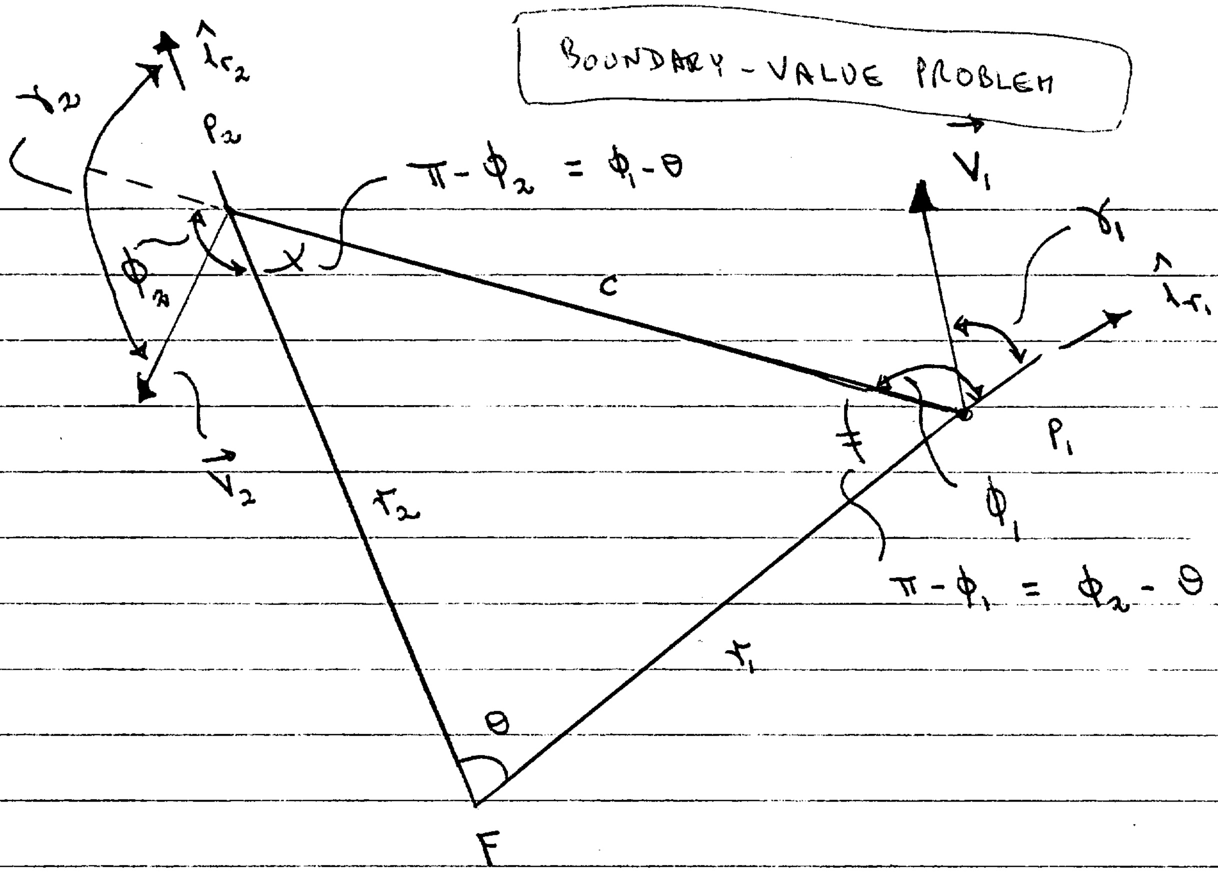
BOUNDARY-VALUE PROBLEM



$$V_r = \dot{r} = \frac{\sqrt{\mu}\,\sigma}{r} = \frac{h}{r}\cot\gamma = \frac{he}{p}\sin f$$

$$V_\theta = r\dot{f} = \frac{h}{r} = \frac{\sqrt{\mu p}}{r}$$

$$r_1 V_{\theta 1} = r_2 V_{\theta 2} = h = \sqrt{\mu p}$$

$$V_{r1} + V_{r2} = \frac{he}{p}\left\{\sin f_1 + \sin(f_1+\theta)\right\}$$

$$= \sqrt{\frac{\mu}{p}}\, e \left\{ \sin\left(\left[f+\tfrac{\theta}{2}\right]-\tfrac{\theta}{2}\right) + \sin\left(\left[f+\tfrac{\theta}{2}\right]+\tfrac{\theta}{2}\right)\right\}$$

$$= \sqrt{\frac{\mu}{p}}\, e \left\{ 2\sin\left(f+\tfrac{\theta}{2}\right)\cos\tfrac{\theta}{2}\right\} = \sqrt{\frac{\mu}{p}}\, e\left\{2\sin\left(f+\tfrac{\theta}{2}\right)\sin\tfrac{\theta}{2}\right\}\cot\tfrac{\theta}{2}$$

$$V_{r_1} + V_{r_2} = \sqrt{\frac{\mu}{p}} \left\{ e\cos f_1 - e\cos(f_1+\theta) \right\} \cot\frac{\theta}{2}$$

$$e\cos f = \frac{p}{r} - 1$$

$$V_{r_1} + V_{r_2} = \left\{ \frac{\sqrt{\mu p}}{r_1} - \frac{\sqrt{\mu p}}{r_2} \right\} \cot\frac{\theta}{2}$$

$$V_{r_1} + V_{r_2} = \left[V_{\theta_1} - V_{\theta_2} \right] \cot\frac{\theta}{2}$$

$$r_2 \sigma_1 + r_1 \sigma_2 = \sqrt{p}\,(r_2 - r_1) \cot\frac{\theta}{2}$$

$$\boxed{r_2 \cot\gamma_1 + r_1 \cot\gamma_2 = (r_2 - r_1)\cot\frac{\theta}{2}}$$

"PURELY GEOMETRIC RELATIONSHIP"

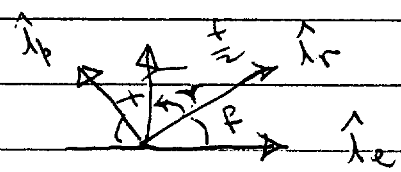
$$\boxed{r_2 \cot\gamma_1 + r_1 \cot\gamma_2 = (r_2 - r_1)\cot\frac{\theta}{2}}$$

PARABOLA

$$r = \frac{p}{1+\cos f} = \frac{p}{2}\sec^2\frac{f}{2}$$

$$\vec{v} = -\frac{2\mu}{h}\sin\frac{f}{2}\cos\frac{f}{2}\,\hat{\lambda}_e + \frac{2\mu}{h}\cos^2\frac{f}{2}\,\hat{\lambda}_p$$

$$= \frac{2\mu}{h}\cos\frac{f}{2}\left\{-\sin\frac{f}{2}\,\hat{\lambda}_e + \cos\frac{f}{2}\,\hat{\lambda}_p\right\}$$



$$\boxed{\gamma = \frac{\pi}{2} + \frac{f}{2} - f = \frac{\pi}{2} - \frac{f}{2}}$$

$$\gamma_1 - \gamma_2 = \frac{\theta}{2}$$

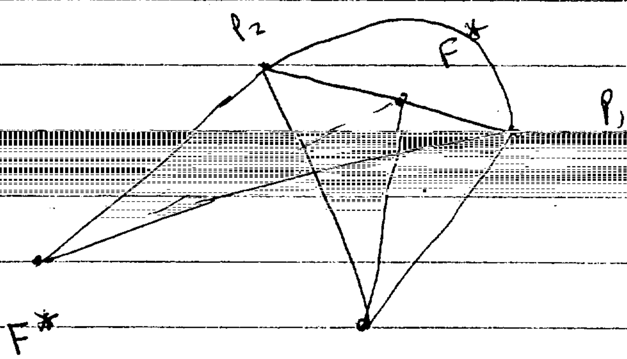
MINIMUM-ENERGY ORBIT

$$\mathcal{E} = \begin{cases} \dfrac{-\mu}{2a} < 0 & 0 < e < 1 \\ 0 & e = 1 \\ \dfrac{-\mu}{2a} > 0 & e > 1 \end{cases}$$

The min-energy orbit connecting the 2 points is the ellipse with the smallest value of a

$$P_1F + P_1F^* = 2a \qquad P_1F^* = 2a - r_1$$

$$P_2F + P_2F^* = 2a \qquad P_2F^* = 2a - r_2$$



$P_1F_* + P_2F_* \geq P_1P_2 = c$

$$4a - r_1 - r_2 = P_1F^* + P_2F^*$$

$$4a_m - r_1 - r_2 = c$$

$$2a_m = \tfrac{1}{2}(r_1 + r_2 + c)$$

$s = \frac{1}{2}(r_1 + r_2 + c)$ <u>semiperimeter</u>

$$P_1F^* = s - r_1 = \frac{1}{2}(r_2 + c - r_1)$$

$$P_2F^* = s - r_2 = \frac{1}{2}(r_1 + c - r_2)$$

$$\boxed{\theta = \pi}$$

$$p = r_1 (1 + e\cos f_1) \qquad p = r_2 (1 + e\cos(f_1 + \pi))$$

$$p = r_2 (1 - e\cos f_1) \qquad r_1 + r_2 = c$$

$$\frac{p}{r_1} - 1 = 1 - \frac{p}{r_2}$$

$$p \left\{ \frac{r_1 + r_2}{r_1 r_2} \right\} = 2$$

$$p = \frac{2 r_1 r_2}{r_1 + r_2} = \frac{2 r_1 r_2}{c}$$

all orbits: $p = \dfrac{2 r_1 r_2}{c}$ <u>SAME</u> PARAMETER

$$V_{\theta 1}^2 = r_1^2 \dot{f}^2 = \frac{\mu p}{r_1^2} = \frac{2\mu r_2}{c r_1} \qquad V_{\theta 2}^2 = \frac{\mu p}{r_2^2} = \frac{2\mu r_1}{c r_2}$$

$$V_{r1}^2 + \frac{2\mu r_2}{c r_1} = \frac{2\mu}{r_1} - \frac{\mu}{a}$$

$$V_{r1}^2 = \frac{2\mu \{ r_1 + r_2 - r_2 \}}{c r_1} - \frac{\mu}{a} = \mu \left\{ \frac{2}{c} - \frac{1}{a} \right\}$$

$$V_{r2}^2 = V_{r1}^2 = \mu \left\{ \frac{2}{c} - \frac{1}{a} \right\}$$

$$p = \frac{2r_1 r_2}{c}$$

$$V_{r1}^2 = V_{r2}^2 = \mu\left\{\frac{2}{c} - \frac{1}{a}\right\}$$

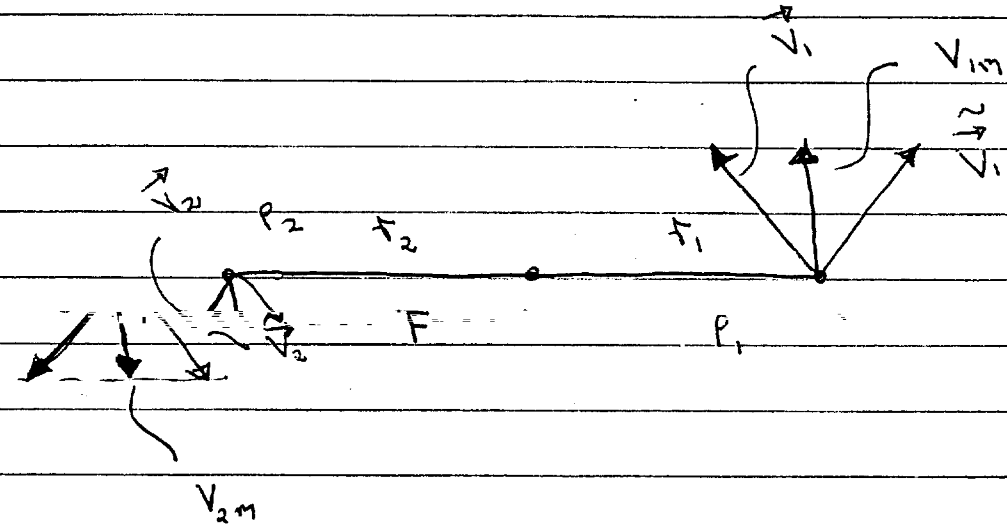
$$V_{\theta 1}^2 = \frac{2\mu r_2}{c r_1} \qquad V_{\theta 2}^2 = \frac{2\mu r_1}{c r_2}$$

MINIMUM ENERGY ORBIT $\theta = \pi$ P_1, P_2
APSIDES $V_{r1} = 0$ $V_{r2} = 0$ $(2a = c)$

$$V_{\theta 1}^2 = \mu\left\{\frac{2}{r_1} - \frac{1}{(\frac{1}{2}c)}\right\} = V_{m1}^2$$

$$V_{\theta 2}^2 = \mu\left\{\frac{2}{r_2} - \frac{1}{(\frac{1}{2}c)}\right\} = V_{m2}^2$$

$$\boxed{\theta = \pi}$$



$$V_{\theta 1}^2 = V_{m1}^2 = \frac{2\mu r_2}{r_1 c} \qquad V_{\theta 2}^2 = V_{m2}^2 = \frac{2\mu r_1}{r_2 c}$$

$$V_{r1}^2 = V_{r2}^2 = \mu\left\{\frac{2}{c} - \frac{1}{a}\right\}$$

$$V_{r1} + V_{r2} = 0 \qquad V_{r1}^2 = V_{r2}^2 = \mu\left\{\frac{1}{a_m} - \frac{1}{a}\right\}$$

FOR EACH VALUE OF $a > a_m$ (ellipses)
$a < 0$ (hyperbolae) [($a \to \pm\infty$) PARABOLAE]

2 POSSIBLE VALUES OF V_{r1}, V_{r2}

$\rightarrow \vec{V}_1, \tilde{\vec{V}}_1 ; \vec{V}_2, \tilde{\vec{V}}_2$

$$\boxed{\theta \neq \pi}$$

$$\vec{r}_2 = F\vec{r}_1 + G\vec{v}_1 \quad \ldots \quad (1)$$

$$\vec{v}_2 = F_t\vec{r}_1 + G_t\vec{v}_1$$

$$FG_t - GF_t = 1$$

$$\vec{r}_1 = G_t\vec{r}_2 - G\vec{v}_2 \quad \ldots \quad (2)$$

$$F = 1 - \frac{r_2}{p}(1-\cos\theta) \qquad G = \frac{r_1 r_2 \sin\theta}{\sqrt{\mu p}}$$

$$G_t = 1 - \frac{r_1}{p}(1-\cos\theta)$$

$$\vec{v}_1 = \frac{1}{G}\left\{\vec{r}_2 - F\vec{r}_1\right\} = \frac{\sqrt{\mu p}}{r_1 r_2 \sin\theta}\left\{\vec{r}_2 - \vec{r}_1 + \frac{r_2}{p}(1-\cos\theta)\vec{r}_1\right\}$$

$$\vec{v}_2 = \frac{1}{G}\left\{G_t\vec{r}_2 - \vec{r}_1\right\} = \frac{\sqrt{\mu p}}{r_1 r_2 \sin\theta}\left\{(\vec{r}_2 - \vec{r}_1) - \frac{r_1}{p}(1-\cos\theta)\vec{r}_2\right\}$$

$$\hat{\lambda}_{r_1} = \frac{1}{r_1}\vec{r}_1 \qquad \hat{\lambda}_{r_2} = \frac{1}{r_2}\vec{r}_2 \qquad \hat{\lambda}_c = \frac{\vec{r}_2 - \vec{r}_1}{c}$$

$$\vec{v}_1 = V_c\hat{\lambda}_c + V_\rho\hat{\lambda}_{r_1} \qquad \vec{v}_2 = V_c\hat{\lambda}_c - V_\rho\hat{\lambda}_{r_2}$$

$$V_c = \frac{c\sqrt{\mu p}}{(r_1 r_2 \sin\theta)} \qquad V_\rho = \sqrt{\frac{\mu}{p}}\,\frac{(1-\cos\theta)}{\sin\theta}$$

$$V_c V_g = \frac{\mu c}{r_1 r_2} \frac{[1 - \cos\theta]}{\sin^2\theta}$$

$$= \frac{\mu c}{r_1 r_2} \frac{2\sin^2\frac{\theta}{2}}{\left[4\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}\right]}$$

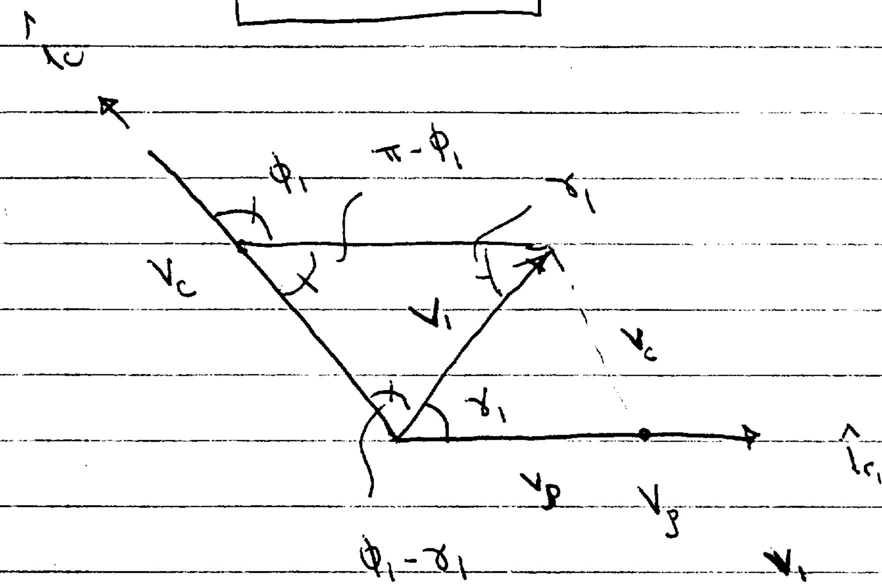
$$\boxed{V_c V_g = \frac{\mu c}{2 r_1 r_2} \sec^2(\theta/2)}$$

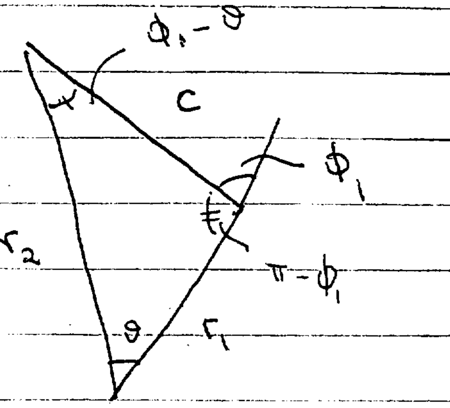
$$\vec{V}_1 = V_c \hat{\lambda}_c \ldots V_g \hat{\lambda}_{r_1} + V \hat{\lambda}$$

$$\vec{V}_2 = V_c \hat{\lambda}_c - V_g \hat{\lambda}_{r_2}$$

NOTICE THAT $V_g \neq V_r$!!

AT P1





$$\frac{V_1}{\sin\phi_1} = \frac{V_c}{\sin\gamma_1} = \frac{V_\rho}{\sin(\phi_1-\gamma_1)}$$

$$V_c^2 + V_\rho^2 - 2V_cV_\rho\left(\cos(\pi-\phi_1)\right) = V_1^2$$

$$\frac{c}{\sin\theta} = \frac{r_2}{\sin\phi_1} = \frac{r_1}{\sin(\phi_1-\theta)}$$

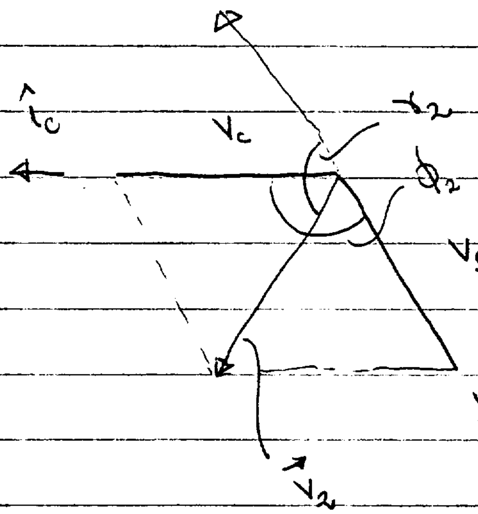
$$c^2 = r_1^2 + r_2^2 - 2r_1r_2\cos\theta$$

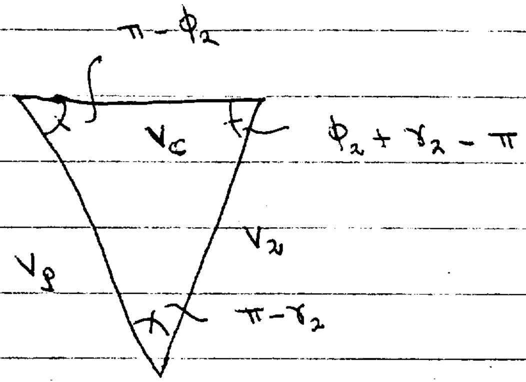
$$V_cV_\rho = \frac{\mu c}{2r_1r_2}\sec^2\frac{\theta}{2} = A$$

$$V_1^2 = V_\rho^2 + \frac{A^2}{V_\rho^2} + 2A\cos\phi_1 = 2\mathcal{E} + \frac{2\mu}{r_1}$$

$$V_2^2 = V_\rho^2 + \frac{A^2}{V_\rho^2} + 2A\cos\phi_2 = 2\mathcal{E} + \frac{2\mu}{r_2}$$

At ρ_2

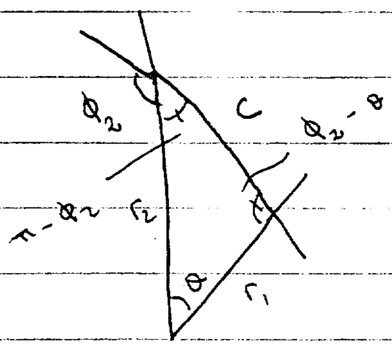




$$\frac{V_c}{\sin\gamma_2} = \frac{-V_\rho}{\sin(\phi_2+\gamma_2)} = \frac{V_2}{\sin\phi_2}$$

$$V_2^2 = V_c^2 + V_\rho^2 - 2V_cV_\rho\cos(\pi-\phi_2)$$

$$V_2^2 = V_c^2 + V_\rho^2 + 2V_cV_\rho\cos\phi_2$$



$$\frac{r_1}{\sin\phi_2} = \frac{r_2}{\sin(\phi_2-\theta)} = \frac{c}{\sin\theta}$$

PARAMETER p

$$\frac{V_c}{V_\rho} = \frac{cp}{r_1 r_2 (1-\cos\theta)}$$

$$p = \frac{V_c}{V_\rho}\left(\frac{r_1 r_2}{c}\right)(1-\cos\theta)$$

$$p_m = \frac{r_1 r_2}{c}(1-\cos\theta)$$

$$\frac{p}{p_m} = \frac{V_c}{V_\rho}$$

Orbit: $\vec{V}_1 = k\hat{\lambda}_{r_1} + l\hat{\lambda}_c$

conjugate orbit $\tilde{\vec{V}}_1 = l\hat{\lambda}_{r_1} + k\hat{\lambda}_c$

$$p_m = \sqrt{p\tilde{p}}$$

$$\frac{\sin\gamma_1}{\sin(\phi_1-\gamma_1)} = \frac{V_c}{V_\rho} = \frac{\sin(\phi_1-\tilde{\gamma}_1)}{\sin\tilde{\gamma}_1}$$

$$\sin\gamma_1 \sin\tilde{\gamma}_1 = \sin(\phi_1-\tilde{\gamma}_1)\sin(\phi_1-\gamma_1)$$

$$\cos(\gamma_1-\tilde{\gamma}_1) - \cos(\gamma_1+\tilde{\gamma}_1) = \cos(\gamma_1-\tilde{\gamma}_1) - \cos(2\phi_1-\gamma_1-\tilde{\gamma}_1)$$

LAW OF COSINES

$$c^2 = r_1^2 + r_2^2 - 2r_1r_2\cos\theta$$

$$= (r_1^2 + r_2^2 - 2r_1r_2) + 2r_1r_2(1-\cos\theta)$$

$$\sin^2\frac{\theta}{2} = \frac{c^2 - (r_1-r_2)^2}{4r_1r_2} = \frac{(s-r_1)(s-r_2)}{r_1r_2}$$

$$\boxed{p_m = \frac{2r_1r_2}{c}\sin^2\frac{\theta}{2} = \frac{2}{c}(s-r_1)(s-r_2)}$$

d perp. distance from F to chord

$$d = r_2\sin(\phi_1-\theta) = \frac{r_1r_2}{c}\sin\theta$$

$$\boxed{p_m = \frac{d}{\sin\theta}(1-\cos\theta) = d\tan\frac{\theta}{2}}$$

$$P_1F_m^* = s - r_1 \qquad P_2F_n^* = s - r_2$$

$$\frac{1}{P_1F_m^*} + \frac{1}{P_2F_n^*} = \frac{2s - r_1 - r_2}{(s-r_1)(s-r_2)} = \frac{c}{(s-r_1)(s-r_2)}$$

$$\frac{1}{p_m} = \frac{1}{2}\left\{\frac{1}{P_1F_m^*} + \frac{1}{P_2F_n^*}\right\}$$

$$\cos(\gamma_1 + \tilde{\gamma}_1) = \cos(2\phi_1 - \gamma_1 - \tilde{\gamma}_1)$$

$$\boxed{\gamma_1 + \tilde{\gamma}_1 = \phi_1}$$

SIMILARLY

$$\frac{\sin(\phi_2 + \gamma_2)}{\sin\gamma_2} = \frac{\sin\tilde{\gamma}_2}{\sin(\phi_2 + \tilde{\gamma}_2)}$$

$$\cos(\gamma_2 + \tilde{\gamma}_2) = \cos(2\phi_2 + \gamma_2 + \tilde{\gamma}_2)$$

$\pi < \gamma_2 + \tilde{\gamma}_2 < 2\pi$ $\qquad$ $2\pi < 2\phi_2 + \gamma_2 + \tilde{\gamma}_2 < 4\pi$

$$\gamma_2 + \tilde{\gamma}_2 = 4\pi - (2\phi_2 + \gamma_2 + \tilde{\gamma}_2)$$

$$\boxed{\gamma_2 + \tilde{\gamma}_2 = 2\pi - \phi_2}$$

$$\frac{d\mathcal{E}}{dV_\rho} = 0$$

$$2V_\rho - \frac{2A^2}{V_\rho^3} = 0$$

$$V_\rho^2 = A = V_c V_\rho$$

@ $\mathcal{E}_{min}$ $\quad V_c = V_\rho = \sqrt{A}$

$$\frac{\sqrt{A}}{\sin(\varphi_1 - \gamma_1)} = \frac{\sqrt{A}}{\sin\gamma_1}$$

MIN. ENERGY ELLIPSE P_1

$$\boxed{V_c = V_\rho = \sqrt{A} \qquad \gamma_1 = \Phi_1/2}$$

$$\frac{\sqrt{A}}{\sin\gamma_2} = \frac{-\sqrt{A}}{\sin(\Phi_2+\gamma_2)}$$

MIN ENERGY ELLIPSE P_2

$$\boxed{V_c = V_\rho = \sqrt{A} \qquad \gamma_2 = \pi - \Phi_2/2}$$

$$V_{m1}^2 = A\left\{2 + 2\cos\phi_1\right\} = 4A\cos^2 \phi_1/_2$$

$$V_{m2}^2 = A\left\{2 + 2\cos\phi_2\right\} = 4A\cos^2 \phi_2/_2$$

$$V_c V_g = A = \frac{1}{4} V_{m1}^2 \sec^2 \frac{\phi_1}{2} = \frac{1}{4} V_{m2}^2 \sec^2 \frac{\phi_2}{2}$$

$$\frac{c}{\sin\theta} = \frac{r_1}{\sin\phi_2} = \frac{r_2}{\sin(\phi_2-\theta)}$$

$$\sin\phi_2 = \frac{r_1}{c}\sin\theta$$

$$\sin\phi_2\cos\theta - \cos\phi_2\sin\theta = \frac{r_2}{c}\sin\theta$$

$$\cos\phi_2 = \frac{r_1}{c}\cos\theta - \frac{r_2}{c}$$

$$\sin(\phi_2+\gamma_2) = \frac{r_1}{c}\sin\theta\cos\gamma_2 + \left\{\frac{r_1}{c}\cos\theta - \frac{r_2}{c}\right\}\sin\gamma_2$$

$$= \frac{1}{c}\left\{r_1\sin(\theta+\gamma_2) - r_2\sin\gamma_2\right\}$$

$$\frac{p}{p_m} = \frac{-(\sin\gamma_2)}{\sin(\phi_2+\gamma_2)} = \frac{c\sin\gamma_2}{\left\{r_2\sin\gamma_2 - r_1\sin(\theta+\gamma_2)\right\}}$$

$$\boxed{\frac{p}{p_m} = \frac{c\sin\gamma_2}{\left\{r_2\sin\gamma_2 - r_1\sin(\theta+\gamma_2)\right\}}}$$

$$\frac{c}{\sin\theta} = \frac{r_2}{\sin\phi_1} = \frac{r_1}{\sin(\phi_1-\theta)}$$

$$\sin\phi_1 = \frac{r_2}{c}\sin\theta$$

$$\sin(\phi_1-\theta) = \frac{r_1}{c}\sin\theta$$

$$\sin\phi_1\cos\theta - \cos\phi_1\sin\theta = -\cos\phi_1\sin\theta + \frac{r_2}{c}\sin\theta\cos\theta = \frac{r_1}{c}\sin\theta$$

$$\cos\phi_1 = \frac{r_2}{c}\cos\theta - \frac{r_1}{c}$$

$$\sin(\phi_1-\gamma_1) = \sin\phi_1\cos\gamma_1 - \cos\phi_1\sin\gamma_1$$

$$= \frac{r_2}{c}\sin\theta\cos\gamma_1 - \left\{\frac{r_2}{c}\cos\theta - \frac{r_1}{c}\right\}\sin\gamma_1 =$$

$$\frac{1}{c}\left\{r_1\sin\gamma_1 + r_2\sin(\theta-\gamma_1)\right\}$$

$$\frac{\sin\gamma_1}{\sin(\phi_1-\gamma_1)} = \frac{c\sin\gamma_1}{\left[r_1\sin\gamma_1 + r_2\sin(\theta-\gamma_1)\right]}$$

$$\frac{p}{p_m} = \frac{V_c}{V_p} =$$

Lambert's Theorem

Parabola

$$r = \frac{p}{2}\sec^2\frac{f}{2} = \frac{p}{2}\left(1 + \tan^2\frac{f}{2}\right)$$

$$v = \sqrt{\frac{2\mu}{r}} \qquad \gamma = \frac{\pi}{2} - \frac{f}{2}$$

$$\sigma = \frac{v r \cos\gamma}{\sqrt{\mu}} = \sqrt{2r}\,\sin\frac{f}{2} = \sqrt{p}\tan\frac{f}{2}$$

$$r = \frac{1}{2}\left(p + \sigma^2\right)$$

Barker's Equation

$$\tan^3\frac{f}{2} + 3\tan\frac{f}{2} = 6\sqrt{\frac{\mu}{p^3}}\,(t - t_p)$$

$$6\sqrt{\mu}\,(t - t_p) = 3p\sigma + \sigma^3$$

$$6\sqrt{\mu}\,(t_2 - t_1) = (\sigma_2 - \sigma_1)\left\{3p + \sigma_2^2 + \sigma_1^2 + \sigma_1\sigma_2\right\}$$

$$= (\sigma_2 - \sigma_1)\left\{3(p + \sigma_1\sigma_2) + (\sigma_2 - \sigma_1)^2\right\}$$

$$p + \sigma_1\sigma_2 = p\left\{1 + \tan\frac{f_1}{2}\tan\frac{f_2}{2}\right\}$$

$$= p\sec\frac{f_1}{2}\sec\frac{f_2}{2}\cos\frac{\theta}{2} = 2\sqrt{r_1 r_2}\,\cos\frac{\theta}{2}$$

$$\sqrt{r_1 r_2}\cos\theta/2 = \quad = \sqrt{r_1 r_2}\left\{\cos\frac{f_2}{2}\cos\frac{f_1}{2} + \sin\frac{f_2}{2}\sin\frac{f_1}{2}\right\}$$

$$= a(1-e)\cos\frac{E_2}{2}\cos\frac{E_1}{2} + a(1+e)\sin\frac{E_2}{2}\sin\frac{E_1}{2}$$

$$= a\cos\left(\frac{E_2-E_1}{2}\right) - ae\cos\left(\frac{E_2+E_1}{2}\right)$$

$$\sqrt{r_1 r_2}\cos\frac{\theta}{2} = a(\cos\psi - \cos\phi)$$

$$c^2 = (r_1+r_2)^2 - 4r_1r_2\cos^2\frac{\theta}{2} = r_1^2 + r_2^2 - 2r_1r_2\cos\theta$$

$$= 4a^2\left\{(1-\cos\psi\cos\phi)^2 - (\cos\psi - \cos\phi)^2\right\}$$

$$= 4a^2\left\{1 + \cos^2\psi\cos^2\phi - \cos^2\phi - \cos^2\psi\right\}$$

$$= 4a^2\left\{\sin^2\phi\left\{1 - \cos^2\psi\right\}\right\}$$

$$= 4a^2\sin^2\phi\sin^2\psi$$

$$c = 2a\sin\phi\sin\psi$$

$$2s = r_1 + r_2 + c = 2a\left\{1 - (\cos\phi\cos\psi - \sin\phi\sin\psi)\right\}$$

$$= 2a\left(1 - \cos(\phi+\psi)\right)$$

$$2(s-c) = r_1 + r_2 - c = 2a\left\{1 - (\cos\phi\cos\psi + \sin\phi\sin\psi)\right\}$$

$$= 2a\left\{1 - \cos(\phi-\psi)\right\}$$

$$6\sqrt{\mu}(t_2 - t_1) = \sqrt{2}\left(\sqrt{s} \mp \sqrt{s-c}\right)\left\{\pm 6\sqrt{s(s-c)} + 2\left\{\sqrt{s} \mp \sqrt{s-c}\right\}^2\right\}$$

$$= \sqrt{2}\left\{s \mp \sqrt{s-c}\right\}\left\{2(\sqrt{s})^2 \pm 2\sqrt{s(s-c)} + 2(\sqrt{s-c})\right.$$

$$= 2\sqrt{2}\left\{(\sqrt{s})^3 \mp \left(\sqrt{s-c}\right)^3\right\}$$

$$= \left(\sqrt{2s}\right)^3 \mp \left(\sqrt{2(s-c)}\right)^3$$

$$6\sqrt{\mu}(t_2 - t_1) = (r_1 + r_2 + c)^{3/2} \mp (r_1 + r_2 - c)^{3/2}$$

SE

$$= a(1 - e\cos E)$$

$$\sin\frac{E}{2}$$

$$\cos\frac{E}{2}$$

$$\sqrt{\frac{\mu}{a^3}}\,(t - t_p)$$

$$- E_1 - e\{\sin E_2 - \sin E_1\}$$

$$E_2 - E_1 - 2e\cos\left(\frac{E_1+E_2}{2}\right)\sin\left(\frac{E_2-E_1}{2}\right)$$

$$\psi = \frac{1}{2}(E_2 - E_1)$$

$$\psi - \sin\psi\cos\phi\}$$

$$E_1) + a(1 - e\cos E_2)$$

$$- e\cos\left(\frac{E_1+E_2}{2}\right)\cos\left(\frac{E_2-E_1}{2}\right)$$

$$2a(1 - \cos\phi\cos\psi)$$

ELLIP

$$r = \frac{a(1-e^2)}{1+e\cos f}$$

$$\sqrt{r}\sin\frac{f}{2} = \sqrt{a(1+e}$$

$$\sqrt{r}\cos\frac{f}{2} = \sqrt{a(1-e}$$

$$E - e\sin E =$$

$$\sqrt{\frac{\mu}{a^3}}\,(t_2 - t_1) = E_2$$

$$= E$$

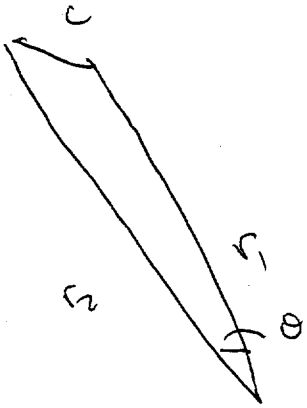
$$\cos\phi = e\cos\left(\frac{E_1+E_2}{2}\right)$$

$$\sqrt{\mu}\,(t_2 - t_1) = 2a^{3/2}\{$$

$$(r_1 + r_2) = a(1 - e\cos$$

$$= 2a(1 -$$

$$=$$



$$S = \tfrac{1}{2}(r_1 + r_2 + c)$$

$$r_1 + r_2 = 2S - c$$
$$= (s + [s-c])$$

$$c^2 = r_1^2 + r_2^2 - 2r_1r_2\cos\theta$$

$$= (r_1 + r_2)^2 - 4r_1r_2\cos^2\tfrac{\theta}{2}$$

$$2r_1r_2\cos\theta/2 = \pm\sqrt{(r_1+r_2-c)(r_1+r_2+c)}$$

$$= \pm\, 2\sqrt{s(s-c)}$$

$$p + \sigma_1\sigma_2 = \pm\, 2\sqrt{s(s-c)}$$

$$2(r_1 + r_2) = 2p + \sigma_1^2 + \sigma_2^2$$

$$= (\sigma_2 - \sigma_1)^2 + 2(p + \sigma_1\sigma_2)$$

$$(\sigma_2 - \sigma_1)^2 = 2\left\{r_1 + r_2 - (p + \sigma_1\sigma_2)\right\}$$

$$= 2\left\{s + (s-c) \mp 2\sqrt{s(s-c)}\right\}$$

$$2\alpha = \phi + \psi \qquad\qquad 2\beta = \phi - \psi$$

$$\sin^2 \frac{\alpha}{2} = \frac{s}{2a} \qquad\qquad \sin^2 \frac{\beta}{2} = \frac{s-c}{2a}$$

$$2\psi = \alpha - \beta$$

$$2 \sin\psi \cos\phi = + \left[\sin(\psi + \phi) - \sin(\phi - \psi) \right]$$

$$= \sin\alpha - \sin\beta$$

$$\sqrt{\mu}\,(t_2 - t_1) = a^{3/2} \left\{ (\alpha - \sin\alpha) - (\beta - \sin\beta) \right\}$$

$$\sin^2 \tfrac{1}{2}\alpha = \frac{s}{2a} \qquad\qquad \sin^2 \tfrac{1}{2}\beta = \frac{s-c}{2a}$$

α, β depend only on a, $r_1 + r_2$, c.

HYPERBOLIC ORBIT

$$\sqrt{\frac{\mu}{(-a)^3}}\,(t - t_p) = e \sinh H - H$$

$$\sqrt{\frac{\mu}{(-a)^3}}\,(t_2 - t_1) = \left\{ e \left\{ \sinh H_2 - \sinh H_1 \right\} - (H_2 - H_1) \right\}$$

"Trig identities"

$$\cosh^2 A - \sinh^2 A = 1 \qquad \cosh(A+B) = \cosh A \cosh B + \sinh A \sinh B$$

$$\cosh 2A = 2\cosh^2 A - 1 \qquad \sinh(A+B) = \sinh A \cosh B + \cosh A \sinh B$$

$$= 2\sinh^2 A + 1 \qquad \sinh(A-B) = \sinh A \cosh B - \cosh A \sinh B$$

$$\sqrt{\frac{\mu}{(-a)^3}}\,(t_2 - t_1) = 2e \cosh\left(\frac{H_1 + H_2}{2}\right) \sinh\left(\frac{H_2 - H_1}{2}\right) - (H_2 - H_1)$$

$$\psi = \frac{1}{2}\left\{ H_2 - H_1 \right\} \qquad \cosh\phi = e \cosh\left(\frac{H_1 + H_2}{2}\right)$$

$$\sqrt{\mu}\,(t_2 - t_1) = 2(-a)^{3/2} \left\{ \cosh\phi \sinh\psi - \psi \right\}$$

$$r = (-a)\left\{ e \cosh H - 1 \right\}$$

$$r_1+r_2 = (-a)\left\{e\cosh H_2 + e\cosh H_1 - 2\right\}$$

$$= (-a)\left\{2e\cosh\frac{H_2+H_1}{2}\cosh\frac{H_2-H_1}{2} - 2\right\}$$

$$= 2(-a)\left\{\cosh\phi\cos\psi - 1\right\}$$

$$\boxed{(r_1+r_2) = 2(-a)\left[\cosh\phi\cosh\psi - 1\right]}$$

$$r_1 r_2 = (-a)^2\left\{e^2(\cosh H_2\cosh H_1) - e(\cosh H_2 + \cosh H_1) + 1\right\}$$

$$= (-a)^2\left\{\frac{1}{2}e^2\left[2\cosh^2\frac{H_2+H_1}{2} + 2\cosh^2\frac{H_2-H_1}{2} - 2\right] - 2e\cosh\frac{H_1+H_2}{2}\cosh\frac{H_2-H_1}{2} + 1\right\}$$

$$\boxed{r_1 r_2 = (-a)^2\left\{\cosh^2\phi - 2\cosh\phi\cosh\psi + e^2\sinh^2\psi + 1\right\}}$$

$$(1+\cos\theta) = r_1 r_2 + (-a)^2(e^2-1)(-\sinh^2\psi)$$

$$= (-a)^2\left\{\cosh^2\phi - 2\cosh\phi\cosh\psi + 1 + \sinh^2\psi\right\}$$

$$\boxed{r_2\cos\frac{\theta}{2} = (-a)\left\{\cosh\phi - \cosh\psi\right\}}$$

$$\frac{r_1 r_2}{2}($$

$$\boxed{\sqrt{r_1 r}}$$

$$c^2 = r_1^2 + r_2^2 - 2r_1r_2\cos\theta$$

$$= (r_1+r_2)^2 - 4r_1r_2\cos^2\frac{\theta}{2}$$

$$= 4(-a)^2\left\{(\cosh\phi\cosh\psi - 1)^2 - (\cosh\phi - \cosh\psi)^2\right\}$$

$$= 4(-a)^2\left\{\cosh^2\phi\cosh^2\psi - \cosh^2\phi\right.$$

$$\left.- \cosh^2\psi + 1\right\}$$

$$= 4(-a)^2\left\{(\cosh^2\phi - 1)(\cosh^2\psi - 1)\right\}$$

$$\boxed{c = 2(-a)\sinh\psi\sinh\phi}$$

$$\alpha = \phi + \psi \qquad \beta = \phi - \psi$$

$$2s = r_1 + r_2 + c = 2(-a)\left\{\cosh(\alpha) - 1\right\}$$

$$2(s-c) = r_1 + r_2 - c = 2(-a)\left\{\cosh(\beta) - 1\right\}$$

$$2\cosh\phi\sinh\psi = \sinh\alpha - \sinh\beta$$

$$\sqrt{\mu}\,(t_2 - t_1) = (-a)^{3/2}\left\{(\sinh\alpha - \alpha) - (\sinh\beta - \beta)\right\}$$

$$\sinh^2\alpha = \frac{s}{2(-a)} \qquad \sinh^2\beta = \frac{(s-c)}{2(-a)}$$