. Furthermore, only (3) may lead to the situation that a large number of entities have the same performance score and further ranking among them becomes impossible. To address these issues, we propose a similar optimization model as follows:

\[
b_{l,i} = \min \prod_{j=1}^{n} I_{ij}^w
\]

subject to

\[
\prod_{j=1}^{n} I_{ij}^w \geq e, \quad i = 1,2,\ldots,m
\]

\[
w_j \geq 0, \quad j = 1,2,\ldots,n
\]

Contrary to Model (3), Model (4) seeks the “worst” set of weights for each entity which are used to aggregate the sub-indicators into a performance score. Externally, Model (4) is very similar to a multiplicitive DEA model with multiple inputs and constant outputs. However, in (4) all the sub-indicators are of the benefit type and it is not appropriate to treat them as “inputs”. In essence, Model (4) attempts to measure how close the entity evaluated is from the worst practice entity under the worst possible weights. It provides a way for further performance comparison among those incomparable entities only based on Model (3).
So far we have provided two performance indexes for each entity which are derived from two DEA-like models, i.e. Model (3) and (4). Since the two indexes are based on weights that are most favourable and least favourable for each entity, they could only reflect partial aspects of an entity in terms of its aggregated performance. It is logical and reasonable to combine them into an overall index. Therefore, we combine the two indexes to form a CI in the following way:

\[
CI_i(\lambda) = \lambda \cdot \frac{gl_i^l - gl_i^{l_{\min}}}{gl_i^{l_{\max}} - gl_i^{l_{\min}}} + (1 - \lambda) \cdot \frac{bl_i^l - bl_i^{l_{\min}}}{bl_i^{l_{\max}} - bl_i^{l_{\min}}}
\]

(5)

where \( gl_i^{l_{\max}} = \max\{gl_i^l, i = 1, \ldots, m\} \), \( gl_i^{l_{\min}} = \min\{gl_i^l, i = 1, \ldots, m\} \), \( bl_i^{l_{\max}} = \max\{bl_i^l, i = 1, \ldots, m\} \), \( bl_i^{l_{\min}} = \min\{bl_i^l, i = 1, \ldots, m\} \), and \( 0 \leq \lambda \leq 1 \) is a control parameter.

In Model (5), we use the linear scaling in the min-max range to let the two indexes become comparable and then use the linear aggregation to combine them together by a control parameter. If \( \lambda = 1 \), \( CI_i \) will become a normalized version of \( gl_i^l \). If \( \lambda = 0 \), \( CI_i \) will become a normalized version of \( bl_i^l \). For other cases, Model (5) makes a compromise between \( gl_i^l \) and \( bl_i^l \). Therefore, we may say that Model (5) provides a more encompassing CI since it takes into account two extreme cases. In application, if decision makers or analysts have no particular preferences, \( \lambda = 0.5 \) seems to be a fairly neutral choice.

It can be easily shown that \( CI_i \) satisfies the property \( 0 < CI_i \leq 1 \). It implies that Model (5) provides a standardized index which lies in the interval \( (0, 1] \). The larger \( CI_i \) is, the better the entity \( i \) performs. If an entity has the largest values in terms of both \( gl_i^l \) and \( bl_i^l \), it will give a CI of “1” no matter what \( \lambda \) is. If an entity has the smallest values in terms of both \( gl_i^l \) and \( bl_i^l \), it will give a CI of “0” no matter what \( \lambda \) is.

Up to now we focus only on the theoretical analysis of Model (3) to (5). Since Model (3) and (4) are two nonlinear programming problems, it may not be easy to solve them directly. However, by taking logarithms with \( e \) as the base, we can obtain their equivalent linear programs as follows:

\[
gl_i' = \max \sum_{j=1}^n w_j l_{ij}'
\]

s.t. \( \sum_{j=1}^n w_j l_{ij}' \leq 1, \ i = 1, 2, \ldots, m \) \hspace{1cm} (6)

\( w_j \geq 0, \ j = 1, 2, \ldots, n \)

\[bl_i' = \min \sum_{j=1}^n w_j l_{ij}'\]

s.t. \( \sum_{j=1}^n w_j l_{ij}' \geq 1, \ i = 1, 2, \ldots, m \) \hspace{1cm} (7)

\( w_j \geq 0, \ j = 1, 2, \ldots, n \)

where \( l_{ij}' = \ln l_{ij} \), \( gl_i' = \ln(gl_i^l) \) and \( bl_i' = \ln(bl_i^l) \).
We can then derive the CI value for each entity by solving Model (6) and (7). Assume that 
\[ g_i^{\text{max}} = \max \{ g_i', i = 1, \ldots, m \}, \quad g_i^{\text{min}} = \min \{ g_i', i = 1, \ldots, m \}, \quad b_i^{\text{max}} = \max \{ b_i', i = 1, \ldots, m \} \quad \text{and} \quad b_i^{\text{min}} = \min \{ b_i', i = 1, \ldots, m \} \], we can transform Model (5) into

\[
CI_i(\lambda) = \lambda \cdot \frac{\exp(g_i') - \exp(g_i^{\text{min}})}{\exp(g_i^{\text{max}}) - \exp(g_i^{\text{min}})} + (1 - \lambda) \cdot \frac{\exp(b_i') - \exp(b_i^{\text{min}})}{\exp(b_i^{\text{max}}) - \exp(b_i^{\text{min}})} \tag{8}
\]

Note that in the previous models all the weights used for aggregation are generated by model itself and no exogenous restrictions on the weights are imposed. In the circumstance, the weights for some sub-indicators may be equal to zero so that these sub-indicators would be ignored in aggregation. In addition, some information on the weights may become available in some cases. Therefore, it is worthwhile to restrict the flexibility of weights in an appropriate manner by incorporating additional information. In DEA literature, this can be done by a number of methods as reviewed by Allen, Athanassopoulos, Dyson, and Thanassoulis (1997). Here we suggest the use of “proportion constraints” proposed by Wong and Beasley (1990) to restrict the flexibility of the weights. In CI construction, the usefulness of “proportion constraints” has been demonstrated by Zhou, Ang, and Poh (2007a) although their study is based on additive DEA type models.

Following Zhou, Ang, and Poh (2007a), we give the “proportion constrains” in multiplicative form as follows:

\[
\left( \prod_{j=1}^{n} I_{y_j} \right)^{L_j} \leq I_y^{w_j} \leq \left( \prod_{j=1}^{n} I_{y_j} \right)^{U_j} \quad j = 1, 2, \ldots, n \tag{9}
\]

where \( L_j \) and \( U_j \) are respectively denote the lower and upper limits for the contribution of the \( j \)-th sub-indicator in CI and satisfy \( 0 \leq L_j < U_j \leq 1 \). By including (9) into the constraints in (3) and (4), we may obtain two general optimization models for data weighting and aggregation. Note that (9) is also a multiplicative model in form. If the variables after transformation are used, we will obtain the following “proportion constraints” in standard form:

\[
L_j \leq \frac{w_j I_{y_j}}{\sum_{j=1}^{n} w_j I_{y_j}} \leq U_j \quad j = 1, 2, \ldots, n \tag{10}
\]

As a result, we can incorporate (10) into Model (6) and (7) before calculating \( g_i' \) and \( b_i' \). In practice, usually it is easier and more practical to let experts make a “limited agreement” on the determination of the weights, which can be done by making a consensus among experts as for the relative importance of each sub-indicator. For instance, if the experts make an agreement that the contribution of the first sub-indicator to the overall index should be larger than 10% but less than 50%, we should let \( L_1 = 0.1 \) and \( U_1 = 0.5 \). In the case that no consensus could be reached in terms of a certain sub-indicator, we can remove the corresponding weight restriction constraint. If no expert information is given, we can let \( L_j = 0 \) and \( U_j = 1 \) which makes the revised models the same as the basic models.

**Case Study**

The Sustainable Energy Index (SEI) is a CI given in Zhou, Ang, and Poh (2007a) that might be
used to assess the performance of an economy towards sustainable energy development. It involves the aggregation of three sub-indicators, namely energy efficiency indicator (EEI), renewable energy indicator (REI) and climate change indicator (CCI). The definitions on the three sub-indicators as well as the sources of data can be found in Zhou, Ang, and Poh (2007a). By using the three sub-indicators, we apply the proposed multiplicative optimization approach to calculate the SEIs for eighteen APEC economies in 2002 with the purpose of illustrating the use of the proposed approach. Table 1 presents the SEI results based on our proposed multiplicative optimization models (SEI-MOM1) and the additive optimization models (SEI-AOM) given in Zhou, Ang, and Poh (2007a). Both of them set the control parameter $\lambda$ equal to 0.5.

Table 1: The SEI results based on multiplicative and additive optimization models

<table>
<thead>
<tr>
<th>Economy</th>
<th>EEI (10^3 US$ per toe)</th>
<th>REI (%)</th>
<th>CCI (10^3 US$ per tons)</th>
<th>SEI-MOM</th>
<th>SEI-AOM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>6.208</td>
<td>5.6</td>
<td>1.425</td>
<td>0.179</td>
<td>0.116</td>
</tr>
<tr>
<td>Canada</td>
<td>4.286</td>
<td>46.8</td>
<td>1.608</td>
<td>0.543</td>
<td>0.477</td>
</tr>
<tr>
<td>Chile</td>
<td>6.950</td>
<td>32.2</td>
<td>2.542</td>
<td>0.514</td>
<td>0.463</td>
</tr>
<tr>
<td>China</td>
<td>8.178</td>
<td>11.1</td>
<td>1.372</td>
<td>0.349</td>
<td>0.214</td>
</tr>
<tr>
<td>Indonesia</td>
<td>8.516</td>
<td>7.8</td>
<td>1.784</td>
<td>0.356</td>
<td>0.240</td>
</tr>
<tr>
<td>Japan</td>
<td>8.647</td>
<td>8.2</td>
<td>2.522</td>
<td>0.367</td>
<td>0.353</td>
</tr>
<tr>
<td>Korea</td>
<td>4.683</td>
<td>0.6</td>
<td>1.437</td>
<td>0.135</td>
<td>0.064</td>
</tr>
<tr>
<td>Malaysia</td>
<td>5.767</td>
<td>4.0</td>
<td>1.442</td>
<td>0.134</td>
<td>0.101</td>
</tr>
<tr>
<td>Mexico</td>
<td>8.424</td>
<td>9.5</td>
<td>2.059</td>
<td>0.359</td>
<td>0.278</td>
</tr>
<tr>
<td>New Zealand</td>
<td>5.473</td>
<td>56.9</td>
<td>2.281</td>
<td>0.658</td>
<td>0.648</td>
</tr>
<tr>
<td>Papua New Guinea</td>
<td>12.381</td>
<td>23.5</td>
<td>5.039</td>
<td>0.815</td>
<td>0.810</td>
</tr>
<tr>
<td>Peru</td>
<td>13.825</td>
<td>53.6</td>
<td>4.510</td>
<td>0.914</td>
<td>1.000</td>
</tr>
<tr>
<td>Philippines</td>
<td>17.758</td>
<td>44.6</td>
<td>4.136</td>
<td>1.000</td>
<td>0.977</td>
</tr>
<tr>
<td>Russia</td>
<td>2.453</td>
<td>11.5</td>
<td>0.652</td>
<td>0.059</td>
<td>0.000</td>
</tr>
<tr>
<td>Taiwan (China)</td>
<td>5.539</td>
<td>2.6</td>
<td>1.391</td>
<td>0.103</td>
<td>0.081</td>
</tr>
<tr>
<td>Thailand</td>
<td>8.204</td>
<td>4.8</td>
<td>1.891</td>
<td>0.315</td>
<td>0.220</td>
</tr>
<tr>
<td>United States</td>
<td>5.901</td>
<td>6.0</td>
<td>1.614</td>
<td>0.157</td>
<td>0.144</td>
</tr>
<tr>
<td>Vietnam</td>
<td>10.790</td>
<td>30.0</td>
<td>2.478</td>
<td>0.626</td>
<td>0.529</td>
</tr>
<tr>
<td>Mean</td>
<td>7.999</td>
<td>20.0</td>
<td>2.232</td>
<td>0.421</td>
<td>0.373</td>
</tr>
</tbody>
</table>

It can be seen from Table 1 that all the economies can be compared with each other based on the SEI values derived from our multiplicative optimization models, which demonstrates that the proposed approach will lead to CIs with higher discriminating power. In addition, we find that the SEI values based on our multiplicative optimization models are extremely highly correlated with the SEI values based on the additive optimization models given in Zhou, Ang, and Poh (2007a). It can also be found that the SEI rankings for a number of economies remained the same no matter multiplicative or additive optimization models are adopted. Interestingly, although Russia always
has the least SEI value under the two approaches, its value is not equal to zero if the proposed multiplicative optimization approach is used.

To examine whether the control parameter has severe effects on the SEI values, we consider all the cases that \( \lambda = 0.1, 0.2, \ldots, 0.9 \). Using the nine \( \lambda \) we can get nine SEI scores for each of the eighteen economies. Figure 2 shows the comparative box plots of the SEI values for the eighteen economies. It can be seen from Figure 2 that the SEI value is very insensitive to \( \lambda \) for most economies. In fact, as shown in Figure 2, the SEI values for two thirds of economies keep almost no changes when \( \lambda \) is changeable. Nevertheless, for such economies as New Zealand and Canada, their SEI values keep increasing when \( \lambda \) becomes larger, which is consistent with the results given in Zhou, Ang, and Poh (2007a). It could be explained by the fact that these economies perform better in terms of Model (4) than in terms of Model (3).

![Figure 2. Boxplots of the SEI values for eighteen APEC economies when \( \lambda \) is changeable](image)

Previous discussions are based on the basic models of the proposed multiplicative optimization approach, i.e. no additional information on the weights is considered. Now we shall consider the case that the flexibility of weights is restricted in the form of (9) and (10). We arbitrarily choose \( L_1 = 0.1, L_2 = L_3 = 0, \) and \( U_1 = U_2 = U_3 = 0.8 \) for use, which indicates that the contribution of the first sub-indicator is not less than 10% while the contributions of all the three sub-indicators cannot be larger than 80% of the aggregated CI. We then apply the resulting models to recalculate the SEI values for these economies by using \( \lambda = 0.5 \). The results obtained, labeled as Scenario 2, as well as the SEI values without restricting the flexibility of weights (Scenario 1) are displayed in Figure 3.
Figure 3. Comparison between the SEIs from basic models and those from the models with weight restrictions

It can be seen from Figure 3 that the SEI values of most economies have decreased once the weights for sub-indicators are restricted in the manner specified. Nevertheless, the SEI ranks for most economies have not many changes. For instance, under Scenario 1 Philippines has the largest SEI value while under Scenario 2 it still has the largest SEI value. In fact, the correlation coefficient between two sets of SEI values is as high as 0.84.

As mentioned earlier, the CI values based on the proposed models are not invariant to the measurement units of sub-indicators. To examine whether the measurement units of sub-indicators have large impacts on the SEI values, we now consider the case when the measurement units of sub-indicators are changed. In an arbitrary way, we reset the units of EEI, REI and CCI as US$/toe, 1/10000, and US$/tons, respectively. We then use the data obtained to recalculate the SEI values for these economies. The results obtained, labeled as Scenario 3, as well as the SEI values under scenario 1 are displayed in Figure 4.

Figure 4. Comparison between the SEI values before and after the measurement units of sub-indicators are changed
It can be seen from Fig. 4 that the SEI values have no obvious changes after the measurement units of sub-indicators are changed. A further examination on the two sets of SEI has found that their Pearson and Spearman correlation coefficients are respectively 0.97 and 0.99, which may be an indication of the robustness of the proposed approach in constructing CIs.

Conclusion

CIs have been widely accepted as a useful tool for assessing systems performance at macro level. However, the quality and reliability of a CI depends heavily on the underlying weighting and aggregation schemes. Past studies have shown that the WP method possesses some desirable properties. A major problem in applying the WP method to construct CIs is the subjectivity in determining the weights for sub-indicators. In this paper, we extend the WP method and propose an optimization-based CI construction approach. The proposed approach requires no prior knowledge of the weights for sub-indicators. The weights used can be generated by solving a series of multiplicative DEA type models that can be transformed into equivalent linear programs. When additional information on the relative importance of sub-indicators becomes available, it can be easily incorporated into the proposed models. Since the proposed approach uses two sets of weights that are most and least favourable for each entity, it may provide a more reasonable and encompassing CI.

To illustrate the use of the proposed approach, we apply it to study the SEI case given in Zhou, Ang, and Poh (2007a). It is found that the SEI values based on our multiplicative optimization approach are highly correlated with the SEI values based on the additive optimization approach by Zhou, Ang, and Poh (2007a). In addition, we find that the control parameter has minor impacts while the weight restrictions have impacts on the SEI values. The sensitivity of SEI values on the change of the measurement units of sub-indicators is also examined. It is found that the two sets of SEI values, before and after the measurement units of sub-indicators are changed, are highly correlated with each other, which may be an indication of the robustness of the proposed approach in constructing CIs. Nevertheless, since the unit invariance property is a desirable property, further research may be carried out to extend the proposed multiplicative optimization approach to let it possess the property.

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References


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