TWO - BODY PROBLEM

$$- \vec{r}_1 \} \qquad (1)$$

$$\vec{r}_1 - \vec{r}_2 \} \qquad (2)$$

$$+m_2)\vec{r}_G \} = 0 \qquad (3)$$

$$)t$$

$$\mu = G(m_1 + m_2)$$

$$= 0 \qquad (4)$$

$$m_1 \frac{d^2\vec{r}_1}{dt^2} = \frac{Gm_1m_2}{|\vec{r}_1 - \vec{r}_2|^3} \{ \vec{r}_2$$

$$m_2 \frac{d^2\vec{r}_2}{dt^2} = \frac{Gm_1m_2}{|\vec{r}_1 - \vec{r}_2|^3} \{$$

$$\frac{d^2}{dt^2} \{ m_1\vec{r}_1 + m_2\vec{r}_2 \} = \frac{d^2}{dt^2} \{ (m$$

$$\vec{r}_G = \vec{r}_{G0} + (\vec{V}_{G0}$$

$$\vec{r} = \vec{r}_1 - \vec{r}_2$$

$$r = |\vec{r}|$$

$$\frac{d^2\vec{r}}{dt^2} + \frac{\mu}{r^3}\vec{r} =$$

$$\frac{d\vec{v}}{dt} = \frac{-\mu}{r^3}\vec{r}$$

$$\vec{r} \times \frac{d\vec{v}}{dt} = \frac{d}{dt}\left\{\vec{r} \times \vec{v}\right\} - \overset{0}{\vec{v} \times \vec{v}} = \vec{0}$$

$$\boxed{\vec{r} \times \vec{v} = \vec{h}\ (\text{const}) = h\,\hat{i}_h} \qquad (5)$$

$\vec{r}$ and $\vec{v}$ lie in the plane normal to $\hat{i}_h$.

$\vec{h}$ ... "angular momentum per unit mass"

$$\frac{d\vec{v}}{dt} \times \vec{h} = \frac{d}{dt}\left\{\vec{v} \times \vec{h}\right\} = -\frac{\mu}{r^3}\,\vec{r} \times \vec{h}$$

$$\vec{r} \times \vec{h} = \vec{r} \times (\vec{r} \times \vec{v}) = (\vec{r} \cdot \vec{v})\,\vec{r} - (\vec{r} \cdot \vec{r})\,\vec{v}$$

$$\frac{1}{r^3}(\vec{r} \times \vec{h}) = \frac{1}{r^3}(\vec{r} \cdot \vec{v})\,\vec{r} - \frac{1}{r}\,\vec{v}$$

$$2(\vec{r} \cdot \vec{v}) = \frac{d}{dt}(\vec{r} \cdot \vec{r}) = \frac{d}{dt}\left\{r^2\right\} = 2r\dot{r}$$

$$\frac{1}{r^3}(\vec{r} \times \vec{h}) = \frac{\dot{r}}{r^2}\,\vec{r} - \frac{1}{r}\,\vec{v} = -\frac{d}{dt}\left\{\frac{1}{r}\,\vec{r}\right\}$$

$$\frac{d}{dt}\left\{ \vec{v}\times\vec{h} - \frac{\mu}{r}\vec{r} \right\} = 0$$

$$\boxed{\vec{v}\times\vec{h} - \frac{\mu}{r}\vec{r} = \mu\vec{e}} \qquad (6)$$

$\vec{e}$ .... "ECCENTRICITY VECTOR"

$$\vec{e} = e\,\hat{\lambda}_e$$

$$\hat{\lambda}_p = \hat{\lambda}_h \times \hat{\lambda}_e$$

$$\vec{e} = \frac{1}{\mu}(\vec{v}\times\vec{h}) - \hat{\lambda}_r$$

$$e^2 = \frac{1}{\mu^2}(\vec{v}\times\vec{h})\cdot(\vec{v}\times\vec{h}) - \frac{2}{\mu r}\vec{r}\cdot(\vec{v}\times\vec{h}) + 1$$

$$(\vec{v}\times\vec{h})\cdot(\vec{v}\times\vec{h}) = |\vec{v}|^2|\vec{h}|^2 - \cancelto{0}{(\vec{v}\cdot\vec{h})^2} = v^2h^2$$

$$\vec{r}\times(\vec{v}\times\vec{h}) = \vec{h}\cdot(\vec{r}\times\vec{v}) = h^2$$

$$\boxed{1 - e^2 = \frac{h^2}{\mu}\left(\frac{2}{r} - \frac{v^2}{\mu}\right)} \qquad (7)$$

Parameter:

$$p = \frac{h^2}{\mu}$$

$$\frac{2}{r} - \frac{V^2}{\mu} = \text{const} = \frac{1}{a}$$

$$\mathcal{E} = \frac{V^2}{2} - \frac{\mu}{r} = \frac{-\mu}{2a}$$

$\therefore$ Energy Integral

$$\boxed{p = a(1-e^2)} \qquad (8)$$

a has physical dimensions of length

$a > 0$ if $e < 1$

$a < 0$ if $e > 1$

$|a| \to \infty$ as $e \to 1$

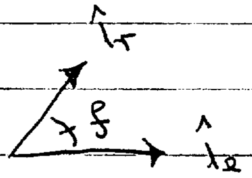
$$\mu \vec{h}\times\vec{e} = \vec{h}\times(\vec{v}\times\vec{h}) - \frac{\mu}{r}\vec{h}\times\vec{r}$$

$$\vec{h}\times(\vec{v}\times\vec{h}) = (\vec{h}\cdot\vec{h})\vec{v} - \overbrace{(\vec{h}\cdot\vec{v})}^{0}\vec{h}$$

$$= h^2\vec{v}$$

$$\boxed{\vec{v} = \frac{\mu}{h^2}\vec{h}\times\left\{\vec{e}+\frac{1}{r}\vec{r}\right\}} \qquad (9)$$

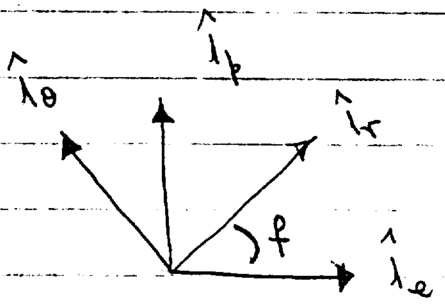
$$\vec{e}\cdot\vec{r} = \frac{1}{\mu}\vec{r}\cdot(\vec{v}\times\vec{h}) - \frac{1}{r}r^2 = \frac{h^2}{\mu} - r = p - r$$



f .... "true anomaly"

$$r + er\cos f = p$$

$$\boxed{r = \frac{p}{[1+e\cos f]} = \frac{a(1-e^2)}{[1+e\cos f]}} \qquad (10)$$



$$\hat{\lambda}_r = (\cos f)\hat{\lambda}_e + (\sin f)\hat{\lambda}_p \quad \Big| \quad \hat{\lambda}_e = (\cos f)\hat{\lambda}_r - (\sin f)\hat{\lambda}_\theta$$

$$\hat{\lambda}_\theta = -(\sin f)\hat{\lambda}_e + (\cos f)\hat{\lambda}_p \quad \Big| \quad \hat{\lambda}_p = (\sin f)\hat{\lambda}_r + (\cos f)\hat{\lambda}_\theta$$

$$\frac{h\vec{v}}{\mu} = \hat{\lambda}_h \times (e\hat{\lambda}_e + \hat{\lambda}_r)$$

$$\frac{h\vec{v}}{\mu} = e\hat{\lambda}_p + \hat{\lambda}_\theta = -(\sin f)\hat{\lambda}_e + (e + \cos f)\hat{\lambda}_p$$

$$= (e\sin f)\hat{\lambda}_r + (1 + e\cos f)\hat{\lambda}_\theta$$

$$\boxed{\frac{h\vec{v}}{\mu} = -(\sin f)\hat{\lambda}_e + (e + \cos f)\hat{\lambda}_p} \qquad (11)$$

$$\boxed{\frac{h\vec{v}}{\mu} = (e\sin f)\hat{\lambda}_r + (1 + e\cos f)\hat{\lambda}_\theta} \qquad (12)$$

LAGRANGE COEFFICIENTS

$$\vec{r} = (r\cos f)\hat{i}_e + (r\sin f)\hat{i}_p$$

$$\vec{V} = \left(-\frac{\mu}{h}\sin f\right)\hat{i}_e + \frac{\mu}{h}(e+\cos f)\hat{i}_p$$

$$Q = \begin{bmatrix} r\cos f & r\sin f \\ -\frac{\mu}{h}\sin f & \frac{\mu}{h}(e+\cos f) \end{bmatrix}$$

$$\begin{matrix} o \\ s f_o) \end{matrix}\Bigg]$$

$$e\cos f\Big\} = \frac{\mu p}{h} = h = \det Q_o$$

$$Q_o = \begin{bmatrix} r_o\cos f_o & r_o\sin f \\ -\frac{\mu}{h}\sin f_o & \frac{\mu}{h}(e+co \end{matrix}$$

$$\det Q = \frac{\mu r}{h}\Big\{\cos^2 f + \sin^2 f + $$

$$\vec{r} = F\,\vec{r}_o + G\,\vec{V}_o$$

$$\vec{V} = F_t\,\vec{r}_o + G_t\,\vec{V}_o$$

$$G \Big]$$

$$G_t \Big]$$

$$Q\, Q_0^{-1}$$

$$\begin{matrix} r\sin f \\ \frac{\mu}{h}(e+\cos f) \end{matrix} \Bigg] \begin{bmatrix} \frac{\mu}{h^2}(e+\cos f_0) & -\frac{r_0}{h}\sin f_0 \\ \frac{\mu}{h^2}(\sin f_0) & \frac{r_0}{h}\cos f_0 \end{bmatrix}$$

$$\sigma = \frac{\vec{r}\cdot\vec{v}}{\sqrt{\mu}} \qquad \theta = f - f_0$$

$$1 \qquad e\sin f = \frac{\sqrt{p}\,\sigma}{r}$$

$$1 \qquad e\sin f_0 = \frac{\sqrt{p}\,\sigma_0}{r_0}$$

$f\cos f_0 + \sin f\sin f_0 +$ [illegible] $\cos f \}$

$$\cos\theta + \frac{p}{r} - 1 \}$$

$$1 + \frac{r}{p}(\cos\theta - 1) = 1 - \frac{r}{p}(1-\cos\theta)$$

$$A = \begin{bmatrix} F \\ F_t \end{bmatrix}$$

$$A = \Theta$$

$$\begin{bmatrix} r\cos f \\ -\frac{\mu}{h}\sin f \end{bmatrix}$$

$$\Theta$$

$$e\cos f = \frac{p}{r} -$$

$$e\cos f_0 = \frac{p}{r_0} -$$

$$F = \frac{\mu r}{h^2}\{\cos$$

$$= \frac{r}{p}\{$$

$$F =$$

$$F_t = \frac{\mu^2}{h^3}\left\{ e\sin f_0 - e\left(\sin\theta\cos f_0 + \cos\theta\sin f_0\right) - \sin\theta \right\}$$

$$= \frac{\sqrt{\mu}}{p\sqrt{p}}\left\{ e\sin f_0\left[1-\cos\theta\right] \text{ [illegible] } \sin\theta\left[1 + e\cos f\right] \text{ [illegible] } \right\}$$

$$= \frac{\sqrt{\mu}}{p\sqrt{p}}\left\{ \frac{\sqrt{p}\,\sigma_0}{r_0}\left[1-\cos\theta\right] - \frac{p}{r_0}\sin\theta \right\}$$

$$F_t = \frac{\sqrt{\mu}}{r_0 p}\left\{ \sigma_0(1-\cos\theta) - \sqrt{p}\sin\theta \right\}$$

$$G = \frac{r r_0}{h}\left(\sin f\cos f_0 - \cos f\sin f_0\right) = \frac{r r_0}{\sqrt{\mu p}}\sin\theta$$

$$G_t = \frac{\mu r_0}{h^2}\left\{ \sin f\sin f_0 + \cos f\cos f_0 + e\cos f_0 \right\}$$

$$= \frac{r_0}{p}\left\{ \cos\theta + \frac{p}{r} - 1 \right\}$$

$$= 1 - \frac{r_0}{p}(1-\cos\theta)$$

$$e \cos f = e \cos\theta \cos f_0 - e \sin\theta \sin f_0$$

$$= \left(\frac{p}{r_0} - 1 \right) \cos\theta - \frac{\sqrt{p}\, \sigma_0}{r_0} \sin\theta$$

$$1 + e \cos f = \frac{1}{r_0} \left\{ r_0 + (p - r_0) \cos\theta - \sqrt{p}\ \sigma_0 \sin\theta \right\}$$

$$r = \frac{p r_0}{\left\{ r_0 + [p - r_0] \cos\theta - \sqrt{p}\, \sigma_0 \sin\theta \right\}}$$

$$F = 1 - \frac{r}{p}(1 - \cos\theta)$$

$$G = \frac{r r_0}{\sqrt{\mu p}} \sin\theta$$

$$F_t = \frac{\sqrt{\mu}}{r_0 p} \left\{ \sigma_0 [1 - \cos\theta] - \sqrt{p} \sin\theta \right\}$$

$$G_t = 1 - \frac{r_0}{p} (1 - \cos\theta)$$

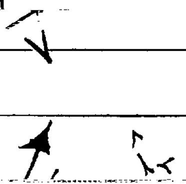
TIME IDENTITY

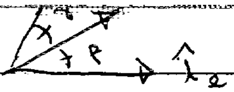
$$r^2 \dot{f} = \frac{p^2 \dot{f}}{[1+e\cos f]^2} = \sqrt{p\mu}$$

$$\frac{\dot{f}}{[1+e\cos f]^2} = \sqrt{\frac{\mu}{p^3}}$$

$$\int_0^{f(t)} \frac{df}{(1+e\cos f)^2} = \sqrt{\frac{\mu}{p^3}}\left(t-t_0\right)$$

FLIGHT-PATH ANGLE





$$\vec{V} = \frac{\mu}{h} e \sin f \,\hat{i}_r + \frac{\mu}{h}(1+e\cos f)\,\hat{i}_\theta$$

$$\cos\gamma = \frac{\mu}{hv} e \sin f \qquad \sin\gamma = \frac{\mu}{hv}(1+e\cos f)$$

$$\sin\gamma = \frac{p\mu}{hrv} = \frac{\sqrt{p\mu}}{vr}$$

$$\sigma = \frac{\vec{r}\cdot\vec{v}}{\sqrt{\mu}} = \frac{vr\cos\gamma}{\sqrt{\mu}} = \frac{\sqrt{\mu}\, re \sin f}{h} = \frac{er\sin f}{\sqrt{p}}$$

$$= \frac{\cos\gamma}{\sqrt{\mu}}\,\frac{\sqrt{p\mu}}{\sin\gamma} = \sqrt{p}\cot\gamma$$

ELLIPTIC ORBIT

$a > 0$ $\qquad$ $0 \le e < 1$

$$p = a(1 - e^2)$$

$$\frac{V^2}{2} - \frac{\mu}{r} = \frac{-\mu}{2a}$$

$$\frac{)}{s f}$$

$$-e)$$

$$1 + e)$$

$$\frac{}{-e)} - \frac{1}{a}\Big\} = \frac{\mu}{a}\Big\{\frac{1+e}{1-e}\Big\}$$

$$\frac{}{+e)} - \frac{1}{a}\Big\} = \frac{\mu}{a}\Big\{\frac{1-e}{1+e}\Big\}$$

$$r = \frac{a(1 - e^2}{1 + e\cos}$$

$$r_p = r_{min} = a(1$$

$$r_a = r_{max} = a($$

$$V^2 = \mu\Big\{\frac{2}{r} - \frac{1}{a}\Big\}$$

$$V_p^2 = V_{max}^2 = \mu\Big\{\frac{2}{a(1}$$

$$V_a^2 = V_{min}^2 = \mu\Big\{\frac{2}{a(1}$$

$$b = a\sqrt{1-e^2}$$

$$x = ae + r\cos f = a\cos E$$

$$y = r\sin f = a\sqrt{1-e^2}\,\sin E$$

$$a\cos E = a\left\{ e + \frac{r}{a}\cos f \right\}$$

$$\cos E = e + \frac{(1-e^2)\cos f}{1+e\cos f} = \frac{e+\cos f}{1+e\cos f}$$

$$\sin E = \frac{\sqrt{1-e^2}\,\sin f}{1+e\cos f}$$

$$(1-e\cos E)\cos f = \cos E - e$$

$$\cos f = \frac{\cos E - e}{1-e\cos E}$$

$$1+e\cos f = \frac{1-e\cos E + e\cos E - e^2}{1-e\cos E} =$$

$$\frac{(1-e^2)}{\left[1-e\cos E\right]}$$

$$r = \frac{a(1-e^2)}{[1 + e\cos f]} = a(1 - e\cos E)$$

$$\sin f = \frac{1}{\sqrt{1-e^2}} \frac{p}{r} \sin E = \frac{\sqrt{1-e^2}\sin E}{1 - e\cos E}$$

$$\cos f \frac{df}{dE} = \frac{[\cos E - e]}{[1 - e\cos E]} \frac{df}{dE} = \frac{\sqrt{1-e^2}\cos E}{(1 - e\cos E)} \frac{- e\sqrt{1-e^2}\sin^2 E}{(1 - e\cos E)^2}$$

$$= \frac{\sqrt{1-e^2}}{[1 - e\cos E]^2} \left\{ \cos E - e(\cos^2 E + \sin^2 E) \right\}$$

$$= \frac{\sqrt{1-e^2}\,[\cos E - e]}{[1 - e\cos E]^2}$$

$$\frac{df}{dE} = \frac{\sqrt{1-e^2}}{[1 - e\cos E]} = \frac{b}{r} \quad -$$

$$\dot{f} = \frac{b}{r}\,\dot{E}$$

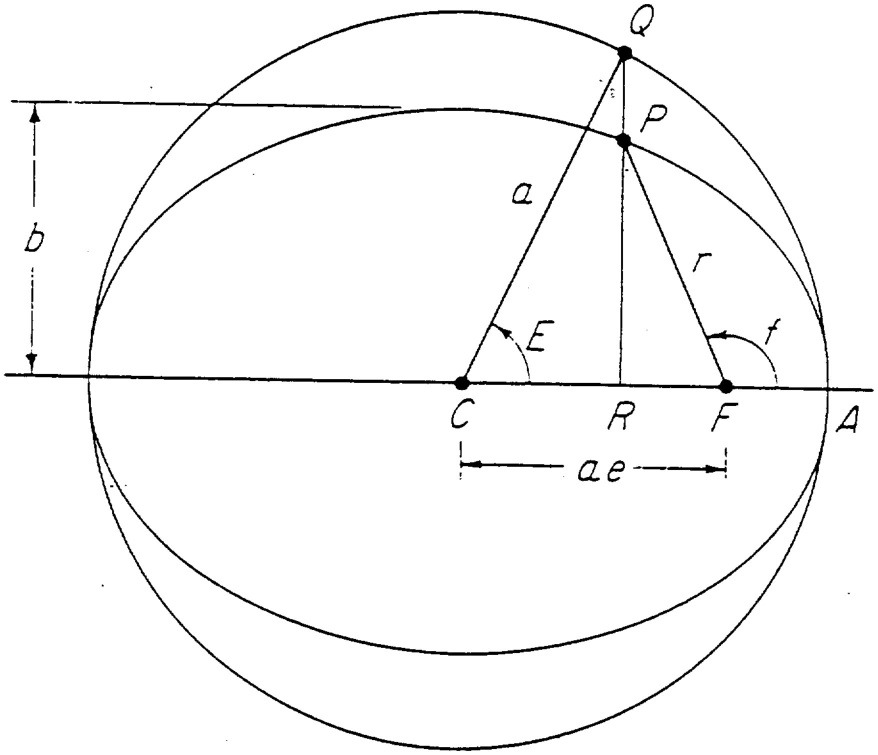
$$r^2\dot{f} = br\,\dot{E} = a^2\left(1 - e\cos E\right)\sqrt{1-e^2}\,\dot{E} = \sqrt{\mu a(1-e^2)}$$

$$\left[1 - e\cos E\right]\dot{E} = \sqrt{\frac{\mu}{a^3}}$$

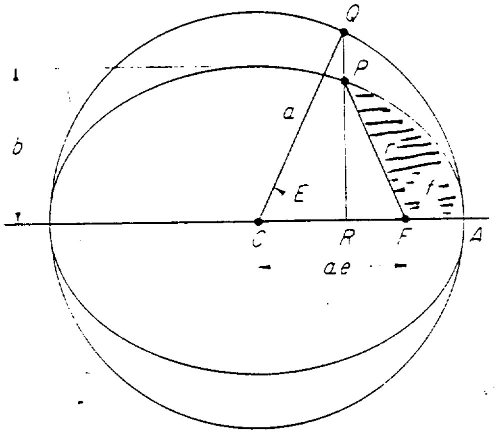
$$\frac{d}{dt}\left\{E - e\sin E\right\} = \sqrt{\frac{\mu}{a^3}}$$

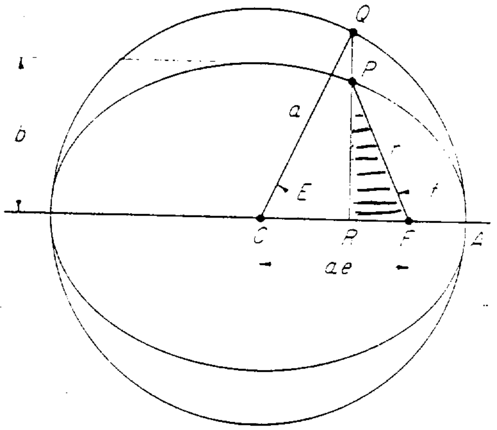
KEPLER TIME EQUATION

$$\boxed{E - e\sin E = \sqrt{\frac{\mu}{a^3}}\left\{t - t_p\right\}}$$


Q
P
a
r
b
E
f
C
R
F
A
ae

KEPLER TIME EQUATION

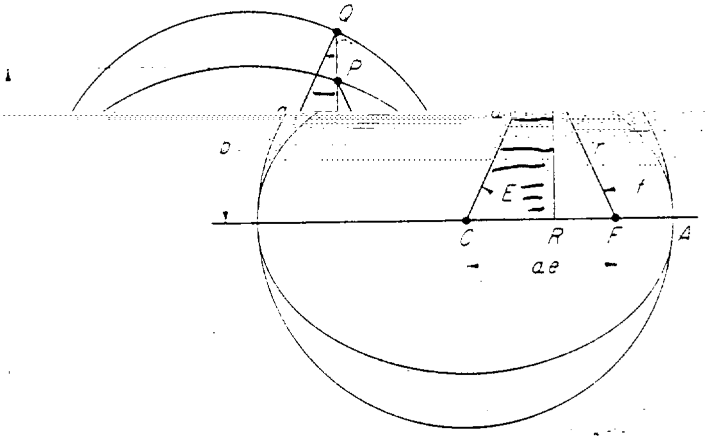


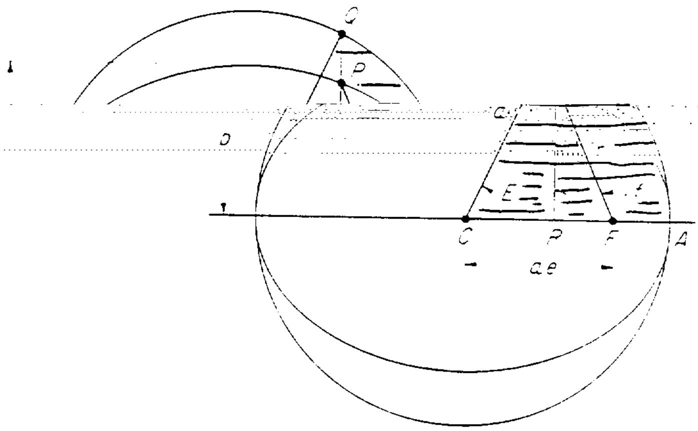


$$\text{AREA PFA} = \frac{h}{2}[t - t_p]$$

$$\text{AREA PRF} = \frac{1}{2}[ae - a\cos E]\left(\frac{b}{a}\right)[a\sin E]$$

$$\text{AREA PRF} = \left(\frac{ab}{2}\right)[e - \cos E]\sin E$$

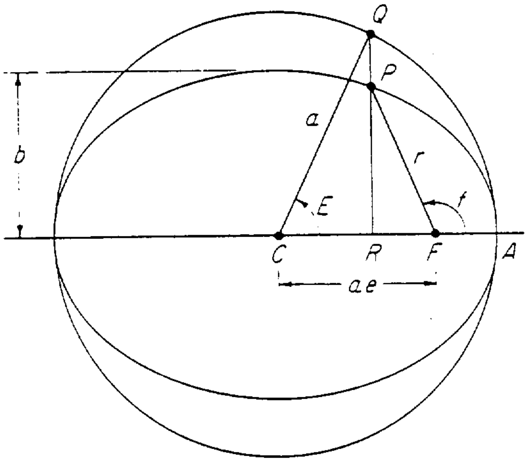




$$\text{AREA QCR} = \frac{a^2}{2}[\cos E\sin E]$$

$$\text{AREA QCA} = \frac{a^2}{2}E$$

$$\text{AREA PRA} = \int_{x_R}^{a[1-e]} \{b \sin E(x)\} dx = \left(\frac{b}{a}\right) \int_{x_R}^{a[1-e]} \{a \sin E(x)\} dx = \left(\frac{b}{a}\right) \text{AREA QRA}$$



$$\text{AREA QRA} = \text{AREA QCA} - \text{AREA QCR} = \frac{a^2}{2}[E - \cos E \sin E]$$

$$\text{AREA PRA} = \text{AREA PRF} + \text{AREA PFA} = \left(\frac{ab}{2}\right)[e - \cos E] \sin E + \left(ab\sqrt{\frac{\mu}{a^3}}\right)\left[t - t_p\right.$$

$$E - \cos E \sin E = [e - \cos E] \sin E + \left(\sqrt{\frac{\mu}{a^3}}\right)[t - t_p]$$

$$E - e \sin E = \left(\sqrt{\frac{\mu}{a^3}}\right)[t - t_p]$$

$$r = a(1 - e\cos E)$$

$$e\cos E = 1 - \frac{r}{a} \qquad e\cos E_0 = 1 - \frac{r_0}{a}$$

$$\sin f = \frac{a}{r}\sqrt{1-e^2}\,\sin E \qquad \cos f = \frac{a}{r}(\cos E - e)$$

$$\sin f_0 = \frac{a}{r_0}\sqrt{1-e^2}\,\sin E_0 \qquad \cos f_0 = \frac{a}{r_0}(\cos E_0 - e)$$

$$e\sin E = \frac{r}{a\sqrt{1-e^2}}(e\sin f) = \frac{r}{a\sqrt{1-e^2}}\,\frac{\sqrt{a(1-e^2)}\,\sigma}{r} = \frac{\sigma}{\sqrt{a}}$$

$$\cos\theta = \cos f\cos f_0 + \sin f\sin f_0$$

$$= \frac{a^2}{rr_0}\left\{(\cos E_0 - e)(\cos E - e) + (1-e^2)\sin E\sin E_0\right\}$$

$$= \frac{a^2}{rr_0}\Big\{(1-e^2)\left[\cos E\cos E_0 + \sin E\sin E_0\right] + e^2\cos E\cos E_0 + e^2$$

$$- e\cos E - e\cos E_0\Big\}$$

$$= \frac{a^2}{rr_0}\left\{(1-e^2)\cos(E-E_0) + e^2 + \left(1-\frac{r}{a}\right)\left(1-\frac{r_0}{a}\right) - \left(1-\frac{r}{a}\right) - \left(1-\frac{r_0}{a}\right)\right\}$$

$$= \frac{a^2}{rr_0}\left\{(1-e^2)\cos(E-E_0) + e^2 - 1 \cancel{- \frac{r}{a}} \cancel{+ \frac{r}{a}} \cancel{- \frac{r_0}{a}} + \cancel{\frac{r_0}{a}} + \frac{rr_0}{a^2}\right\}$$

$$\cos\theta = 1 - \frac{a^2}{r r_0}(1-e^2)\left\{1 - \cos(E-E_0)\right\}$$

$$\sin\theta = \frac{a^2}{r r_0}\sqrt{1-e^2}\left\{\sin E\cos E_0 - e\sin E - \cos E\sin E_0 + e\sin E_0\right\}$$

$$\sin\theta = \frac{a^2}{r r_0}\sqrt{1-e^2}\,\sin(E-E_0) + \frac{a^2 e}{r r_0}\sqrt{1-e^2}\left\{\sin E_0 - \sin E\right\}$$

$$\sin E = \sin(E-E_0)\cos E_0 + \cos(E-E_0)\sin E_0$$

$$e\sin E_0 = \frac{\sigma_0}{\sqrt{a}} \qquad e\cos E_0 = 1 - \frac{r_0}{a}$$

$$\sin\theta = \frac{a^2\sqrt{1-e^2}}{r r_0}\sin(E-E_0) + \frac{a^2\sqrt{1-e^2}}{r r_0}\left\{\frac{\sigma_0}{\sqrt{a}} - \frac{\sigma_0}{\sqrt{a}}\cos(E-E_0) - \left(1-\frac{r_0}{a}\right)\sin(E-E_0)\right\}$$

$$\sin\theta = \frac{a\sqrt{1-e^2}}{r}\sin(E-E_0) + \frac{a\sqrt{p}\,\sigma_0}{r r_0}\left\{1-\cos(E-E_0)\right\}$$

LAGRANGE COEFFICIENTS

$$\cos\theta = 1 - \frac{a^2(1-e^2)}{r r_0}\left\{1 - \cos(E-E_0)\right\}$$

$$\sin\theta = \frac{a\sqrt{1-e^2}}{r}\sin(E-E_0) + \frac{a\sqrt{p}\,\sigma_0}{r r_0}\left\{1 - \cos(E-E_0)\right\}$$

$$1 - \cos\theta = \frac{ap}{r r_0}\left\{1 - \cos(E-E_0)\right\}$$

$$1 - \cos(E-E_0)\}$$

$$)\} + r_0\sqrt{\frac{a}{\mu}}\sin(E-E_0)$$

$$\sin(E-E_0)$$

$$\} = -\frac{\sqrt{\mu a}}{r r_0}\sin(E-E_0)$$

$$- \cos(E-E_0))$$

$$F = 1 - \frac{r}{p}(1-\cos\theta) = 1 - \frac{a}{r_0}\{$$

$$G = \frac{r r_0}{\sqrt{\mu p}}\sin\theta = \frac{a\sigma_0}{\sqrt{\mu}}\{1 - \cos(E-E$$

$$F_t = \frac{\sqrt{\mu}}{r_0 p}\left\{\sigma_0(1-\cos\theta) - \sqrt{p}\sin\theta\right\}$$

$$= \frac{\sqrt{\mu}}{r_0 p}\left\{\frac{ap\sigma_0}{r r_0}\left[1 - \cos(E-E_0)\right] - \frac{\sqrt{a}\,p}{r}\right.$$

$$- \frac{ap\sigma_0}{r r_0}\left[1 - \cos(E-E_0)\right]$$

$$G_t = 1 - \frac{r_0}{p}(1-\cos\theta) = 1 - \frac{a}{r}($$

HALF - ANGLE FORMULAS

When $f = 0$, $E = 0$

When $f = \frac{\pi}{2}$, $E = \arccos(e)$

when $f = \arccos(-e)$, $E = \frac{\pi}{2}$

when $f = \pi$, $E = \pi$

when $E = \frac{3\pi}{2}$, $\cos f = -e$ $\sin f = -\sqrt{1-e^2}$

when $f = \frac{3\pi}{2}$ $\cos E = e$ $\sin E = -\sqrt{1-e^2}$

Between $f = \frac{\pi}{2}$ and $E = \frac{\pi}{2}$, E and f lie in different quadrants

Between $E = \frac{3\pi}{2}$ and $f = \frac{3\pi}{2}$, E and f lie in different quadrants

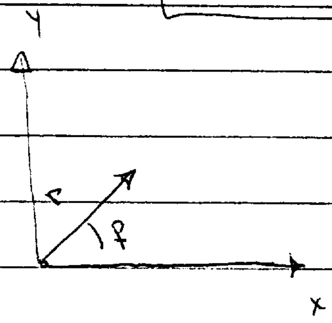
$$2\cos^2\frac{f}{2} = 1 + \frac{\cos E - e}{1 - e\cos E} = \frac{(1-e)(1+\cos E)}{1-e\cos E} = \frac{2a(1-e)}{r}\cos^2\frac{E}{2}$$

$$2\sin^2\frac{f}{2} = 1 - \left\{\frac{\cos E - e}{1-e\cos E}\right\} = \frac{1-e\cos E - \cos E + e}{1-e\cos E} = \frac{(1+e)(1-\cos E)}{1-e\cos E}$$

$$= \frac{2a(1+e)}{r}\sin^2\frac{E}{2}$$

$$\boxed{\tan\frac{f}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}}$$

HYPERBOLIC ORBIT



$e > 1$

$x = r\cos f$

$y = r\sin f$

$$\frac{(x+ea)^2}{a^2} + \frac{y^2}{a^2(1-e^2)} = 1$$

$a < 0 \qquad e > 1 \qquad -a = |a|$

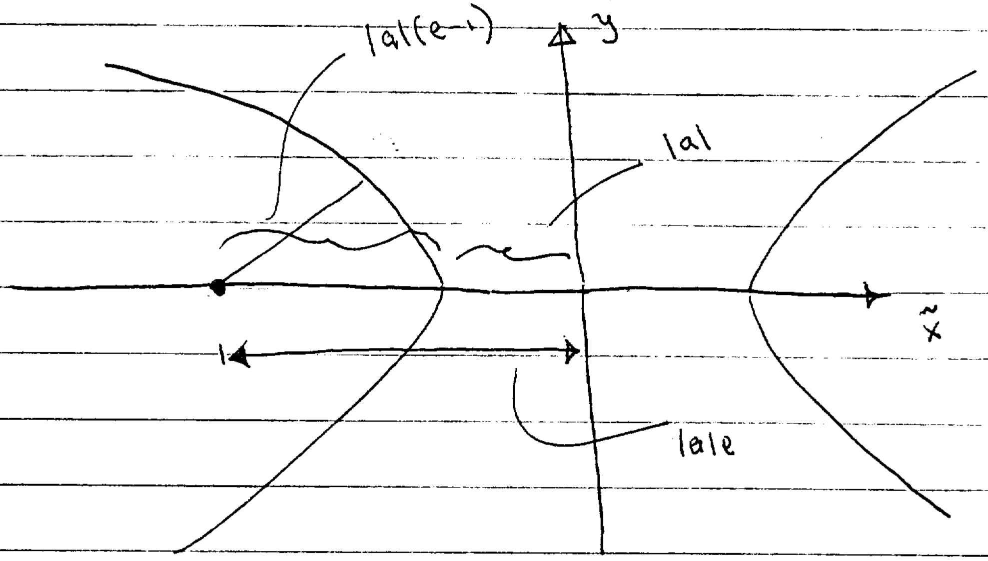
$b = |a|\sqrt{e^2-1}$

$$\frac{(x-e|a|)^2}{a^2} - \frac{y^2}{b^2} = 1$$

Hyperbola $\quad x = r\cos f = e|a| \pm a$ when $f = 0$

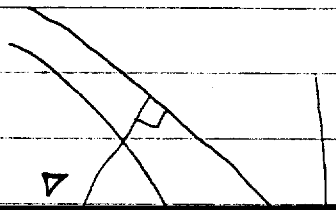
$r_p = e|a| - |a| = (-a)(e-1)$

$\tilde{x} = x + ea = x - e|a|$



$$\frac{\tilde{x}^2}{|a|^2} - \frac{y^2}{b^2} = 1$$

asymptotes $y = \pm \frac{b}{|a|} \tilde{x} = \pm \sqrt{e^2-1}\, \tilde{x}$



$\tan^2\phi = e^2-1$ $\cos\phi = \frac{1}{e}$

$\sin\phi = \frac{\sqrt{e^2-1}}{e}$

$$\frac{V^2}{2} - \frac{\mu}{r} = \frac{-\mu}{2a} = \frac{\mu}{2|a|}$$

$$\frac{V_\infty^2}{2} = \frac{\mu}{2|a|}$$

$$\boxed{V_\infty = \sqrt{\frac{\mu}{|a|}} = \sqrt{\frac{\mu}{(-a)}}}$$

CHECK : $h = V_\infty \Delta$

$$h = \sqrt{\mu|a|(e^2-1)} = \left(\sqrt{\frac{\mu}{|a|}}\right) |a| \sqrt{e^2-1} = V_\infty \Delta \checkmark$$

$$\frac{V_{max}^2}{2} - \frac{\mu}{|a|(e-1)} = \frac{\mu}{2|a|}$$

$$V_{max}^2 = \frac{\mu}{|a|(e-1)} \{2 + e - 1\} = \frac{\mu}{|a|} \frac{(e+1)}{(e-1)}$$

$$\boxed{V_{max} = \sqrt{\frac{\mu}{|a|} \frac{(e+1)}{(e-1)}}}$$

$$r\cos f = \frac{|a|(e^2-1)\cos f}{1+e\cos f}$$

$$r\cos f - e|a| = -|a|\cosh H$$

$$\frac{(e^2-1)\cos f - e(1+e\cos f)}{1+e\cos f} = -\cosh H$$

$$\cosh H = \frac{e+\cos f}{1+e\cos f}$$

$$(e\cosh H - 1)\cos f = e - \cosh H$$

$$\cos f = \frac{e-\cosh H}{e\cosh H - 1}$$

$$r\cos f = r\left\{\frac{e-\cosh H}{e\cosh H - 1}\right\} = |a|\{e - \cosh H\}$$

$$r = |a|\{e\cosh H - 1\} = a\{1 - e\cosh H\}$$

$$r\sin f = b\sinh H = |a|\sqrt{e^2-1}\,\sinh H$$

$$\sin f = \frac{\sqrt{e^2-1}\,\sinh H}{[e\cosh H - 1]}$$

HALF ANGLE FORMULAS

$$\cos f = \frac{e - \cosh H}{e\cosh H - 1} \qquad 1 + \cos f = \frac{(e-1)\left[\cosh H + 1\right]}{\left[e\cosh H - 1\right]}$$

$$1 - \cos f = \frac{(e+1)\left[\cosh H - 1\right]}{e\cosh H - 1}$$

$$\frac{2|a|}{r}(e-1)\cosh^2\frac{H}{2} \qquad 2\cos^2\frac{f}{2} = \frac{2(e-1)\cosh^2\frac{H}{2}}{e\cosh H - 1} =$$

$$\frac{2|a|}{r}(e+1)\sinh^2\frac{H}{2} \qquad 2\sin^2\frac{f}{2} = \frac{2(e+1)\sinh^2\frac{H}{2}}{e\cosh H - 1} =$$

$$\frac{H}{2} \qquad \tan\frac{f}{2} = \sqrt{\frac{e+1}{e-1}}\ \tanh$$

$$\vec{r} = (r\cos f)\,\hat{\lambda}_e + (r\sin f)\,\hat{\lambda}_p$$

$$\vec{v} = -\frac{\mu}{h}(\sin f)\,\hat{\lambda}_e + \frac{\mu}{h}(e+\cos f)\,\lambda_p$$

$$r\cos f = |a|\{e-\cosh H\} = a\{\cosh H - e\}$$

$$r\sin f = |a|\sqrt{e^2-1}\,\sinh H = \sqrt{(-ap)}\,\sinh H$$

$$e+\cos f = \frac{p}{r}\cosh H \;; \qquad \frac{\mu}{h}(e+\cos f) = \frac{\sqrt{\mu p}}{r}\cosh H$$

$$-\frac{\mu}{h}\sin f = -\sqrt{\frac{\mu}{p}}\left\{\frac{|a|\sqrt{e^2-1}}{r}\sinh H\right\} = -\frac{\sqrt{\mu|a|}}{r}\sinh H$$

$$\vec{r} = a\left(\cosh H - e\right)\hat{\lambda}_e + \sqrt{(-ap)}\,\sinh H\,\hat{\lambda}_p$$

$$\vec{v} = -\frac{\sqrt{-a\mu}}{r}\sinh H\,\hat{\lambda}_e + \frac{\sqrt{\mu p}}{r}\cosh H\,\hat{\lambda}_p$$

$$\sigma = \frac{\vec{r}\cdot\vec{v}}{\sqrt{\mu}} = \frac{r}{\sqrt{p}}\left\{e\sin f + \sin f\cos f - \sin f\cos f\right\}$$

$$\sigma = \frac{e\,r\sin f}{\sqrt{p}}$$

$$\cos f = \frac{e - \cosh H}{e\cosh H - 1} \qquad \cos f_0 = \frac{e - \cosh H_0}{e\cosh H_0 - 1}$$

$$\sin f_0 = \frac{\sqrt{e^2-1}\ \sinh H_0}{\left[e\cosh H_0 - 1\right]}$$

$$\sinh H = \frac{\sqrt{e^2-1}\ \sin f}{\left[1 + e\cos f\right]}$$

$$\frac{r}{|a|\sqrt{e^2-1}}\left\{\sqrt{|a|(e^2-1)}\ \frac{\sigma}{r}\right\}$$

$$= \frac{\sigma}{\sqrt{(-a)}}$$

$$1 - \frac{r}{a}$$

$$\sin f = \frac{\sqrt{e^2-1}\ \sinh H}{\left[e\cosh H - 1\right]}$$

$$\cosh H = \frac{e + \cos f}{\left[1 + e\cos f\right]}$$

$$e\sinh H = \frac{r}{|a|\sqrt{e^2-1}}\,(e\sin f) =$$

$$= \frac{\sigma}{\sqrt{|a|}}$$

$$e\cosh H = 1 + \frac{r}{|a|} =$$

$$e\sinh H_0 = \frac{\sigma_0}{\sqrt{(-a)}}$$

$$e\cosh H_0 = 1 - \frac{r_0}{a}$$

$$\cos\theta = \cos f \cos f_0 + \sin f \sin f_0$$

$$= \frac{a^2}{rr_0}\left\{(e-\cosh H)(e-\cosh H_0) + (e^2-1)\sinh H \sinh H_0\right\}$$

$$= \frac{a^2}{rr_0}\left\{(1-e^2)\left\{\cosh H \cosh H_0 - \sinh H \sinh H_0\right\} + e^2\cosh H\cosh H_0 + e^2 - e(\cosh H + \cosh H_0)\right\}$$

$$= \frac{a^2}{rr_0}\left\{(1-e^2)\cosh(H-H_0) + \left(1-\frac{r}{a}\right)\left(1-\frac{r_0}{a}\right) + e^2 - \left(1-\frac{r}{a}\right) - \left(1-\frac{r_0}{a}\right)\right\}$$

$$= \frac{a^2}{rr_0}\left\{(1-e^2)\cosh(H-H_0) + e^2 - 1 + \frac{rr_0}{a^2}\right\}$$

$$\cos\theta = 1 + \frac{a^2}{rr_0}(e^2-1)\left\{1-\cosh(H-H_0)\right\}$$

$$1 - \cos\theta = \frac{a^2}{rr_0}(e^2-1)\left\{\cosh(H-H_0) - 1\right\}$$

$$(e - \cosh H_0) - (e - \cosh H)\sinh H_0 \Big\}$$

$$h\, H \cosh H_0 - \cosh H \sinh H_0 \}$$

$$e \sinh H - e \sinh H_0 \Big\}$$

$$H_0) \cosh H_0 + e \cosh(H - H_0) \sinh H_0$$

$$H_0)\left(1 - \frac{r_0}{a}\right) + \cosh(H - H_0)\left(\frac{\sigma_0}{\sqrt{(-a)}}\right)$$

$$\mathrm{inh}(H - H_0)\left[1 - \frac{r_0}{a} - 1\right] + \frac{\sigma_0}{\sqrt{(-a)}}\left\{\cosh(H - H_0) - 1\right\}\Big\}$$

$$h(H - H_0) + \frac{|a|\sqrt{p}\,\sigma_0}{r r_0}\Big\{\cosh(H - H_0) - 1\Big\}$$

$$= \frac{|a|^2\sqrt{e^2-1}}{r r_0}\Big\{ \sinh H$$

$$= \frac{|a|^2\sqrt{e^2-1}}{r r_0}\Big\{ - \{ \sin$$

$$+$$

$$e \sinh H = e \sinh(H -$$

$$= \sinh(H -$$

$$\sin\theta = \frac{|a|^2\sqrt{e^2-1}}{r r_0}\Big\{$$

$$\sin\theta = \frac{|a|\sqrt{e^2-1}}{r} \sin$$

LAGRANGE COEFFICIENTS

$$1-\cos\theta = \frac{a^2}{r r_0}\left[e^2-1\right]\left\{\cosh(H-H_0)-1\right\}$$

$$\sin\theta = \frac{|a|\sqrt{e^2-1}}{r}\sinh(H-H_0) + \frac{|a|\sqrt{p}\,\sigma_0}{r r_0}\left\{\cosh(H-H_0)-1\right\}$$

$$F = 1-\frac{r}{p}(1-\cos\theta) = 1-\frac{|a|}{r_0}\left\{\cosh(H-H_0)-1\right\}$$

$$= 1-\frac{a}{r_0}\left\{1-\cosh(H-H_0)\right\}$$

$$G = \frac{r r_0}{\sqrt{\mu p}}\sin\theta = \frac{|a|\sigma_0}{\sqrt{\mu}}\left\{\cosh(H-H_0)-1\right\} + r_0\sqrt{\frac{|a|}{\mu}}\sinh(H-H_0)$$

$$= \frac{a\sigma_0}{\sqrt{\mu}}\left\{1-\cosh(H-H_0)\right\} + r_0\sqrt{\frac{(-a)}{\mu}}\sinh(H-H_0)$$

$$F_t = \frac{\sqrt{\mu}\,\sigma_0}{r_0 p}(1-\cos\theta) - \frac{\sqrt{\mu}}{r_0\sqrt{p}}\sin\theta = \left\{\frac{\sqrt{\mu}\,|a|\,\sigma_0}{r r_0^2} - \frac{\sqrt{\mu}\,a}{r r_0^2}\sigma_0\right\}\left\{\cosh(H-H_0)-1\right\}$$

$$- \frac{\sqrt{\mu}\sqrt{|a|}\sinh(H-H_0)}{r r_0}$$

$$F_t = -\frac{\sqrt{\mu(-a)}}{r r_0}\sinh(H-H_0)$$

$(H-H_0)-1\}$

$$G_t = 1-\frac{r_0}{p}(1-\cos\theta) = 1-\frac{|a|}{r}\{\cos$$

$H-H_0)\}$

$$= 1-\frac{a}{r}\{1-\cosh($$

PARABOLIC ORBIT

$$= \qquad \frac{p}{1+\cos f}$$

$$- \frac{\mu}{r} = 0 \qquad \frac{V^2}{2}$$

$$V = \sqrt{\frac{2\mu}{r}} = \sqrt{\frac{2\mu}{p}\left(1+\cos f\right)}$$

$$= 2\sqrt{\frac{\mu}{p}} \cos\frac{f}{2}$$

$f \rightarrow \pm \pi$ as $r \rightarrow \infty$

PARABOLIC ORBIT

$e = 1$

$$r = \frac{p}{1 + \cos f} = \frac{p}{2} \sec^2 \frac{f}{2}$$

$$r^2 \dot{f} = \frac{p^2}{4} \left(\sec^2 \frac{f}{2} \right)^2 \dot{f} = \sqrt{\mu p}$$

$$\frac{p^2}{2} \left(1 + \tan^2 \frac{f}{2} \right) \frac{d}{dt} \left(\tan \frac{f}{2} \right) = \sqrt{\mu p}$$

$$\frac{d}{dt} \left\{ \tan \frac{f}{2} + \frac{1}{3} \left(\tan \frac{f}{2} \right)^3 \right\} = 2 \sqrt{\frac{\mu}{p^3}}$$

$$\tan^3 \frac{f}{2} + 3 \tan \frac{f}{2} = 2B$$

$$B = 3 \sqrt{\frac{\mu}{p^3}} \left(t - t_p \right)$$

$$\boxed{\left(\tan \frac{f}{2} \right)^3 + 3 \left(\tan \frac{f}{2} \right) = 6 \sqrt{\frac{\mu}{p^3}} \left\{ t - t_p \right\}}$$

– BARKER'S EQUATION

PARABOLIC ORBIT

$r = \frac{1}{2} p \sec^2 \frac{f}{2}$

[redacted]

$\vec{r} = (r\cos f)\hat{\lambda}$ … $+ (r\sin f)\hat{\lambda}_p$

$= \frac{p}{2}\left\{1 - \tan^2\frac{f}{2}\right\}\lambda_e + p\tan\frac{f}{2}\lambda_p$

$\vec{v} = -\frac{\mu}{h}\sin f\,\hat{\lambda}_e + \frac{\mu}{h}(1+\cos f)\hat{\lambda}$

$= \frac{2\mu}{h}\cos\frac{f}{2}\left\{-\sin\frac{f}{2}\hat{\lambda}_e + \cos\frac{f}{2}\hat{\lambda}_p\right\}$ … $= -\frac{\sqrt{\mu p}}{r}\tan\frac{f}{2}\hat{\lambda}_p + \frac{\sqrt{\mu p}}{r}\hat{\lambda}_p$

$\sigma = \frac{r\sqrt{\mu}}{h}\sin f = 2\frac{r}{\sqrt{p}}\sin\frac{f}{2}\cos\frac{f}{2} =$ … $\sqrt{p}\tan\frac{f}{2}$

$p = 2r - \sigma^2 = 2r_0^2 - \sigma_0^2$

$F = 1 - \sec^2\frac{f}{2}\sin^2\frac{\theta}{2} = 1 - \sec^2\frac{f}{2}\left(\sin\frac{f}{2}\right.$ … $\left.\cos\frac{f_0}{2} - \cos\frac{f}{2}\sin\frac{f_0}{2}\right)^2$

$= 1 - \cancel{\sec^2\frac{f}{2}}\,\cancel{\cos^2}\frac{f_0}{2}\,\cancel{\cos^2\frac{f}{2}}\left\{\tan f_{/2} - \tan\frac{f_0}{2}\right\}^2$

$$\boxed{F = 1 - \frac{x^2}{2r_0}}$$

$$G = \frac{r r_0}{\sqrt{\mu p}} \sin\theta = \frac{p^2}{4\sqrt{\mu p}} \sec^2\frac{f}{2} \sec^2\frac{f_0}{2} \left\{ \sin f \cos f_0 - \cos f \sin f_0 \right\}$$

$$= \frac{p^{3/2}}{4\sqrt{\mu}} \left\{ 2\sin\frac{f}{2}\cos\frac{f}{2}\left(\cos^2\frac{f_0}{2} - \sin^2\frac{f_0}{2}\right) \right.$$

$$\left. - 2\sin\frac{f_0}{2}\cos\frac{f_0}{2}\left(\cos^2\frac{f}{2} - \sin^2\frac{f}{2}\right) \right\} \sec^2\frac{f_0}{2}\sec^2\frac{f}{2}$$

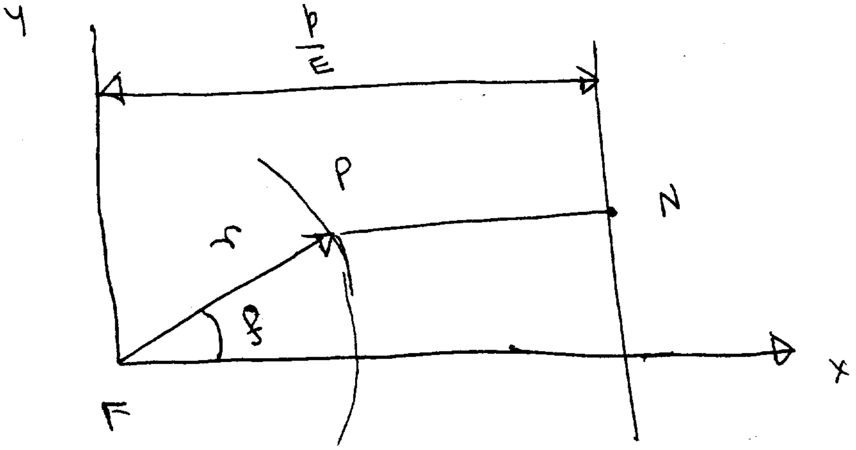
$$G = \frac{p^{3/2}}{2\sqrt{\mu}} \left\{ \tan\frac{f}{2}\left(1 - \tan^2\frac{f_0}{2}\right) - \tan\frac{f_0}{2}\left(1 - \tan^2\frac{f}{2}\right) \right\}$$

$$= \frac{p^{3/2}}{2\sqrt{\mu}} \left(\tan\frac{f}{2} - \tan\frac{f_0}{2}\right) \left\{ 1 + \tan\frac{f_0}{2}\tan\frac{f}{2} \right\}$$

$$= \frac{x}{2\sqrt{\mu}} \left\{ p\sec^2\frac{f_0}{2} + p\tan\frac{f_0}{2}\left(\tan\frac{f}{2} - \tan\frac{f_0}{2}\right) \right\}$$

$$\boxed{G = \frac{x}{2\sqrt{\mu}} \left\{ 2r_0 + \sigma_0 x \right\}}$$

GEOMETRICAL PROPERTIES



FOCUS-DIRECTRIX PROPERTY

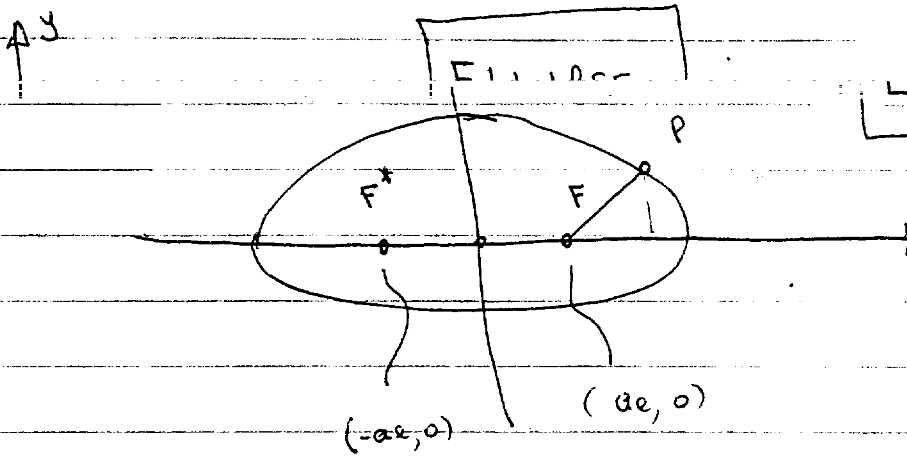
$$PN = \frac{p}{e} - r\cos f = \frac{p\{1+e\cos f - e\cos f\}}{e[1+\cos f]}$$

$$= \frac{r}{e}$$

$$PF = r$$

$$\boxed{\frac{PF}{PN} = e}$$

FOCAL RADII PROPERTY



$$(PF)^2 = (x-ae)^2 + y^2$$

$$(PF^*)^2 = (x+ae)^2 + y^2$$

$$(PF^*)^2 = (PF)^2 + 4aex$$

$$r = \frac{a(1-e^2)}{1+e\cos f} \qquad r - a = -a$$

$$x = ae + r\cos f$$

$$a - ex = a(1-e^2) -$$

$$= r$$

$$(PF^*)^2 = (a-ex)^2 + 4aex = (a$$

$$PF^* = a + ex \qquad \boxed{PF + PF}$$

LLIPSE

x

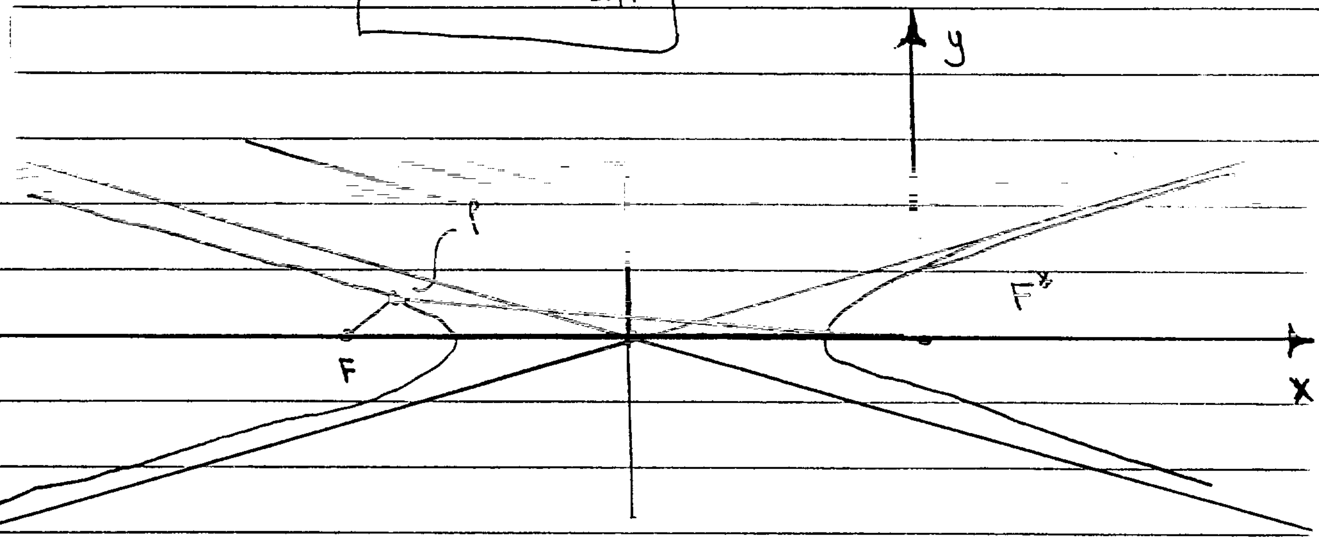
$$\frac{e^2 - a e\cos f}{1+e\cos f}$$

$$\frac{a(1-e^2)(e\cos f)}{(1+e\cos f)}$$

$$+ex)^2$$

$$\boxed{* = 2a}$$

HYPERBOLA



$PF = r = a - ex$

$(PF)^2 = (x - ae)^2 + y^2$

$(PF^*)^2 = (x + ae)^2 + y^2$

$$(PF^*)^2 = (a - ex)^2 + 4aex$$

$$= (a + ex)^2$$

$$PF^* = -(a + ex)$$

$$\boxed{PF^* - PF = -2a}$$

ELLIPSE

$PF + PQ = 2a$

$PF + PF^* = 2a$

$PF^* = PQ$

Δ F^*PQ IS ISOCELES

P LIES ON $\perp$ BISECTOR OF LINE QF^*.

SUPPOSE A SECOND POINT P' OF ELLIPSE LIES ON $\perp$ BISECTOR

$P'Q = P'F^*$ AND $P'Q' = P'F^* = 2a - P'F$

$P'QQ'$ IS AN ISOCELES Δ

BUT FQQ' IS ALSO AN ISOCELES Δ

$P'Q'F$ COLINEAR

2 DISTINCT ISOCELES Δ'S WITH SAME BASE & $\angle$.

CONTRADICTION

NO SECOND POINT P'

PERP. BISECTOR OF QF^* IS TANGENT TO ELLIPSE AT P.

HYPERBOLA

$PF^* - PF = -2a$

$PF^* - PQ = -2a$

$PF = PQ$

PERP. BISECTOR OF Q.F

PASSES THROUGH P.

SUPPOSE PERP BISECTOR ALSO PASSES

THROUGH ANOTHER POINT

$P'F = P'Q$ AND $P'F =$

$P'QQ'$ ISOCELES △ F^*QQ'

AND Q'

-2 DISTINC

WITH SAM

— P' DO

P' OF HYPERBOLA

P'Q'

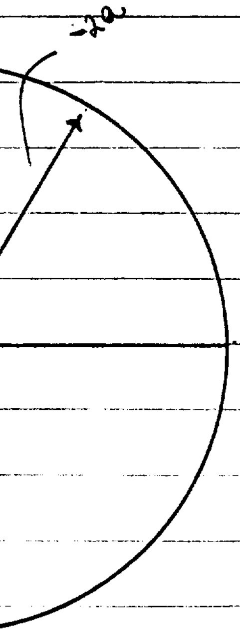
ISOCELES △

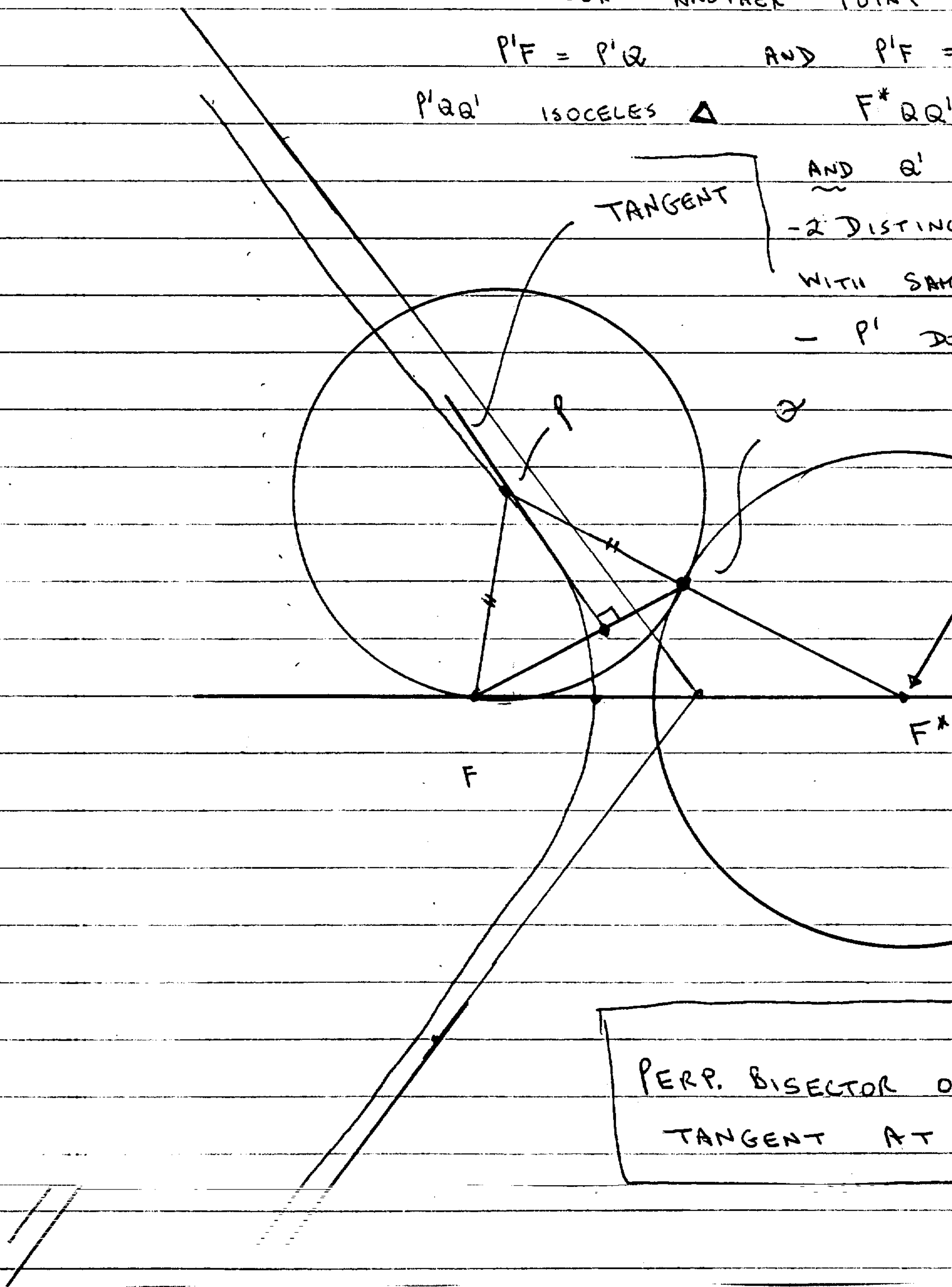
LIES ON $P'F^*$

T ISOCELES △'S

E BASE & ∠

OES NOT EXIST





F QF is
P.

PERP. BISECTOR O
TANGENT AT