# Static Stability and Control

## Longitudinal Stability

$C_{m_\alpha} < 0$ or $C_{m_{C_L}} < 0$

for longitudinal stability.

What contribute to $C_{m_\alpha}$ of an airplane?

Wing, tail, fuselage.

## Wing contribution.

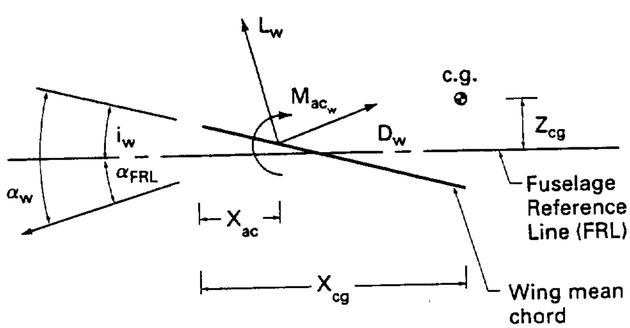



**FIGURE 2.7**
Wing contribution to the pitching moment.

Sum up the moments about the aircraft cg.

$\sum$ Moments $= M_{cg_w}$

Now,

$$M_{cg_w} = L_w \cos(\alpha_w - i_w)\,[x_{cg} - x_{ac}]$$
$$+ D_w \sin(\alpha_w - i_w)\,[x_{cg} - x_{ac}]$$
$$+ L_w \sin(\alpha_w - i_w)\, z_{cg}$$
$$- D_w \cos(\alpha_w - i_w)\, z_{cg} + M_{ac_w}$$

To get $C_{m_{\alpha_w}}$ we divide both sides by $\frac{1}{2}\rho V_c^2 S\bar{c}$

Then,

$$C_{m_{\alpha_w}} = C_{L_w}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right)\cos(\alpha_w - i_w)$$
$$+ C_{D_w}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right)\sin(\alpha_w - i_w)$$
$$+ C_{L_w}\frac{z_{cg}}{\bar{c}}\sin(\alpha_w - i_w) - C_{D_w}\frac{z_{cg}}{\bar{c}}\cos(\alpha_w - i_w)$$
$$+ C_{m_{ac_w}}$$

Comments:

1. Usually, $(\alpha_w - i_w)$ is small $\Rightarrow$
   $\sin(\alpha_w - i_w) \simeq \alpha_w - i_w$
   $\cos(\alpha_w - i_w) \simeq 1$
2. $C_{L_w} \gg C_{D_w}$ [ $C_L$ and $C_D$ mainly from wing. Example.
   Business jet $\quad C_L = 0.737,\ C_D = 0.095$ at Mach $= 0.2$
   $C_L = 0.4,\ C_D = 0.04$ at Mach $= 0.8$ ]

3. $z_{cg} \ll x_{cg} - x_{ac}$ usually.

4. Define $\bar{x}_{cg} = \dfrac{x_{cg}}{\bar{c}}$ ; $\bar{x}_{ac} = \dfrac{x_{ac}}{\bar{c}}$ ; ..

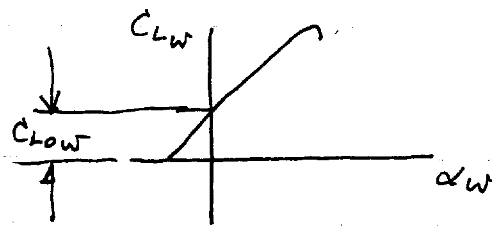
$\Rightarrow$ $\bar{x}_{ac_w} = \dfrac{x_{ac_w}}{\bar{c}}$

We can obtain a simple expression for $C_{m_{\alpha_w}}$ as

$$\boxed{C_{m_{\alpha_w}} = C_{m_{ac_w}} + C_{L_w}\left(\bar{x}_{cg} - \bar{x}_{ac_w}\right)} \qquad (1)$$

Now, $C_{L_w}$ can be written as

$$C_{L_w} = C_{L_{0_w}} + C_{L_{\alpha_w}} \alpha_w \qquad (2)$$




use (2) in (1) to get

$$C_{m_{\alpha_w}} = C_{m_{0_w}} + C_{m_{\alpha_w}} \alpha_w$$

WHERE

$$C_{m_{0_w}} = C_{m_{ac_w}} + C_{L_{0_w}}\left(\bar{x}_{cg} - \bar{x}_{ac_w}\right)$$

and

$$C_{m_{\alpha_w}} = C_{L_{\alpha_w}}\left(\bar{x}_{cg} - \bar{x}_{ac}\right)$$

## Tail Contribution

(Conventional Tail)

Comments:

1. We will again consider moments about the a/c cg.
2. The airflow loses some energy in travelling from the wing to the tail. Hence, velocity of airflow at the tail is not the same as that of the wing. We relate them through an efficiency factor.

$$\eta = \frac{\text{dyn. pressure at tail}}{\text{dyn. pressure at wing}} = \frac{\frac{1}{2}\rho V_t^2}{\frac{1}{2}\rho V_w^2} = \frac{\bar{q}_t}{\bar{q}_w}$$

$$\eta = 0.8 \sim 1.2$$

$>1$ if tail is located in the wake of a jet engine or propeller slip stream.

3. The tail angle of attack is less than that of the wing due to "downwash" at the wing

$$\alpha_t = \alpha_w - i_w - \epsilon + i_t$$

$\epsilon \equiv$ downwash angle

$i_t \equiv$ Tail incidence angle (positive measured upwards from fuselage reference line)

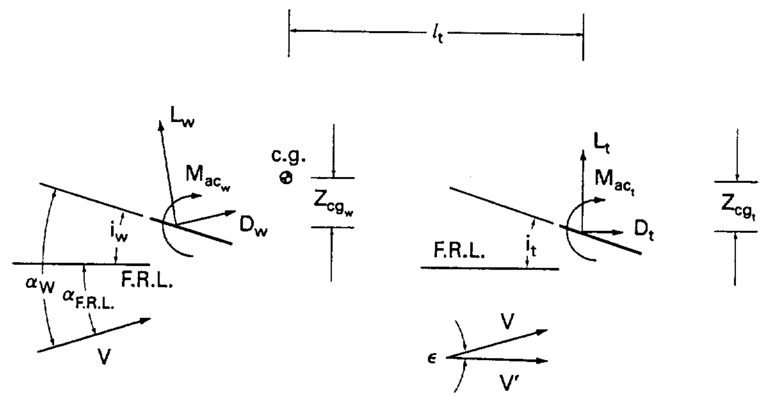



**FIGURE 2.9**
Aft tail contribution to the pitching moment.

$$M_t = -l_t\left[L_t \cos(\alpha_{FRL}-\epsilon) + D_t \sin(\alpha_{FRL}-\epsilon)\right]$$

$$-Z_{cg}\left[D_t \cos(\alpha_{FRL}-\epsilon) - L_t \sin(\alpha_{FRL}-\epsilon)\right]$$

$$+M_{ac_t}$$

Notes:

1) $(\alpha_{FRL}-\epsilon)$ is small $\Rightarrow \sin(\alpha_{FRL}-\epsilon) \approx \alpha_{FRL}-\epsilon$

$\cos(\alpha_{FRL}-\epsilon) \approx 1$

2) $C_{L_t} \gg C_{D_t}$

3) $Z_{cg} \ll l_t$ usually

4) a symmetric airfoil is chosen for tail usually.

$\Rightarrow C_{m_{ac_t}} = 0 \Rightarrow M_{ac_t} = 0$.

:

$$\Rightarrow \quad M_t = -l_t L_t$$

$$= -l_t C_{L_t} \frac{1}{2}\rho V_t^2 S_t$$

To get $C_{m_{\alpha_t}}$ get $C_{m_t}$ first.

$$C_{m_t} = \frac{M_t}{\frac{1}{2}\rho V_c^2 S\bar{c}} = -l_t C_{L_t} \frac{\frac{1}{2}\rho V_t^2 S_t}{\frac{1}{2}\rho V_c^2 S\bar{c}}$$

$$\doteq -C_{L_t}\eta V_H \quad \text{where} \qquad (1)$$

$$V_H \equiv \text{Tail volume ratio} = \frac{S_t l_t}{S\bar{c}}$$

Now, what is $C_{L_t}$?

$$C_{L_t} = C_{L_{\alpha_t}}\alpha_t = C_{L_{\alpha_t}}(\alpha_w - i_w - \epsilon + i_t)$$

$\epsilon$ is usually expressed as $\epsilon = \epsilon_0 + \epsilon_\alpha \alpha_w$
where $\epsilon_\alpha \equiv \frac{\partial\epsilon}{\partial\alpha}$

$\epsilon_0 \equiv$ downwash angle at zero angle of attack

$$\Rightarrow \quad C_{L_t} = C_{L_{\alpha_t}}\left((1-\epsilon_\alpha)\alpha_w - i_w - \epsilon_0 + i_t\right) \qquad (2)$$

use (2) in (1) to get

$$\Rightarrow \quad C_{m_{cg_t}} = C_{m_{0_t}} + C_{m_{\alpha_t}}\alpha_w \quad \text{where}$$

$$C_{m_{0_t}} = \eta V_H C_{L_{\alpha_t}}(\epsilon_0 + i_w - i_t)$$

and

$$\boxed{C_{m_{\alpha_t}} = -\eta V_H C_{L_{\alpha_t}}(1-\epsilon_\alpha)}$$

Note: • usually $i_t < 0$
• $C_{m_{\alpha_t}}$ more negative if $\boxed{l_t}$ is increased
or $S_t$ is increased.
or $C_{L_{\alpha_t}}$ is increased.

## Stick-Fixed Neutral Point

Total pitching moment coefficient, $C_{m_{cg}}$ of the a/c

$$C_{m_{cg}} = C_{m_w} + C_{m_t} + C_{m_f}$$

$$= C_{m_0} + C_{m_\alpha} \alpha_w$$

where

$$\boxed{C_{m_0} = C_{m_{0_w}} + C_{m_{0_f}} + \eta C_{L_{\alpha_t}} V_H (\epsilon_0 + i_w - i_t)}$$

Has to be greater than zero for stability and flight.

$$\boxed{C_{m_\alpha} = C_{L_{\alpha_w}} (\bar{X}_{cg} - \bar{X}_{ac_w}) + C_{m_{\alpha_f}} - \eta V_H C_{L_{\alpha_t}} (1 - \epsilon_\alpha)}$$

Has to be less than zero for stability.

Note:

$C_{m_\alpha}$ depends on aerodynamic characteristics and geometric characteristics..

Question: How do we find the limit of how geometry affects?

Set $C_{m_\alpha} = 0$ to find how far could $\bar{X}_{cg}$ go!

• $C_{m_\alpha} = 0$ at a location of cg called the neutral point. or airplane aerodynamic center.

That is

$$C_{m_\alpha} = 0 \Big|_{\bar{X}_{cg} = \bar{X}_{NP}}$$

$$\Rightarrow \quad \boxed{\bar{X}_{NP} = \bar{X}_{ac_w} - \frac{C_{m_{\alpha_f}}}{C_{L_{\alpha_w}}} + \eta V_H \frac{C_{L_{\alpha_t}}}{C_{L_{\alpha_w}}}(1 - \epsilon_\alpha)}$$

Note that now $C_{m_{cg}}$ can be written as

$$C_{m_{cg}} = C_{m_0} + \cancel{C_m} \, C_{L_{\alpha_w}} (\bar{X}_{cg} - \bar{X}_{NP}) \alpha_w$$

# Fuselage Contribution

## Multhopp's Method

$\boxed{C_{m_{0f}}}$

$$C_{m_{0_f}} = \frac{k_2 - k_1}{36.5S\bar{c}} \int_0^{l_f} w_f^2(\alpha_{0_w} + i_f)\, dx \tag{2.29}$$

which can be approximated as

$$C_{m_{0_f}} = \frac{k_2 - k_1}{36.5S\bar{c}} \sum_{x=0}^{x=l_f} w_f^2(\alpha_{0_w} + i_f)\, \Delta x \tag{2.30}$$

where $k_2 - k_1$ = the correction factor for the body fineness ratio
$S$ = the wing reference area
$\bar{c}$ = the wing mean aerodynamic chord
$w_f$ = the average width of the fuselage sections
$\alpha_{0_w}$ = the wing zero-lift angle relative to the fuselage reference line
$i_f$ = the incidence of the fuselage camber line relative to the fuselage reference line at the center of each fuselage increment. The incidence angle is defined as negative for nose droop and aft upsweep.
$\Delta x$ = the length of the fuselage increments

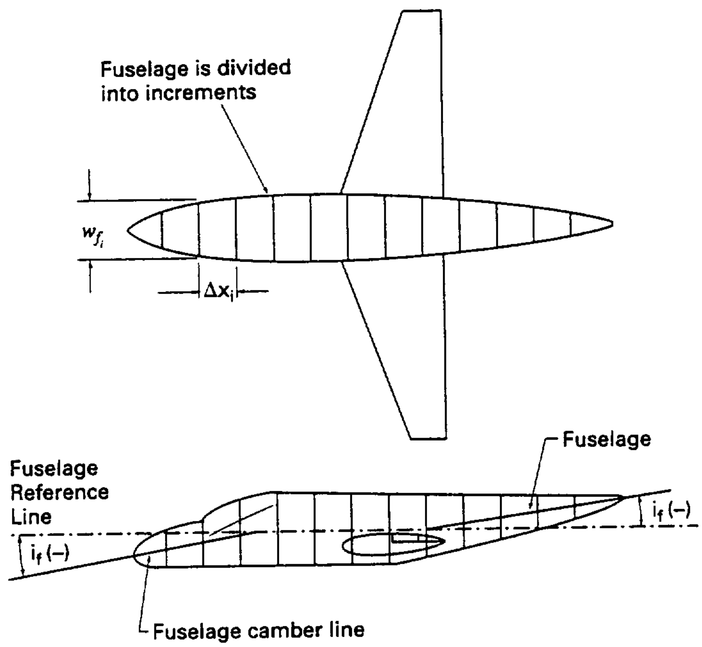



**FIGURE 2.11**
Procedure for calculating $C_{m_0}$ due to the fuselage.

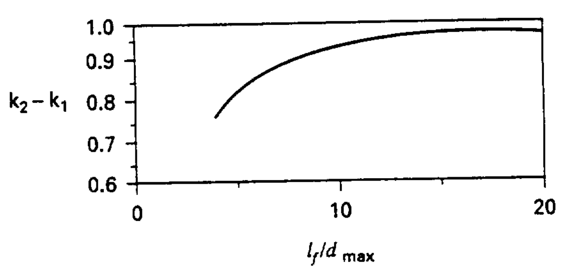



**FIGURE 2.12**
$k_2 - k_1$ versus $l_f/d$.

where $S$ = the wing reference area and $\bar{c}$ = the wing mean aerodynamic chord.

The fuselage again can be divided into segments and the local angle of attack of each section, which is composed of the geometric angle of attack of the section plus the local induced angle due to the wing upwash or downwash for each segment, can be estimated. The change in local flow angle with angle of attack, $\partial\varepsilon_u/\partial\alpha$, varies along the fuselage and can be estimated from Figure 2.13. For locations ahead of the wing, the upwash field creates large local angles of attack; therefore, $\partial\varepsilon_u/\partial\alpha > 1$. On the other hand, a station behind the wing is in the downwash region of the wing vortex system and the local angle of attack is reduced. For the region behind the wing, $\partial\varepsilon_u/\partial\alpha$ is assumed to vary linearly from 0 to $(1 - \partial\varepsilon/\partial\alpha)$ at the tail. The region between the wing's leading edge and trailing edge is assumed

$C_{m_{\alpha_f}}$

$$C_{m_{\alpha_f}} = \frac{1}{36.5S\bar{c}} \int_0^{l_f} w_f^2 \frac{\partial \varepsilon_u}{\partial \alpha} dx \qquad (\text{deg}^{-1})$$

which can be approximated by

$$C_{m_{\alpha_f}} = \frac{1}{36.5S\bar{c}} \sum_{x=0}^{x=l_f} w_f^2 \frac{\partial \varepsilon_u}{\partial \alpha} \Delta x$$

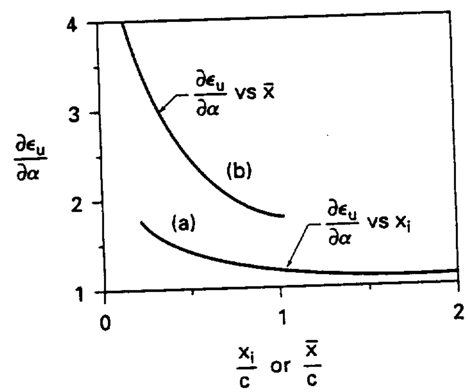



**FIGURE 2.13**
Variation of local flow angle along the fuselage.

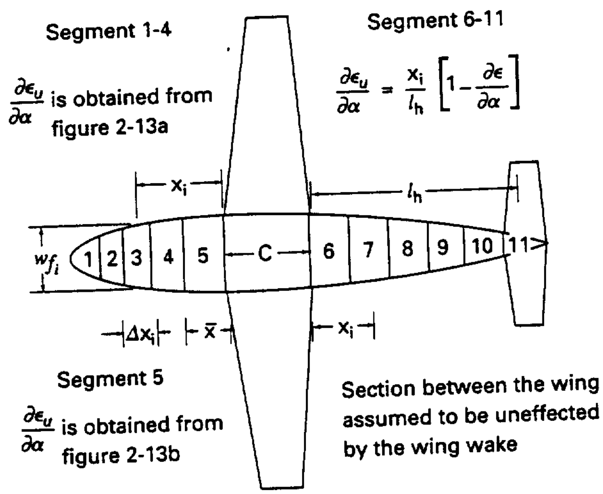



**FIGURE 2.14**
Procedure for calculating $C_{m_\alpha}$ due to the fuselage.

to be unaffected by the wing's flow field, $\partial \varepsilon_u / \partial \alpha = 0$. Figure 2.14 is a sketch showing the application of Equation (2.32).

### Stick Fixed Neutral Point

The total pitching moment for the airplane can now be obtained by summing the wing, fuselage, and tail contributions:

$$C_{m_{cg}} = C_{m_0} + C_{m_\alpha}\alpha \tag{2.33}$$

where
$$C_{m_0} = C_{m_{0_w}} + C_{m_{0_f}} + \eta V_H C_{L_{\alpha_t}}(\varepsilon_0 + i_w - i_t) \tag{2.34}$$

$$C_{m_\alpha} = C_{L_{\alpha_w}}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) + C_{m_{\alpha_f}} - \eta V_H C_{L_{\alpha_t}}\left(1 - \frac{d\varepsilon}{d\alpha}\right) \tag{2.35}$$

Notice that the expression for $C_{m_\alpha}$ depends upon the center of gravity position as well as the aerodynamic characteristics of the airplane. The center of gravity of an airplane varies during the course of its operation; therefore, it is important to know if there are any limits to the center of gravity travel. To ensure that the airplane possesses static longitudinal stability, we would like to know at what point $C_{m_\alpha} = 0$. Setting $C_{m_\alpha}$ equal to 0 and solving for the center of gravity position yields

$$\frac{x_{NP}}{\bar{c}} = \frac{x_{ac}}{\bar{c}} - \frac{C_{m_{\alpha_f}}}{C_{L_{\alpha_w}}} + \eta V_H \frac{C_{L_{\alpha_t}}}{C_{L_{\alpha_w}}}\left(1 - \frac{d\varepsilon}{d\alpha}\right) \tag{2.36}$$

In obtaining equation 2.36, we have ignored the influence of center of gravity movement on $V_H$. We call this location the stick fixed neutral point. If the airplane's

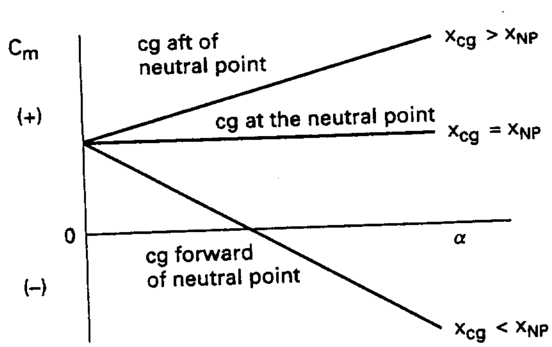



**FIGURE 2.15**
The influence of center of gravity position on longitudinal static stability.